Chapter II:

Reminders on the fundamental laws of electricity

II.1 Introduction: In this chapter, we will discuss the main electric dipoles and the fundamental laws that govern them.

II.2 Continuous regime:

In continuous mode, the current and voltage quantities are constant over time.

II.2.1 Electric dipole

An electric dipole is a single component or a set of components, connected to two (02) terminals (see figure 2.1). We place a meaning for the Koran.

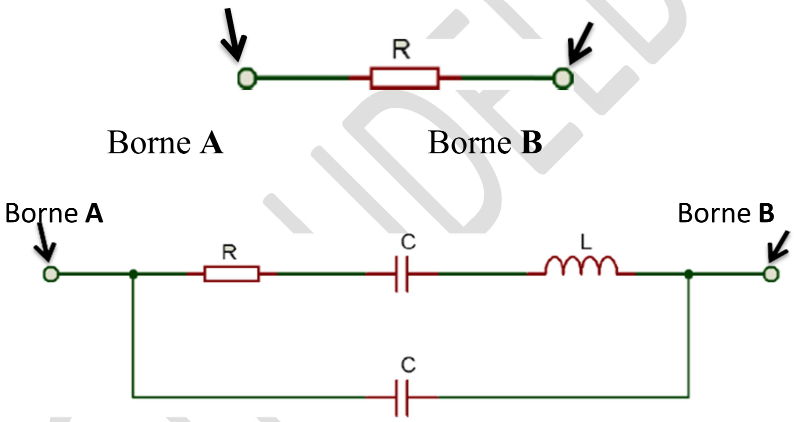


Figure 2.1 Electric dipoles

Receiver convention: current i and voltage u are oriented in opposite directions.

Generator convention: current i and voltage u are oriented in the same direction.

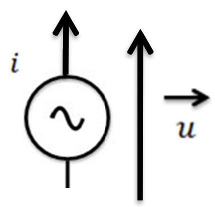


Figure 2.3 Generator conventions.

Passive dipole: It is a dipole which consumes electrical energy and does not contain any energy source. Examples include: resistance, inductance, bulb…..

Active dipole: It is a dipole which contains a source of energy. For example, we can cite battery, or direct current electric motor.

II.2.2 Properties of dipoles:

A- Polarity

A dipole is polarized when its terminals cannot be swapped, for example: chemical capacitor, direct current generator, diode, etc. If the terminals are reversed, operation can be disrupted of the circuit. For a non-polarized dipole, the permutation of their terminals does not influence the operation of the circuit. The resistor is a non-polarized dipole.

B- Linearity

A dipole is linear when it meets the mathematical criteria of linearity. The current/voltage characterization is a straight line. A pure resistor is a linear dipole, on the other hand the diode is a non-linear dipole.

II.2.3 Association of dipoles:

In an electrical circuit, dipoles can be associated in series or in parallel.

Dipoles in series:

Dipoles are associated in series when they are connected one after the other. The current i is common to all dipoles. The voltage u is the sum of the voltages across each dipole.

Dipoles in parallel:

The voltage u is common to all dipoles. The total current i is the sum of the currents across each dipole.

II.2.4 Association of elementary dipoles R, L and C :

A- Association of resistances (R) in series

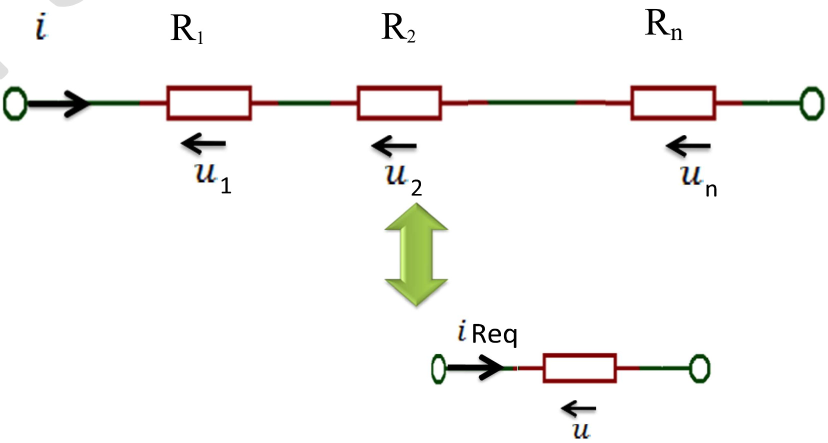


Figure 2.4 Association des résistances en série.

