République Algérienne Démocratique et Populaire Ministère de L'Enseignement Supérieur et de la Recherche Scientifique

University of Badji-Mokhtar Annaba Faculty of Technology Sciences and Technology Department 2023/2024

2nd year (2023-2024), semester 3
Electrotechnique –Automatique – Electromécanique –Télécom- Electronique Cours Electrotechnique Fondamentale 1

Chapitre 1 : Rappels mathématiques sur les nombres complexes (NC) : 1 semaine
Forme cartésienne, NC conjugués, Module, Opérations arithmétiques sur les NC (addition, ...), Représentation géométrique, Forme trigonométrique, Formule de Moivre, racine des NC, Représentation par une exponentielle d'un NC, Application trigonométrique des formules d'Euler, Application à l'électricité des NC.

Chapitre 2 : Rappels sur les lois fondamentales de l'électricité : 2 semaines
Régime continu : dipôle électrique, association de dipôles R, C, L.
Régime harmonique : représentation des grandeurs sinusoïdales, valeurs moyennes et efficaces, représentation de Fresnel, notation complexe, impédances, puissances en régime sinusoïdal (instantanée, active, apparente, réactive), Théorème de Boucherot.
Régime transitoire : circuit RL, circuit RC, circuit RLC, charge et décharge d'un condensateur.

Chapitre 3 : Circuits et puissances électriques : 3 semaines
Circuits monophasés et puissances électriques. Systèmes triphasés : Equilibré et déséquilibré (composantes symétriques) et puissances électriques.

Chapitre 4 : Circuits magnétiques : 3 semaines
Circuits magnétiques en régime alternatif sinusoïdal. Inductances propre et mutuelle. Analogie électrique magnétique.

Chapitre 5 : Transformateurs : 3 semaines
Transformateur monophasé idéal. Transformateur monophasé réel. Autres transformateurs (isolement, à impulsion, autotransformateur, transformateurs triphasés).

Chapitre 6 : Introduction aux machines électriques : 3 semaines
Généralités sur les machines électriques. Principe de fonctionnement du générateur et du moteur. Bilan de puissance et rendement.

Semestre: 3
Unité d'enseignement: UEF 2.1.2
Matière 2:Electrotechnique fondamentale 1
VHS: 45h00 (Cours: 1h30, TD: 1h30)
Crédits: 4
Coefficient: 2

Chapter I:

Mathematical Reminders About Complex Numbers

I.1 Introduction:

Complex numbers are a kind of two-dimensional vectors whose components are the so-called real part and imaginary part. Complex numbers are useful in physics, as well as in the mathematics, because they open a new dimension that allows us to arrive at the results much faster. Using complex numbers allows sometimes to obtain analytical results that are impossible to obtain in other way, such as exact values of some definite integrals.

I.2 Form of Complex numbers:

Complex numbers can be introduced in the component form
$\mathbf{z} = \boldsymbol{u} + \boldsymbol{iv}$, Where u ∈ Reals,v ∈ Reals and multiplication of complex numbers is defined by imposing the property $\boldsymbol{i^2 = -1}$

Where u and v are real numbers, the real and imaginary parts (components) of z. That is,
$\mathbf{u} = \boldsymbol{Re}[z]$, **v=Im** $[z]$
To keep components of z apart, a special new number i is introduced, the so-called imaginary one. The modulus or absolute value of a complex number is defined by

$$|\mathbf{z}| = \sqrt{\boldsymbol{u^2 + v^2}}$$

Complex conjugate z* of a complex number z $= u + iv$ is defined by:

$$\mathbf{z} = \boldsymbol{u - iv}$$

I.3 Addition and subtraction of complex numbers:

Addition and subtraction of complex numbers are defined component-by-component

$$\mathbf{z_1 \pm z_2} = \boldsymbol{u_1 \pm u_2 + i(v_1 \pm v_2)},$$

So that the commutation and association properties are fulfilled,

$$\mathbf{z_1 + z_2} = \boldsymbol{z_2 + z_1}$$

$$(\mathbf{z_1 + z_2}) + \boldsymbol{z_3} = \boldsymbol{z_2 + (z_1 + z_3)}$$

I.4 Product of a complex number :

$$\mathbf{z_1\, z_2} = \boldsymbol{(u_1 + iv_1)(u_2 + iv_2) = u_1u_2 - v_1v_2 + i(u_1v_2 + u_2v_1)}$$

$$\boldsymbol{Re}[\mathbf{z_1\, z_2}] = \boldsymbol{u_1u_2 - v_1v_2} \text{ , } \mathbf{Im}\,[\mathbf{z_1\, z_2}] = \boldsymbol{u_1v_2 + u_2v_1}$$

Product of a complex number and its complex conjugate is real

$$\mathbf{z}\,\mathbf{z}^* = (u+iv)(u-iv) = u^2 - i^2v^2 = u^2+v^2 = [\mathbf{z}]^2$$

I.5 Division of complex numbers :

Division of complex numbers can be introduced via their multiplication and division of reals by eliminating complexity in the denominator

$$\frac{\mathbf{z_1}}{\mathbf{z_2}} = \frac{\mathbf{z_1}\,\mathbf{z_2^*}}{\mathbf{z_2}\,\mathbf{z_2^*}} = \frac{\mathbf{z_1}\,\mathbf{z_2^*}}{[\mathbf{z}]^2} = \frac{u_1u_2+v_1v_2+i(v_1u_2-u_1v_2)}{u^2+v^2}$$

I.6 Trigonometric / Exponential form :

