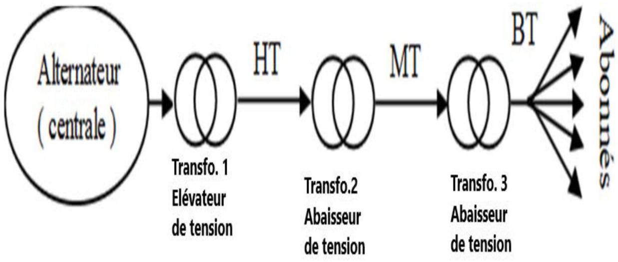
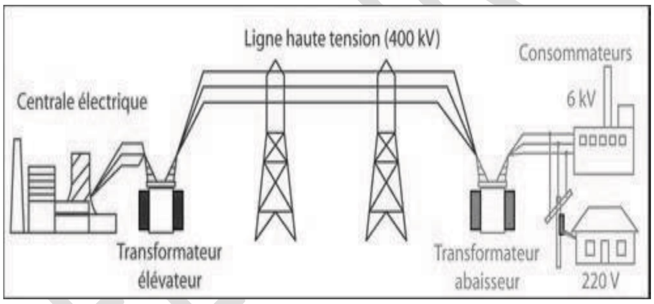
Chapter III:

Circuits and Electrical Power

III.1 Introduction:

In 1884, engineers Lucien Gaulard and John Gibbs developed a high-power transformer using three-phase current and electricity has been produced since the end of the 19th century from different primary energy sources. The first power plants ran on wood. Today, production can be done from fossil energy (coal, natural gas or oil), nuclear energy, hydroelectric energy, solar energy, wind energy and biomass.





The most convenient solution for industrially producing electrical energy is to drive an alternator by a turbine, all rotating around an axis. Naturally, these installations produce sinusoidal voltages.

III.2 Sinusoidal power:

III.2.1 Active power:

This is the energy actually recoverable by the load. It is called active power because it is what is really useful (for example, in a motor, it is the active power which is transformed into mechanical power). The active power is the average value of the instantaneous power expressed in Watt $[w]$.

$$\mathbf{p} = \frac{1}{T}\int_0^T u(t)i(t)dt = UIcos\varphi$$

III.2.2 Reactive power:

Reactive power appears when the installation contains inductive or capacitive receivers. Its unit is: Volt-Ampere-Reactive $[VAR]$.

$$Q=UI\sin\varphi$$

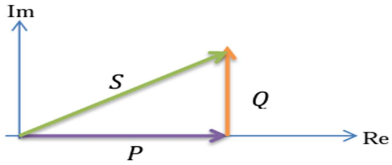
- If $\varphi>0$, The reactive power is positive then the receiver is inductive.
-If $\varphi<0$, The reactive power is negative then the receiver is capacitive.

III.2.3 Apparent power:

It is equal to the vector sum of the two active and reactive powers and it makes it possible to determine the value of the current absorbed by the load. Expressed in VA].

$$S = UI$$

Relationship between power (Triangle of powers)



$$S^2 = P^2 + Q^2$$

$$S = \sqrt{P^2 + Q^2}$$

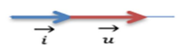
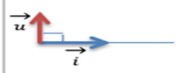
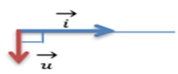
Noticed :

- The power supplied by the generator is equal to the power absorbed by the receiver.
- A resistor does not consume reactive power.
- The coil (la bobine) does not consume active power. It consumes reactive power.
- The capacitor does not consume active power. It is a reactive power generator.

III.2.4 Power factor :

Power factor is a factor (without units) measuring the active power production efficiency of the system.

$$\cos(\varphi) = \frac{P}{S}$$

Dipôles passifs	Temporelle	Module	Impédance Complexes	Représentation de Fresnel	Puissance active	Puissance réactive
résistor de résistance R(Ω)	$u(t) = R.i(t)$	$U = RI$	$z_R = R,$ $\varphi = 0$		RI^2	0
bobine parfaite d'inductance pure L en Henry (H)	$u(t) = l\frac{di(t)}{dt}$	$U = l\omega I$	$z_L = jl\omega$ $\varphi = \frac{\pi}{2}$		0	$l\omega I^2$
condensateur parfait de capacité C en Farad (F)	$u(t) = c\frac{du(t)}{dt}$	$U = \frac{1}{c\omega}I$	$z_c = -j\frac{1}{c\omega}$ $\varphi = -\frac{\pi}{2}$		0	$\frac{1}{c\omega}I^2$