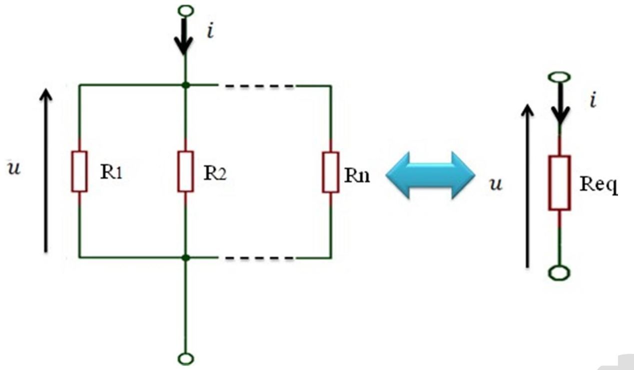
$$u = u_1 + u_2 + u_3 + \cdots + u_n = (R_1 + R_2 + R_3 + \cdots + R_n) = R_{eq}\, i$$

The equivalent resistance is then equal to the sum of the resistances placed in series. Its unit is Ω.

$$R_1 + R_2 + R_3 + \cdots + R_n = \sum^n R_n$$

B- Association of resistances (R) in parallel

In parallel, the voltage is common to all resistors. The current which enters the whole is given, according to the law of nodes, by:



$$i = i_1 + i_2 + i_3 + \cdots + i_n$$
$$= \frac{u}{R_1} + \frac{u}{R_2} + \frac{u}{R_2} + \cdots + \frac{u}{R_n}$$
$$= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}\right) u$$
$$= \frac{1}{R_{eq}} . u$$

The equivalent admittance is equal to the sum of the reciprocals of the resistances placed in parallel. Its unit is $\Omega{-1}$.

$$Y_{eq} = \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n} = \sum_1^n \frac{1}{R_n}$$

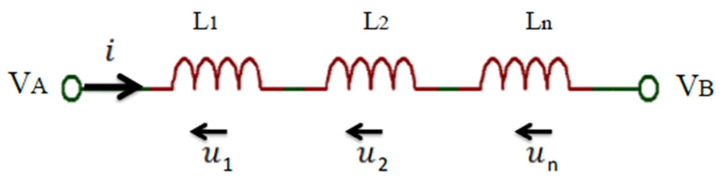
- Case of 2 resistors placed in parallel $\quad R_{eq} = \frac{R_1 R_2}{R_1 + R_1}$

- Case of n resistors placed in parallel $\quad R_{eq} = \frac{R}{n}$

C-Association of inductors (L) in series

Associating inductors in series means increasing the total number of turns. The voltage across an inductor crossed by a current of variable intensity as a function of time is given by:

$$u_L = L\frac{di}{dt}$$



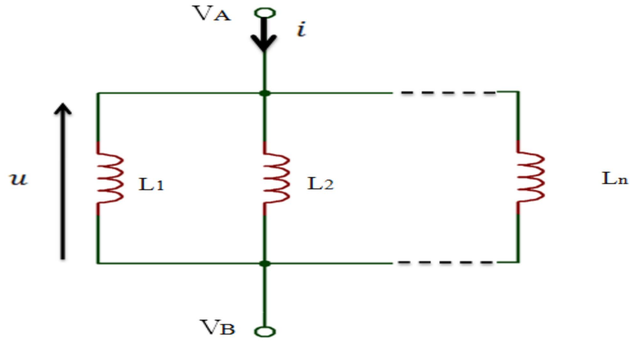
$$V_A - V_B = u_1 + u_2 + u_3 + \cdots + u_n$$
$$= L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt} + \cdots + L_n\frac{di}{dt}$$
$$= (L_1 + L_2 + L_3 + \cdots + L_n)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$