Similarly to 2 d vectors, complex numbers can be represented by their modulus (length) ρ and angle (phase) *Φ* as:

$$\mathbf{z} = \rho(Cos[\emptyset] + i\,Sin[\emptyset])\ ;\ \rho = [z]\ .\ \mathrm{Cos}\ [\emptyset] = \frac{a}{|z|}\ ;\ \mathrm{Sin}\ [\emptyset] = \frac{b}{|z|}$$

This formula can be brought into a more compact and elegant shape

$$\mathbf{z} = \rho e^{i\emptyset}\ ;\ e^{i\emptyset} = Cos[\emptyset] + i\,Sin[\emptyset]$$

This formula can be proven by expanding the three functions in power series, using $i^2 = -1$ and grouping real and imaginary terms on the left. The exponential representation makes multiplication and division of complex numbers very easy

$$\mathbf{z_1 z_2} = \rho_1 e^{i\emptyset_1}\rho_2 e^{i\emptyset_2} = \rho_1\,\rho_2\,e^{i(\emptyset_1+\emptyset_2)}$$

In particular, $\mathbf{Re}[\mathbf{z_1z_2}] = \rho_1\,\rho_2 Cos[(\emptyset_1+\emptyset_2]$; $\mathbf{Im}[\mathbf{z_1z_2}] = \rho_1\,\rho_2 Sin[(\emptyset_1+\emptyset_2]$

That is much easier than the component formula above. Similarly

$$\frac{\mathbf{z_1}}{\mathbf{z_2}} = \frac{\boldsymbol{\rho_1}}{\boldsymbol{\rho_2}} e^{i(\emptyset_1-\emptyset_2)}$$

Squaring a complex number z yields

$$z^2 = \rho^2(Cos[\emptyset] + i\,Sin[\emptyset])^2 = \rho^2(Cos[\emptyset]^2 - Sin[\emptyset]^2 + 2\,i\,Cos[\emptyset]\,Sin[\emptyset])$$

$$z^2 = \rho^2 e^{2i\emptyset} = \rho^2(\,Cos[2\emptyset] + i\,Sin[2\emptyset])$$

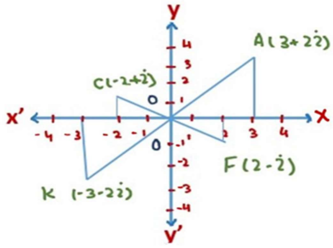
Equating the real and imaginary parts of these two formulas, one obtains the trigonometric identities

$$Cos[2\emptyset] = Cos[\emptyset]^2 + Sin[\emptyset]^2\ ;\ Sin[2\emptyset] = 2Sin[\emptyset]Cos[\emptyset]$$

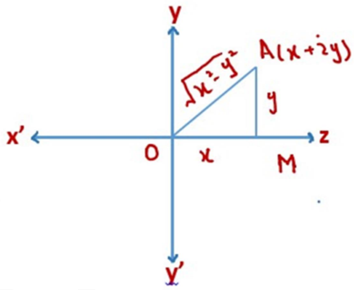
One can derive formulas for Sin and Cos of any multiple arguments with this method.

I.7 Geothermal representation of complex numbers :

In this representation, the x-axis is called the real axis and the y-axis is called the imaginary axis. The coordinate plane itself is called the complex plane or z-plane. By way of illustration , several complex numbers have been shown below in fig.



The figure representing one or more complex numbers on the complex plane is called an argand diagram. Points on the x-axis represent real numbers whereas the points on the y-axis represent imaginary numbers. x and y are the coordinates of a point. It represents the complex number x + iy . The real number $\sqrt{(a^2+b^2)}$ is called the modulus of the complex number a + ib.



In the right-angled triangle OMA, we have, **by Pythagoras theorem**.

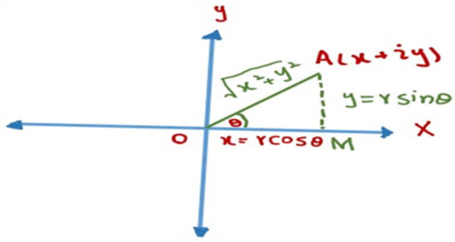
$$|\overline{OA}|^2 = |\overline{OM}|^2 + |\overline{MA}|^2$$

$$\therefore |OA| = \sqrt{x^2+y^2}$$

$$\overline{MA} \perp \overrightarrow{Ox}$$

$$\therefore \overline{OM} = x \, , \overline{MA} = y$$

The polar form of a Complex number consider adjoining representing the complex number z = x + i



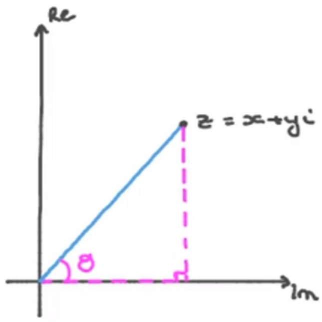
From the diagram, we see that x = r. cosθ and y = r sinθ where r = |z| and θ is called arguments of z.

I.8 Moivre formula :

$$re^{i\theta} = r(\cos\theta + i\sin\theta)$$

$$r^n e^{in\theta} = r^n(\cos n\theta + i\sin n\theta)$$

Formule de De Moivre



$$z = r(\cos\theta + i\sin\theta)$$

I.9 Euler's formula :

$$\cos(x) = \frac{e^{ix}+e^{-ix}}{2} \qquad \sin(x) = \frac{e^{ix}-e^{-ix}}{2i}.$$