III.3 Boucherot's Theorem: The total active power consumed by a system (see figure below) is the sum of the active powers consumed by each element

$$P_T = \sum_{i=1}^{n} P_i = P_1 + P_2 + P_3 + \cdots + P_n$$

The total reactive power consumed by a system is the sum of the reactive powers consumed by each element

The apparent power consumed by a system is calculated from the relationship:

$$S_T = \sqrt{P^2 + Q^2}$$

$$Q_T = \sum_{i=1}^{n} Q_i = Q_1 + Q_2 + Q_3 + \cdots + Q_n$$

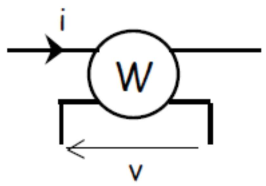
Impédance	*Loi d'Ohm*	*Déphasage*	*Facteur de puissance*	*Puissance active*	*Puissance réactive*	*Puissance apparente*
Z = R	*U = Z I*	*φ = 0* *φ = 0 rads*	*Cos φ = 1* ***Sin φ*** *Sin φ = 0*	*P = U I* *P = R I²* *P = U²/R*	*Q = 0 VAR*	*S = P* *S = U I*
Z = L ω	*U = L ωI*	*φ = 90* *φ = π/2 rads*	*Cos φ = 0* ***Sin φ*** *Sin φ = 1*	*P = 0 W*	*Q = UIsinφ* *Q = UI* *Q = LωI²*	*S = Q* *S = L ωI²*
Z = 1/C ω	*U = I/C ω*	*φ = - 90* *φ = - π/2 rads*	*Cos φ = 0* ***Sin φ*** *Sin φ = - 1*	*P = 0 W*	*Q = UIsinφ* *Q = - UI* *Q = - U² Cω*	*S = Q* *S = - U² Cω*

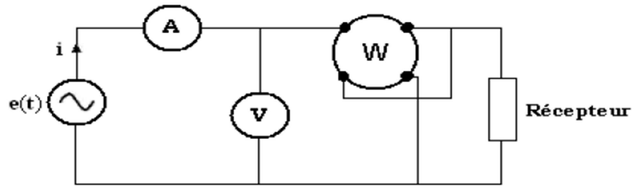
III.4 Power measurement:

- $\mathbf{P = U \cdot I \cdot \cos\varphi \quad [W]}$
- $\mathbf{Q = U \cdot I \cdot \sin\varphi \quad [VAR]}$
- $\mathbf{S = \sqrt{P^2 + Q^2} = U \cdot I \quad [VA]}$

III.4.1 Measure active power P :

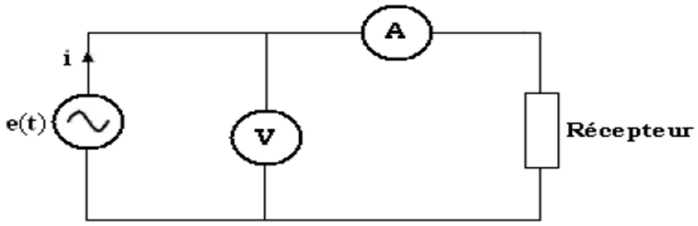
To measure P, simply connect a wattmeter according to the downstream assembly. It has at least four terminals: two terminals for measuring voltage and two terminals for measuring current. There are therefore two connections to make: a parallel connection, like a voltmeter, to measure the voltage, and a series connection, like an ammeter, to measure the current.





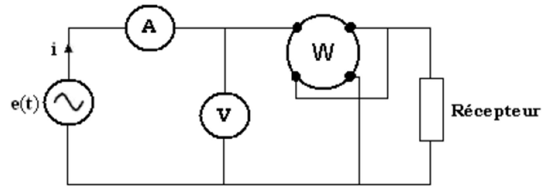
III.4.2 Measuring apparent power S :

To measure S, you must use an ammeter and a voltmeter in order to determine the effective values of the current and the voltage according to the diagram of the following assembly:



III.4.3 Measuring reactive power Q :