The equivalent inductance is then equal to the sum of the inductances placed in series.
(It is assumed that the current has the same direction of flow in the coils).

$$L_1 + L_2 + L_3 + \cdots + L_n = \sum_1^n L_n$$

D-Association of inductors (L) in parallel

In parallel, the voltage is common to all inductors. The current that enters the whole is (law of knots):



$$i = i_1 + i_2 + i_3 + \cdots + i_n$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt} + \cdots + \frac{di_3}{dt}$$

$$= \frac{V_A - V_B}{L_1} + \frac{V_A - V_B}{L_2} + \frac{V_A - V_B}{L_3} + \cdots + \frac{V_A - V_B}{L_n}$$

$$= (V_A - V_B)\left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}\right)$$

The equivalent admittance is equal to the sum of the inductances placed in parallel:

$$= (V_A - V_B)\frac{1}{L_{eq}}$$

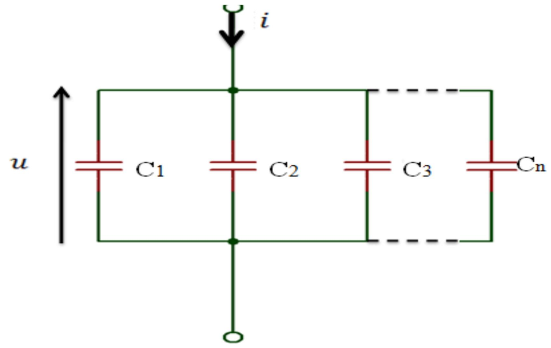
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n} = \sum_1^n \frac{1}{L_n}$$

E) Association of capacitors (C) in parallel

A capacitor is characterized by its capacitance, denoted C and expressed in Farads (symbol F). The voltage across a capacitor crossed by a current of variable intensity as a function of time is:

$$u_c = \frac{1}{C}\int i dt$$

Here the current is common to all capacitors. The voltage across the assembly is:



$$i = i_1 + i_2 + i_3 + \cdots + i_n$$

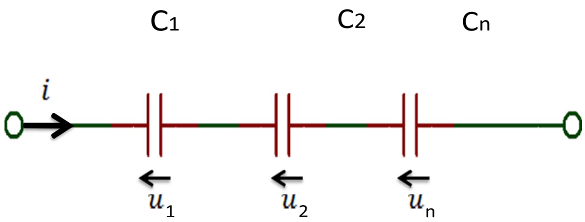
$$= C_1\frac{du}{dt} + C_2\frac{du}{dt} + C_3\frac{du}{dt} + \cdots + C_n\frac{du}{dt}$$

$$= (C_1 + C_2 + C_3 + \cdots + C_n)\frac{du}{dt}$$

$$C_1 + C_2 + C_3 + \cdots + C_n = \sum_1^n C_n$$

F) Association of capacitors (C) in series

In parallel, the voltage is common to all capacitors. The current (see figure 2.11) which enters the whole is (law of knots):



$$u = u_1 + u_2 + u_3 + \cdots + u_n$$

$$= \frac{1}{C_1}\int i.dt + \frac{1}{C_2}\int i.dt + \frac{1}{C_3}\int i.dt + \cdots + \frac{1}{C_n}\int i.dt$$

$$\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}\right).\int i.dt$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n} = \sum_1^n \frac{1}{C_n}$$

II.3 Harmonic regime (Sinusoidal):

We call sinusoidal regime (or harmonic regime) the state of a system for which the variation over time of the quantities characterizing it is sinusoidal. The electrical circuit, in this case, is powered by a sinusoidal alternating voltage V(t) and traversed by a sinusoidal alternating current i(t).

