University of Badji-Mokhtar Annaba
Faculty of Technologie
Computer Science Department

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Mathematical Analysis 1

Series of exercies 2: the Complex numbers

Exercise 1:
Write the following complex numbers in algebraic form (cartesian form) :
$z_1 = \dfrac{2}{1-i\sqrt{3}}, \quad z_2 = \dfrac{i-4}{-3+2i}, \quad z_3 = (1-i)^3, \; z_4 = \dfrac{2\exp\left(i\frac{\pi}{6}\right)}{\exp\left(i\frac{\pi}{3}\right)}$

z_5, is the number of module 2 and argument $\frac{\pi}{3}$.

Exercise 2:
Compute the module and argument of : $v = \dfrac{\sqrt{6}-i\sqrt{2}}{2}$ and $w = 1 - i$.
Deduce the module and argument of $\dfrac{v}{w}$.

Exercise3:
1• Solve the equation in different ways: $z^2 = \dfrac{\sqrt{3}+i}{2}$.

Deduce the values of $\cos\left(\dfrac{\pi}{12}\right), \sin\left(\dfrac{\pi}{12}\right)$.

2• Compute the cube roots of $-8i$.

Exercise 4:
Solve in $\mathbb{C}$ the following equations:
1) $Z^2 + Z + 1 = 0$,
2) $Z^2 + Z + \dfrac{1}{4} - \dfrac{3}{4}i = 0$
3) $(1+2i)\,Z^2 - (9+3i)\,Z + 10 - 5i = 0$
4) $Z^4 - 30Z^2 + 289 = 0$

Exercise 5:
Let the equation $(E) : Z^3 - iZ + 1 - i = 0$
1• Show that (E) admits a real root.
2• Determine the solution of E.

Exercise 6:
Using the Moivre 's formula, determine the trigonometric form of $(1+i)^n$ for $\forall n \in \mathbb{N}$.
Deduce a simple expression of $(1+i)^n + (1-i)^n$.

Exercise 7:
Expand $\cos(4x)$ and $\sin(4x)$ in a polynomial in terms of $\cos x$ and (or $\sin x$).

Supplementary Exercises

Exercise 8:

We give θ_0 a real such that : $\cos(\theta_0) = \dfrac{2}{\sqrt{5}}$ and $\sin(\theta_0) = \dfrac{1}{\sqrt{5}}$.

Calculate the modul and the argument of the following complexs numbers (in terms of θ_0)

$$a = 3i(2+i)(4+2i)(1+i) \quad \text{and} \quad b = \frac{(4+2i)(-1+i)}{(2-i)\,3i}$$

Exercise 9:

Calculate the square roots of : $\dfrac{1+i}{\sqrt{2}}$.

Deduce the values of $\cos\left(\dfrac{\pi}{8}\right), \sin\left(\dfrac{\pi}{8}\right)$.

Exercise 10:

Show that : $|Z| = 1$ and $Z \neq 1 \Rightarrow i\left(\dfrac{Z+1}{Z-1}\right) \in \mathbb{R}$.

Exercise 11:

Let $x \in \mathbb{R}$

1) Calculate $\cos^2(x)\sin^3(x)$ in terms of $\sin(x)$.

2) Linearize $\cos^4(x)$.

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Exercice 1:
Mettre sous forme algébrique (cartésienne) les nombres complexes suivants :
$z_1 = \dfrac{2}{1-i\sqrt{3}}$, $z_2 = \dfrac{3+6i}{3-4i}$, $z_3 = (1-i)^3$, $z_4 = \dfrac{2\exp\left(i\frac{\pi}{6}\right)}{\exp\left(i\frac{\pi}{3}\right)}$

z_5, le nombre de module 2 et d'argument $\frac{\pi}{3}$.

Exercice 2:
Calculer le module et l'argument de : $v = \dfrac{\sqrt{6}-i\sqrt{2}}{2}$ et $w = 1-i$.
En déduire le module et l'argument de $\dfrac{v}{w}$.

Exercice3:
1• Résoudre de façons différentes l'équation : $z^2 = \dfrac{\sqrt{3}+i}{2}$.

En déduire les valeurs de $\cos\left(\dfrac{\pi}{12}\right), \sin\left(\dfrac{\pi}{12}\right)$.

2• Calculer les racines cubiques de $-8i$.

Exercice 4:
Résoudre dans $\mathbb{C}$ les équations suivantes:
1) $Z^2 + Z + 1 = 0$,
2) $Z^2 + Z + \dfrac{1}{4} - \dfrac{3}{4}i = 0$
3) $(1+2i)\,Z^2 - (9+3i)\,Z + 10 - 5i = 0$
4) $Z^4 - 30Z^2 + 289 = 0$

Exercice 5:
Soit l'équation $Z^3 - iZ + 1 - i = 0$
1• Montrer que (E) admet une racine réelle.
2• Déterminer les solutions de E.

Exercice 6:
En utilisons la formule de Moivre, déterminer la forme trigonométrique de $(1+i)^n$ pour $\forall n \in \mathbb{N}$.
En déduire une expression simple de $(1+i)^n + (1-i)^n$.

Exercice 7:
Exprimer $\cos(4x)$ et $\sin(4x)$ en fonction de $\cos x$ et $\sin x$.