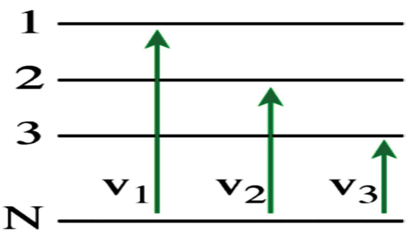
To measure reactive power Q, simply connect an ammeter, a voltmeter and a wattmeter. Then calculate taking into account the type of receiver:

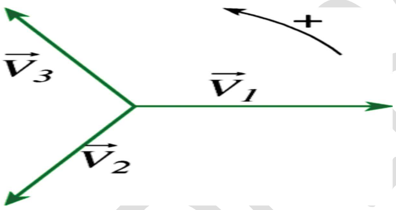


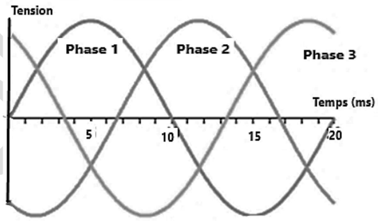
- for a resistive receiver $Q = 0$
- for an inductive receiver $Q > 0$
- for a capacitive receiver $Q < 0$

III.5 Three-phase alternating current power :

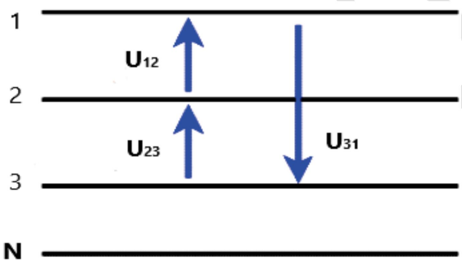
Three-phase receivers: These are receivers made up of three identical dipoles, Z impedance.
Balanced: because the three elements are identical.

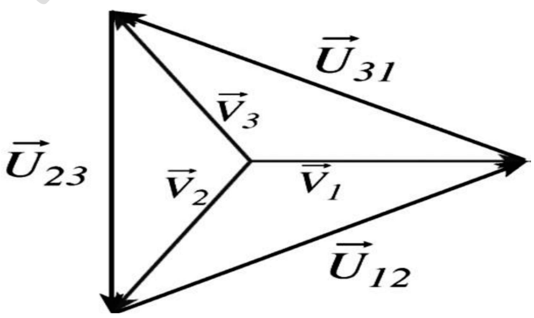


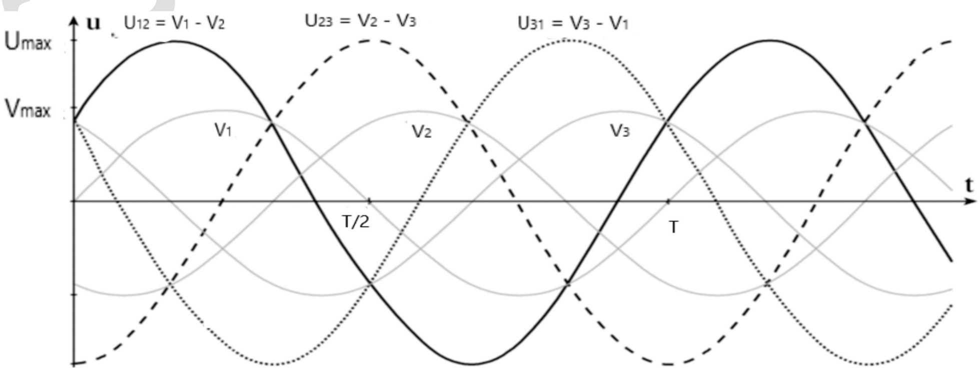




Composite voltages between phases u_{12} , u_{23} , u_{31}







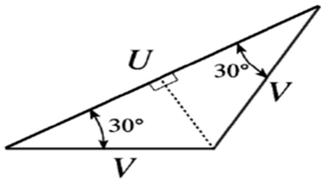
$$\vec{U}_{12} = \vec{V}_1 - \vec{V}_2 \;;\quad \vec{U}_{23} = \vec{V}_2 - \vec{V}_3 \;,\quad \vec{U}_{31} = \vec{V}_3 - \vec{V}_1$$

$$u_{12}(t) = U\sqrt{2}\sin(\omega t + \frac{\pi}{6}) \qquad u_{23}(t) = U\sqrt{2}\sin(\omega t - \frac{\pi}{2}) \qquad u_{31}(t) = U\sqrt{2}\sin(\omega t - \frac{7\pi}{6})$$

Relationship between U and V

Finalement :