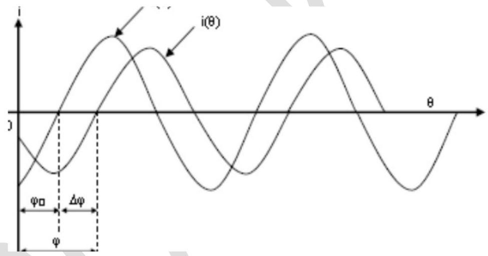
II.3.1 Alternating current:

A sinusoidal alternating current is a periodic **bidirectional** current. The same is true for a sinusoidal alternating voltage. Voltage: $u(t) = U_M \sin(wt + \Phi u)$ and Current, $i(t) = I_M \sin(wt + \Phi_i)$.

With:

u(t) Instantaneous value (Valeur instantanée), U_M Maximum value (Valeur maximale) (V);

$(wt + \Phi_u)$ Instantaneous phase (Phase instantanée)(rd); (w) Pulsation. (Φ_u) and (Φ_i) Phase shift relative to the phase origin; $\Delta\phi = \Phi u - \Phi i$ is the phase shift between current and voltage.



II.3.2 Average values of sinusoidal current (Valeurs moyennes du courant sinusoïdal) :

We have: $i(t) = I_M \sin(wt + \Phi_i)$

$$I_{moy} = \frac{1}{T}\int_0^T i(t)dt = \frac{1}{T}\int_0^T I_M \sin\ \omega t\, dt = \frac{I_M}{T}\left[\frac{-\cos\omega t}{\omega}\right]_0^t = -\frac{I_M}{T\,\omega}\left[\cos\omega t - \cos 0\right] = -\frac{I_M}{2\pi}[1-1] = 0$$

II.3.3 RMS values of sinusoidal current (Valeurs efficaces du courant sinusoïdal)

$$I_{eff}^2 = \frac{1}{T}\int_0^T I^2(t)\,dt = \frac{2}{T}\int_0^{T/2} I^2(t)dt = \frac{2}{T}\int_0^{T/2} I_M^2 \sin^2\omega t\, dt = \frac{2I_M^2}{T}\int_0^{T/2}\frac{1-\cos 2\omega t}{2}dt = \frac{2I_M^2}{2T}\left[t - \frac{\sin}{2\omega}\right]_0^{\frac{T}{2}} = \frac{I_M^2}{2}$$

Alors $I_{eff} = \frac{I_M}{\sqrt{2}}$

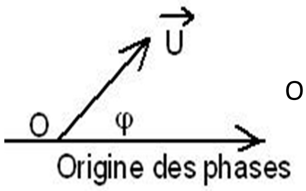
The same for the tension: $U_{eff} = \frac{U_M}{\sqrt{2}}$

II.3.4 Fresnel representation :

Or a sinusoidal quantity $\mathbf{Seff}\ \sqrt{2}\sin(wt + \varphi)$. This quantity can be represented at each moment by a vector $\bar{G}$ appelé vecteur de Fresnel associé à la grandeur sinusoïdale g(t). We choose an axis of origin of the phases and we represent the vector. The vector rotates with a constant speed ω in the trigonometric direction, the interest of the Fresnel representation is to separate the temporal part (wt) from the part phase (φ). Let the signal be:

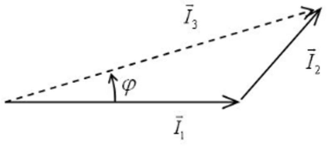
$$\mathbf{S(t) = S_M \sin(wt + \varphi) = \sqrt{2}\, S_{eff} \sin(wt + \varphi)}$$

which can be a voltage or a current. This signal can be represented by a vector $\overrightarrow{OM}$ the module S_{eff} placed relative to the axis (OX) origin of the phases, such that φ = angle between axis (OX) and the vector $\overrightarrow{OM}$

