

Mathematical reminders

Part 1

Dimensional equations

1.1. Generalities about Physical Quantities عموميات عن المقادير الفيزيائية

A Physical Quantity is a property that can be measured or calculated, and usually equals a value followed by a unit.

There are two types of physical quantities: fundamental quantities and derived quantities.

1.1.1. Fundamental (Basic) quantities المقادير الأساسية

The seven basic quantities are given in Table 1:

Table1: the seven fundamental quantities

Physical quantity	Symbol
Length	l (x, d)
Mass	m
Time	t
Electric current	i
Temperature	T
Luminous intensity	j (I _v)
Amount of substance	n

1.1.2. Derived Quantities المقادير المشتقة

These quantities are expressed as a combination of the seven fundamental quantities.

Examples:

Velocity v: $v = l/t$.

1.2. System of units نظام الوحدات

The four fundamental units thus chosen define the MKSA system whose initials mean meter, kilogram, second and ampere respectively. The international system of units SI comprises seven basic units: meter, kilogram, second, ampere, kelvin, mole and candela.

All other units called derived units are obtained by combining these basic units of the international system.

1.3. Dimensional equations معادلات الأبعاد

1.3.1. Dimension البعد

The dimension of a physical quantity G is noted by the expression **[G]**. For example, if G has the dimension of a length, it is said to be homogeneous to a length, so the relation $[G] = L$

The dimension and unity must therefore be coherent with each other. A quantity has a single dimension but can be expressed in several units.

Table 2: dimension of fundamental quantities and their units in the SI system

Basic size	Dimension	Unit Name (SI)	unit symbol (SI)
Length	L	meter	m
Mass	M	kilogram	kg
Time	T	second	s
Electric current	I	ampere	A
Temperature	θ	kelvin	K
Amount of substance	N	mole	mol
Luminous intensity	J	candela	cd

1.3.2. Dimensional equations معادلات الأبعاد

The dimension of a derived quantity G is expressed by the product of powers of the fundamental dimensions. It is written:

$$[G] = M^{\alpha} L^{\beta} T^{\gamma} I^{\delta}$$

The dimensional equations allows to:

- Determine the unit composed of a quantity according to the fundamental quantities.
- Check if a formula is homogeneous and detect errors in calculations.
- Perform unit conversions.

Example: velocity: $v = \frac{x}{t}$

The velocity dimension: $[v] = \left[\frac{x}{t} \right] = \frac{[x]}{[t]} = \frac{L}{T} = L T^{-1}$

Table 3: Dimensional equations of derived quantities and their units in SI

Derived quantity	The expression	dimensional equation	IS Unit	Commonly used unit
Acceleration	$a=l/t^2$	LT^{-2}	$m.s^{-2}$	
Force	$F=ma$	MLT^{-2}	$Kg.m.s^{-2}$	newton (N)
Pressure	$p=F/S$	$ML^{-1}T^{-2}$	$kg.m^{-1}.s^{-2}$	pascal (Pa)
Energy, Work	$W=Fl$	ML^2T^{-2}	$kg.m^2.s^{-2}$	joule (J)
Power	$P=W/t$	ML^2T^{-3}	$kg.m^2.s^{-3}$	watt (W)
Electric charge	$Q=it$	IT	A.S	coulomb (C)
Electric field	$E=F/q$	$MLT^{-3}I^{-1}$	$kg.m.s^{-3}.A^{-1}$	Volt/meter (V/m)
Potential (voltage)	$U=El$	$ML^2T^{-3}I^{-1}$	$kg.m^2.s^{-3}.A^{-1}$	volt (V)
Electrical capacity	$C=q/U$	$M^{-1}L^{-2}T^4I^2$	$Kg^{-1}.m^{-2}.s^4.A^2$	farad (F)
Resistance	$R=U/i$	$ML^{2T^{-3}I^{-2}}$	$kg.m^2. s^{-3}.A^{-2}$	ohm (Ω)

Note: The functions: $\sin(x)$, $\cos(x)$, $\tan(x)$, $\ln(x)$, $\log(x)$ and e^x are dimensionless .

1.3.3. Homogeneity of dimensions تجانس الأبعاد

Dimensional equations are used to verify the homogeneity of formulas, that is, both its members have the same dimension.