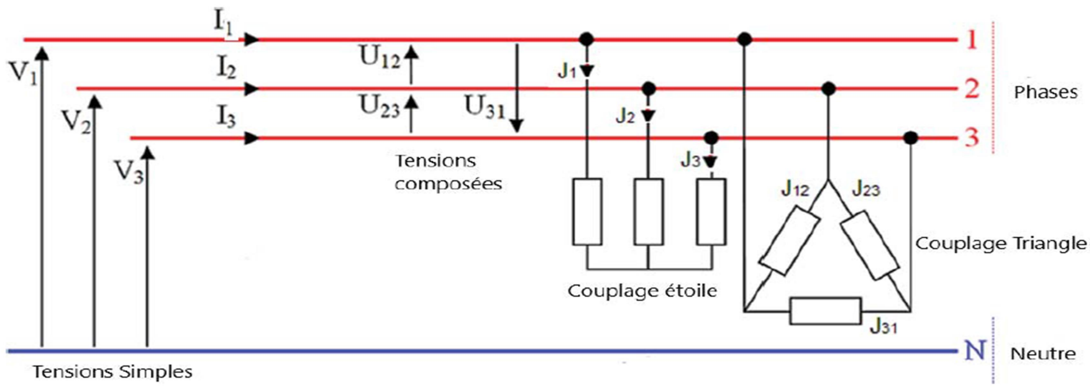
$$U = 2V\cos 30 \quad \text{soit} \quad U = 2V\frac{\sqrt{3}}{2}$$



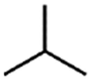
Currents per phase: these are the currents which pass through the Z elements of the receiver three-phase. Symbol J

Line currents: these are the currents which pass through the wires of the three-phase network. Symbol: I.

The network and the receiver can be connected in two different ways: in a star or in a triangle.



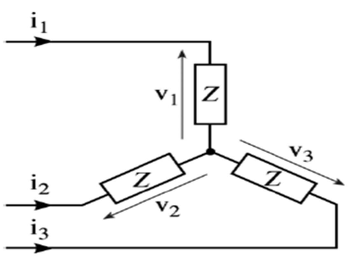
III.5.1 Star coupling

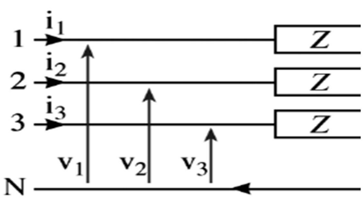


As they are the same impedances, therefore i1 + i2 + i3 = 0, therefore in=0. The current in the neutral wire is zero. The neutral wire is therefore not necessary.

Noticed :

For a balanced three-phase system, the neutral wire is useless.



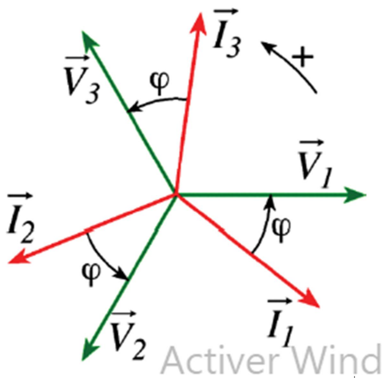


Line currents are equal to currents per phase.

$i_1 = j_1$; $i_2 = j_2$; $i_3 = j_3$

In addition, the load and the network are balanced, so

$I_1 + I_2 + I_3 = I = J$



For a receiver phase $P_1 = VI\cos\varphi \quad avec\ \varphi(\vec{I},\vec{V})$

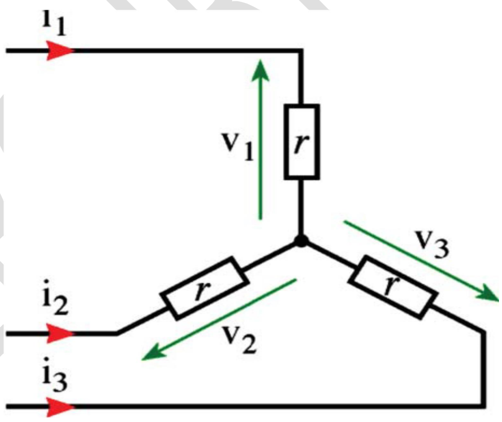
For the complete receiver $P = 3\,VI\cos\varphi$ moreover $V = \frac{U}{\sqrt{3}}$

Finally for the star coupling

$$P = \sqrt{3}\,UI\cos\varphi$$
$$Q = \sqrt{3}\,UI\sin s\,\varphi$$
$$S = UI$$

Power factor: $K = \cos\varphi$

Losses by Joule effect: Let us consider that the resistive part of the receiver.