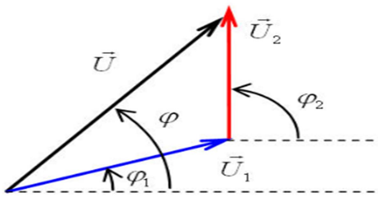


OM

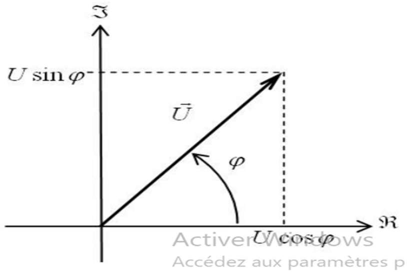
In electricity, this representation will easily make it possible to find the sum vector of two other.



$\vec{I}_3 = \vec{I}_1 + \vec{I}_2$



$\vec{U} = \vec{U}_1 + \vec{U}_2$



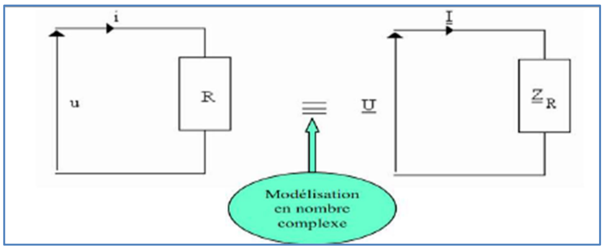
II.3.5 Complex notation:

Let's be the Tension: **$u(t) = U_M \sin(wt + \varphi_u)$** : and the curent, **$i(t) = I_M \sin(wt + \varphi_i)$** . We can associate complex numbers with them in the form

$\underline{U} = U e^{j\varphi_u}$ and $\underline{I} = I e^{j\varphi_i}$ $\underline{U} = \underline{Z} \times \underline{I}$ $\underline{I} = \underline{Y} \times \underline{U}$

II.3.6 Determination of elementary dipole impedances (RLC):

A. Case of an Ohmic resistance



Ohm's law **u(t)=R i(t)** So **$i(t) = \frac{u(t)}{R}$**

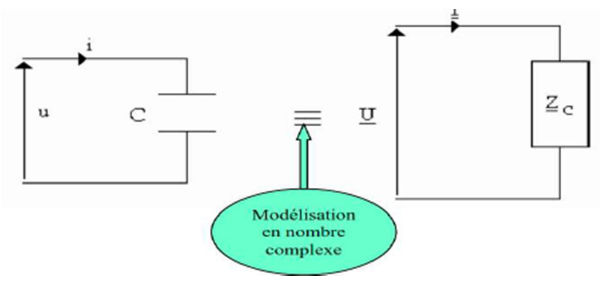
$$i(t) = \frac{U\sqrt{2}\sin(\omega t + \varphi_u)}{R}$$

$$i(t) = \frac{U}{R}\sqrt{2}\sin(\omega t + \varphi_u) \quad \underline{I} = \frac{U}{R} e^{j\varphi_u}$$

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{U e^{j\varphi_u}}{\frac{U}{R} e^{j\varphi_u}} = R e^{j0} \quad \text{donc } \underline{Z}_R = R \text{ et } \arg(\underline{Z}_R) = 0$$

Resistive impedance is purely real. Voltage and current are in phase.

B. Case of a capacitor:



$$u(t) = \frac{1}{c}\int i(t)dt \quad alors\ i(t) = C\frac{du(t)}{dt}$$

$$i(t) = C\omega U\sqrt{2}\cos(\omega t + \varphi_u)$$

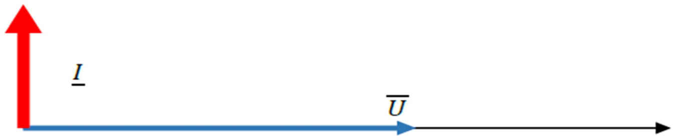
$$= C\omega U\sqrt{2}\sin\left(\omega t + \varphi_u + \frac{\pi}{2}\right)$$

$$\underline{I} = C\omega U e^{j(\varphi_u + \frac{\pi}{2})}$$

$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{Ue^{j\varphi_u}}{C\omega Ue^{j\varphi_u+\frac{\pi}{2}}} = \frac{1}{C\omega}e^{-j\frac{\pi}{2}}$ $\underline{Z_C} = -j\frac{1}{C\omega}$ et $\arg\left(\underline{Z_C}\right) = -\frac{\pi}{2} = \varphi_C$

