University of Badji-Mokhtar Annaba
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Mathematical Analysis 1

Serie 5: The derivative

<u>Exercise1</u>

Study the differentiability of the following functions in point x_0

1• $f(x) = x^2 + |x+1|, \ x_0 = 1, -1$

2• $g(x) = \begin{cases} \dfrac{x}{1+e^{1/x}} \text{ if } x \in \mathbb{R}^* \\ 0 \text{ if } x = 0 \end{cases}, x_0 = 0$ $\boxed{\times}$

<u>Exercise 2</u>: Let f be a function defined by

$$f(x) = \begin{cases} x^2 \sin \dfrac{1}{x}, & x \neq 0, \\ 0, & x = 0 \end{cases}$$

1) Study the continuity and the derivability of f on $\mathbb{R}$.

2)Show that f' is not continuous at 0.

<u>Exercise 3:</u>

1.Calculate the derivatives of functions defined by

$$f(x) = \ln(e^x); \quad g(x) = \ln(\sin^2 x); \quad h(x) = x + \sqrt{1+x^2}$$

Show that : $h'(x) = \dfrac{h(x)}{\sqrt{1+x^2}}$

2.Calculate the n^{th} derivative of

$f(x) = \sin \alpha x; \qquad g(x) = x^3 \ln(1+x),$

<u>Exercise 4:</u>

Let f be a function defined by $f(x) = e^{x^2} \cos x$

Show that for $\forall \alpha > 0$ the equation $f'(x) = 0$ admits at least one solution on $[-\alpha, \alpha]$

<u>Exercise 5:</u>

1.Show that for all real x, y we have: $|\sin x - \sin y| \leq |x - y|$

2.Show that for all $x > 0$ $\qquad \dfrac{x}{x+1} < \ln(1+x) < x.$

<u>Exercise 6:</u>

Calculate the limit using the hospital rule

$$\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}, \quad \lim_{x \to 1} \frac{x^x - 1}{\ln x - x + 1}, \quad \lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$$