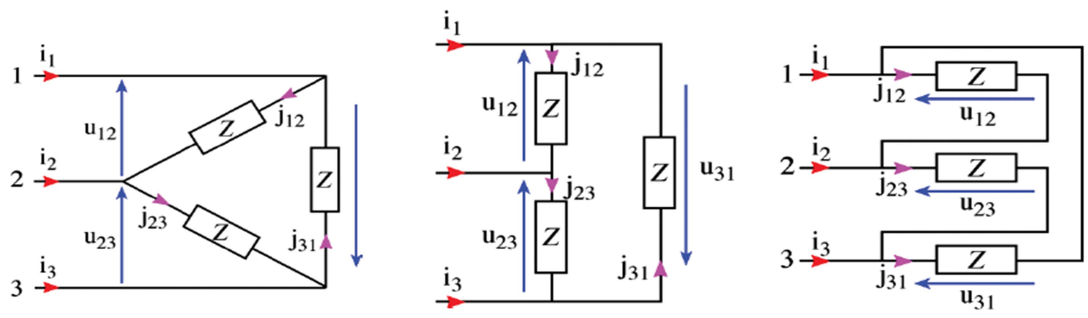
For a receiver phase : $P_{J1} = r\,I^2$

For a receiver complet : $P = 3P_{J1} = 3\,r\,I^2$

Resistance seen between two terminals : R = 2 r

Finally for the star coupling: $P = \frac{3}{2}\,R\,I^2$

III.5.2 Triangle coupling △



As they are the same impedances,
Here in no case is the neutral wire necessary. $i_1 + i_2 + i_3 = 0$, et $J_{12} + J_{23} + J_{31} = 0$

Relationships between currents :

$$i_1 = j_{12} - j_{31} \rightarrow \vec{I}_1 = \vec{J}_{12} - \vec{J}_{31}$$
$$i_2 = j_{23} - j_{12} \rightarrow \vec{I}_2 = \vec{J}_{23} - \vec{J}_{12}$$
$$i_3 = j_{31} - j_{23} \rightarrow \vec{I}_3 = \vec{J}_{31} - \vec{J}_{23}$$

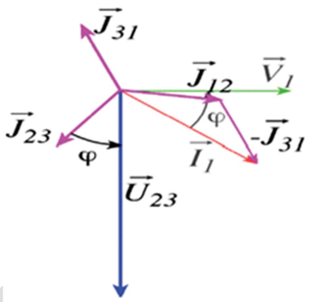
The three-phase system is balanced: $\mathbf{I_1 = I_2 = I_3 = I}$ **et** $\mathbf{J_{12} = J_{23} = J_{31} = J}$

For triangle coupling, the relationship between I and J is the same as the relationship between V and U. For triangle coupling:

A- Puissances :

For a receiver phase: $P_1 = UJ\cos\varphi \;\; avec\; \varphi(\vec{U}, \vec{J})$

For the complete receiver: $P = 3\,P_1 = UJ\cos\varphi$ de plus $J = \frac{I}{\sqrt{3}}$



Finally for the star coupling: $P = \sqrt{3}\,UI\cos\varphi$

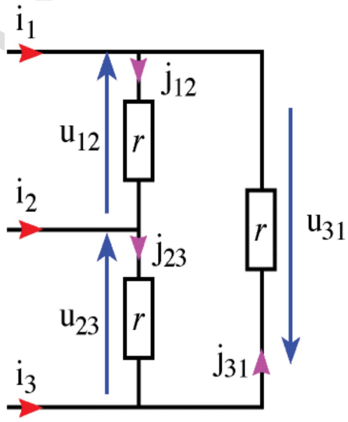
In the same way: $Q = \sqrt{3}\,UI\sin\varphi$

And : $S = UI$

Power factor: $K = \cos\varphi$

B- Pertes par effet Joule :

Consider that the resistive part of the receiver.