Capacitive impedance is pure imaginary capacitive reactance $X_C = \frac{-1}{C\omega}$

Current is quadrature ahead (en avance) of voltage



$\underline{U} = -j\frac{1}{C\omega}\underline{I}$

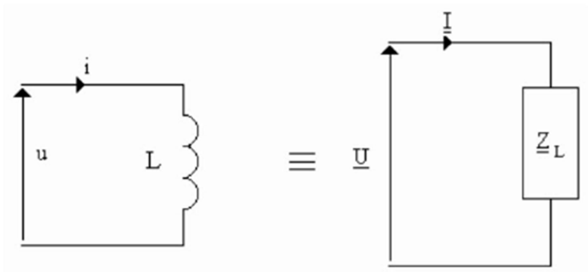
b. Cas d'une Bobine :

$u(t) = L\frac{di(t)}{dt}$ alors $i(t) = \frac{1}{L}\int u(t)dt$

$i(t) = -\frac{1}{L\omega}U\sqrt{2}\cos(\omega t + \varphi_u) = \frac{1}{L\omega}U\sqrt{2}\sin\left(\omega t + \varphi_u - \frac{\pi}{2}\right)$ donc $\underline{I} = \frac{U}{L\omega}e^{j(\varphi_u - \frac{\pi}{2})}$

The impedance of a coil is purely inductive of inductive reactance $X_L = L\omega$ (Ω).

The current is in quadrature lagging behind the voltage of $\varphi = \frac{\pi}{2}$ donc $\underline{U} = jL\omega\underline{I}$



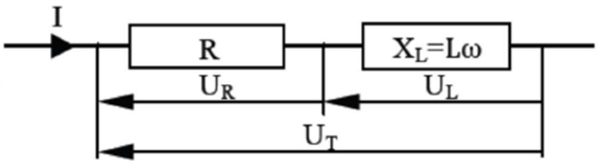
Noticed : $\underline{Z} = |Z|e^{j\varphi} = Ze^{j\varphi} = R + jX$

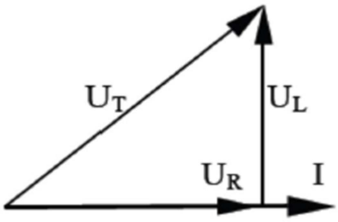
- If X =0 Impedance is resistive and $\boldsymbol{\varphi} = \mathbf{0}$
- If R =0 et X >0 Impedance is purely inductive and $\boldsymbol{\varphi} = \frac{\pi}{2}$
- If R =0 et X <0 Impedance is purely capacitive and $\boldsymbol{\varphi} = -\frac{\pi}{2}$

$\sin(-\alpha) = -\sin\alpha$	$\cos(-\alpha) = \cos\alpha$
$\sin(\alpha + 2n\pi) = \sin\alpha$	$\cos(\alpha + 2n\pi) = \cos\alpha$
$\sin(\alpha + \pi) = -\sin\alpha$	$\cos(\alpha + \pi) = -\cos\alpha$
$\sin(\pi - \alpha) = \sin\alpha$	$\cos(\pi - \alpha) = -\cos\alpha$
$\sin\left(\alpha + \frac{\pi}{2}\right) = \cos\alpha$	$\cos\left(\alpha + \frac{\pi}{2}\right) = -\sin\alpha$
$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$	$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$

Circuit RL:

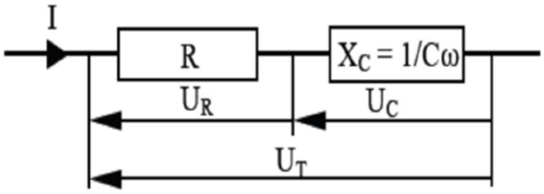
$U_T = U_R + U_L$, $U_T = R.I + jX_L I = R.I + jL\omega I = Z.I$: $Z = R + jL\omega$

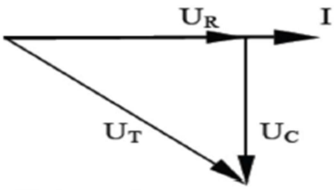




Circuit RC:

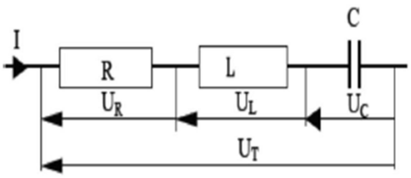
$U_T = U_R + U_C$, $U_T = R.I + jX_C I = R.I - \frac{j}{c\omega} I = Z.I$ avec $Z = R - \frac{j}{c\omega}$

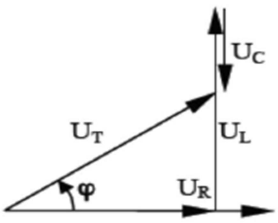


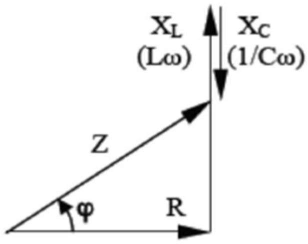


Circuit RLC:

$U_T = U_R + U_L + U_C = R.I + jL\omega - \frac{j}{c\omega} I$







II.7 Transitional regime:

These are the particular evolutions of electrical quantities which appear during sudden modifications of the characteristics of an electrical circuit. In general they do not occur repeatedly, otherwise we speak of a periodic maintained regime.

II.7.1 Transitional regime RL :

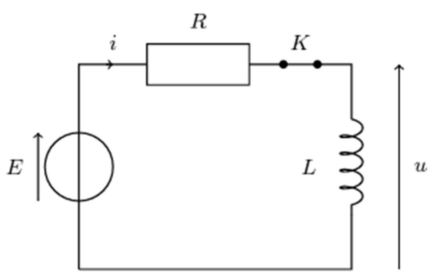
Consider the circuit in the figure. At t=0, we close the switch K. For t<0:i(t)=0. For t>0, the mesh law is written:

$$E = Ri + L\frac{di}{dt} \Rightarrow \frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$$

The solution to this equation is written:

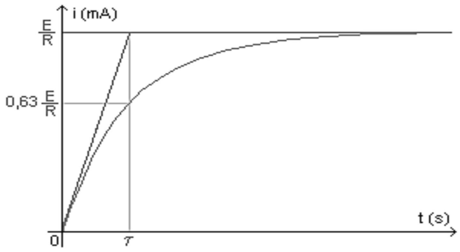
$$i(t) = Ae^{-t/\tau} + \frac{E}{R}$$

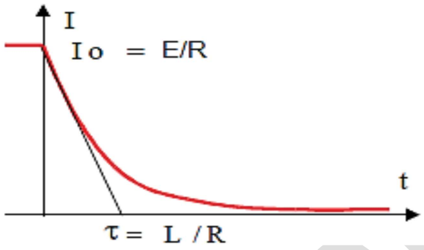
: $A = -\frac{E}{R}$.



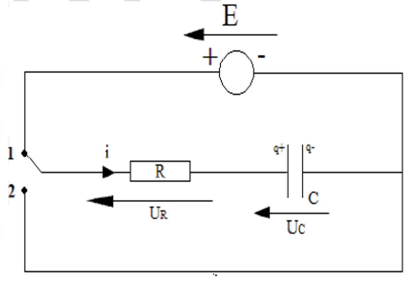
With i(t=0)=0. So

$$i(t) = \frac{E}{R}\left(1 - e^{-t/\tau}\right) \text{ et } u(t) = L\frac{di}{dt} = Ee^{-t/\tau}$$