For a receiver phase: $P_{J1} = r\,I^2$

For the complete receiver: $P = 3P_{J1} = 3\,r\,J^2$

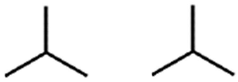
Resistance seen between two terminals: $R = \frac{2r r}{2r + r} = \frac{2}{3} r$

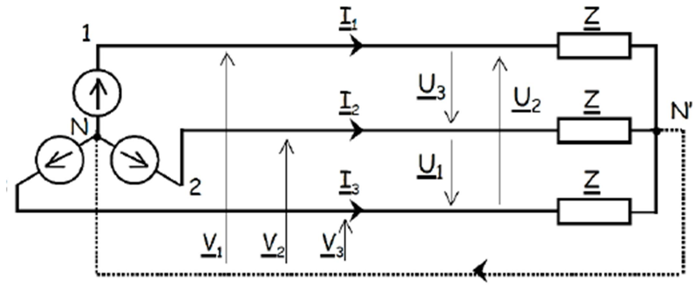
Finally for the star coupling : $P = \frac{3}{2}\;R\,I^2$

	Couplage étoile	Couplage triangle
Relation entre U et V	$U = V\sqrt{3}$	$U = V\sqrt{3}$
Relation entre I et J	$I = J$	$I = J\sqrt{3}$
Déphasage	$\varphi\,(\vec{I},\,\vec{V})$	$\varphi\,(\vec{J},\,\vec{U})$
Puissance active	$P = 3.P_1 = 3VI\cos\varphi$ $P = \sqrt{3}UI\cos\varphi$	$P = 3.P_1 = 3UJ\cos\varphi$ $P = \sqrt{3}UI\cos\varphi$
Pertes joules	$P = 3rI^2$ $P = \frac{3}{2}RI^2$	$P = 3rJ^2$ $P = \frac{3}{2}RI^2$
Résistance équivalente	$R = 2r$	$R = \frac{2}{3}r$
Puissance réactive	$Q = \sqrt{3}UI\sin\varphi$	$Q = \sqrt{3}UI\sin\varphi$
Puissance apparente	$S = \sqrt{3}UI$	$S = \sqrt{3}UI$
Facteur de puissance	$k = \cos\varphi$	$k = \cos\varphi$

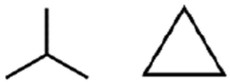
III.6 Different types of Generator-Receiver coupling

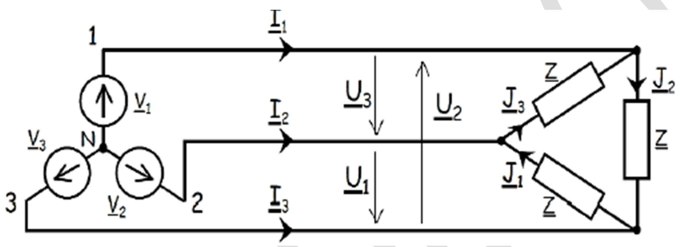
III.6.1 Star – star coupling



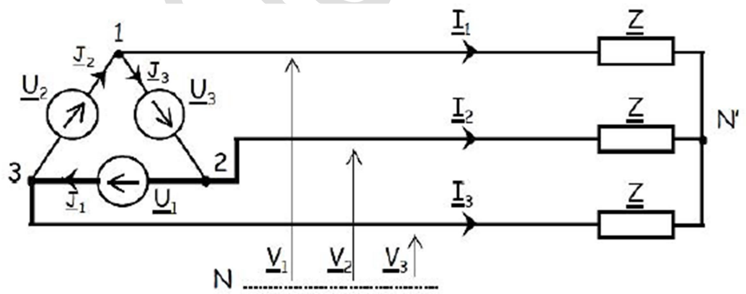


III.6.2 Star – triangle coupling

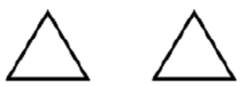


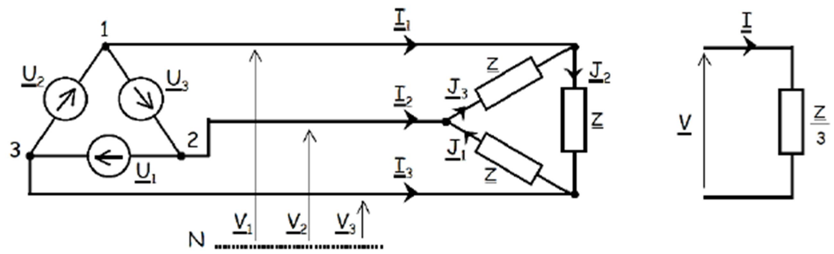


III.6.3 Triangle – Star coupling



III.6.3 Couplage triangle – triangle

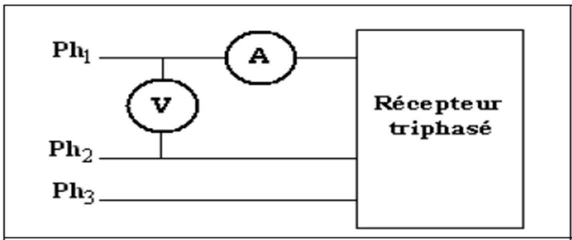




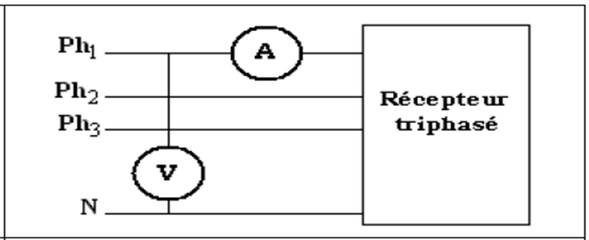
III.6 Power measurement:

III.6.1 Measurement of apparent power S:

To measure S, you must use a voltmeter and an ammeter to determine the simple or compound voltage and the current crossing a power line (we assume that the available three-phase system is direct balanced) according to the two arrangements in the following figure:



Ligne sans fil de neutre $\left(S = \sqrt{3} \cdot U \cdot I\right)$

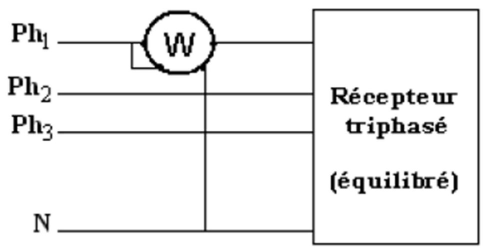


Ligne avec fil de neutre $(S = 3 \cdot V \cdot I)$

III.6.2 Measurement of active power P:

III.6.2.1 Single wattmeter method with neutral wire:

When the receiver is balanced, a single wattmeter can measure the active power absorbed. The principle diagram is given by the following figure:



The power meter, as plugged in, measures power :

$$P_1 = V \cdot I \cdot \cos\varphi$$

The power absorbed by the balanced three-phase receiver is:

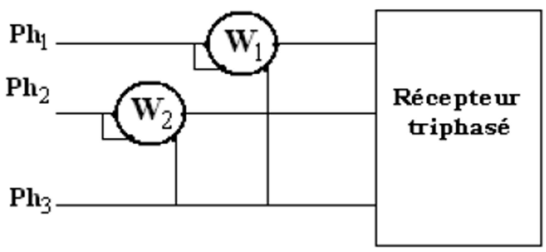
$$P = 3 \cdot P_1$$

Indeed, we can write: $P = 3 \cdot P_1 = 3 \cdot V \cdot I \cdot \cos\varphi = \sqrt{3} \cdot U \cdot I \cdot \cos\varphi$

This measure requires that the neutral wire be accessible.

III.6.2.2 Two wattmeter method:

For an unbalanced system or a balanced system where the neutral is not accessible, the active power is measured using two wattmeters. The assembly diagram is as follows:



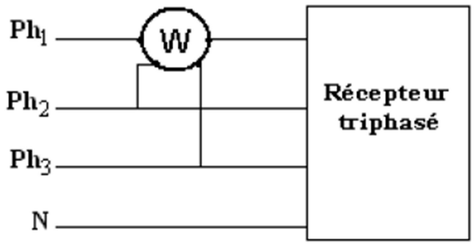
Si on appelle **P1** et **P2** les puissances mesurées par les wattmètres W1 et W2, on détermine la puissance active absorbée par la charge à l'aide de la relation :

$$P = P_1 + P_2$$

III.6.3 Reactive power measurement Q

III.6.3.1 Single wattmeter method:

To measure reactive power using a single wattmeter, simply mount the voltage circuit between phase 2 and 3 wires as shown in the following figure:



The reactive power is given by the following expression:

$$Q = \sqrt{3} \cdot P_1$$

With P_1 the power measured by the wattmeter W

III.6.3.2 Two wattmeter method:

This is the same method used for measuring active power. But we can determine the reactive power by the following relationship:

$$Q = \sqrt{3}\left(P_1 - P_2\right)$$