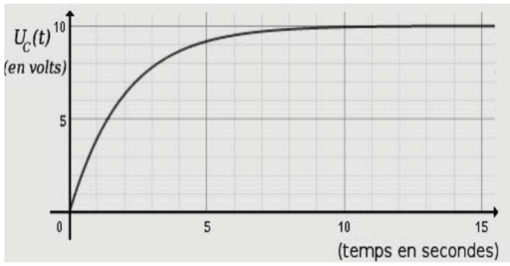
II.7.2 RC transitional regime:



The RC circuit is made up of a generator, a resistor and a capacitor. In their series configuration, RC circuits make it possible to produce low-pass electronic filters or high pass. The time constant τ of an RC circuit is given by the product of the value of these two elements which make up the circuit τ = RC.

II.7.2.1 Capacitor charge:

We flip the switch to position 1 thus, we apply a voltage E across the capacitor.



$$E = R\,I + U_c \quad , \quad I = c\,\frac{dU_c}{dt}$$

$$E = RC\,\frac{dU_c}{dt} + U_c \;\rightarrow\; \frac{dU_c}{dt} + \frac{1}{RC}U_c = \frac{E}{RC}$$

The solution to this equation is:

$$U_C(t) = A\,e^{-\frac{t}{\tau}} + E$$

If t=0 U_c = 0 alors A = - E So

$$U_C(t) = E\left(1 - e^{-\frac{t}{\tau}}\right)$$

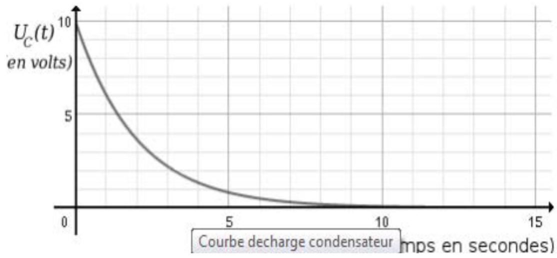
II.7.2.2 Capacitor discharge:

We flip the switch to position 2 thus, we apply a voltage E across the capacitor.

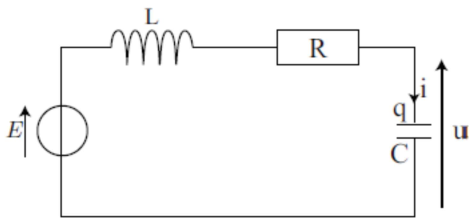
$$R\,I - U_c = 0 \;, \quad I = c\,\frac{dU_c}{dt}$$

$$U_C(t) = E\,e^{-\frac{t}{\tau}}$$

The solution to this equation is:



3.7.3 RLC transitional regime:



According to the law of meshes we have:

$$-E + RI + L\frac{dI}{dt} + \frac{1}{C}\int Idt = 0$$

Let's look for the differential equation, consider the voltage and current across the capacitor:

$$I = \frac{dq}{dt} \;, \qquad q = CU_c \; alors \; I = C\frac{dU_c}{dt}$$

$$E = RC\frac{dU_c}{dt} + LC\frac{d^2U_c}{dt^2} + U_c \quad \rightarrow \quad \ddot{U_c} + \frac{R}{L}\dot{U}_c + \frac{1}{LC}U_c = \frac{E}{LC}$$

We set $\omega_0^2 = \frac{1}{LC}$ proper pulsation as the frequency at which this system oscillates when it is in free evolution.

$2\lambda = \frac{\omega_0}{Q} = \frac{R}{L} \rightarrow Q = \sqrt{\frac{L}{C}} \times \frac{1}{R}$ Damping rate of an oscillator

$$\Omega = \sqrt{\lambda^2 - \omega_0^2}$$

1. si $\lambda < \omega_0$ *régime pseudo − périodique −* Courbe 1
2. si $\lambda > \omega_0$ *régime apériodique* - Courbe 2
3. si $\lambda = \omega_0$ *régime Critique* - Courbe 3