Chapter 2

Dynamics of a particle

II.1. Introduction

In the previous chapter we studied the movement of bodies without taking into account the causes which provoke the movement. In this chapter (dynamics) we study the causes of movement, which are forces.

Dynamics is the analysis of the relationship between forces applied to a body and changes in the movement of this body. It explains the relationship that exists between forces and other quantities.

II.2. Definition

II.2.1. Concept of force

The movement is the result of the interaction between the particle and its environment. This interaction is the force (vector quantity).

The unit of force in SI is the Newton: $1N = 1\ Kg.m.s^{-2}$

There are two main categories of forces:

a- Contact forces: friction forces, tension forces, etc.

b- Forces at a distance: gravitational forces, electric forces, magnetic forces.

Example: a body slides on a horizontal surface by a wire.

II.2.2. Mass:

Mass is a scalar physical quantity that represents the quantity of matter which makes up a particle, and it represents the inertia of the body.

II.2.3. Material point

We call a material point or point mass a mechanical system that can be modelled by a geometric point M with which its mass m is associated.

Material system: is a set of material points.

We freely choose the system we study. Anything other than the system being studied is called the exterior.

II.2.4. Isolated or pseudo-isolated system

A system is isolated if it is not subject to any external force.

A system is pseudo isolated if the Σ of the external forces applied to this system is zero:

$$\Sigma F = 0$$

II.3. Momentum

A movement of a body does not depend only on the speed but also on its mass, two different masses which move at the same speed do not arrive in the same way. For this we introduce a quantity which is the momentum $\vec{P}$.

The momentum relative to the reference frame R of a material point M, of mass m and speed $\vec{v}$ is given by: $\vec{P} = m\vec{v}$.

Unit: kg.m/s; dimension: [momentum] = MLT^{-1}

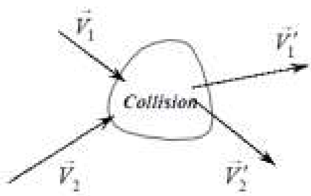
II.3.1 Conservation of quantity of movement:

If we have a system composed of *N* particles of masses m_i and speeds $\vec{V_i}$, then the total momentum of the system is given by:

$$\vec{P} = \sum_{i=1}^{N} \vec{P_i} = \vec{P_1} + \vec{P_2} + \vec{P_3} + \cdots \ldots$$

For an isolated system this momentum is constant:

$$\vec{P} = \sum_{i=1}^{N} \vec{P_i} = cst$$



II.3.2. Case of two particles in collision

Consider a system of two particles m_1 and m_2.

Before the collision the speeds are noted V_1 and V_2.

$$\vec{P} = \vec{P_1} + \vec{P_2} = m_1\vec{v}_1 + m_2\vec{v}_2$$

After the collision the speeds are noted as V'_1 and V'_2.

$$\vec{P'} = \vec{P'_1} + \vec{P'_2} = m_1\vec{v'}_1 + m_2\vec{v'}_2$$

Conservation of momentum:

$$\vec{P} = \vec{P}' \Rightarrow \vec{P_1} + \vec{P_2} = \vec{P'_1} + \vec{P'_2} \Rightarrow \vec{P_1} - \vec{P'_1} = \vec{P'_2} - \vec{P'_2}$$

$$\Rightarrow \Delta\vec{P_1} = -\Delta\vec{P_2}$$

An interaction produces an ***exchange of momentum***. The quantity of movement"**lost**" by one particle is equal to the momentum "**gained**" by the other.

II.4. Fundamental Laws of Dynamics القوانين الأساسية في الديناميك

II.4.1. 1st Newton's law 'Principle of Inertia'مبدأالعطالة

a. Statement of principle:نص المبدأ

If the material body is not subjected to any force or the vector resultant of the applied forces is zero, it is:

- in a uniform rectilinear movement (*v = cst* and *a=0*)حركة مستقيمة منتظمة
- at rest (في السكون), if it was initially at rest (*v=0*).

This property of all bodies to resist change in speed (zero acceleration) is called inertia.

Example: The movements of passengers caused by vehicles when starting and braking. تحركات الركاب الناجمة عن المركبات عند الانطلاق والفرملة.

b. Galilean frame of reference معلم غاليلي

An inertia reference frame is a reference frame in which the principle of inertia is realized. *i.e*, it keeps its inertia: it remains at rest if it is at rest and it keeps its uniform rectilinear movement as long as $\sum \vec{F} = \vec{0}$.

Note: *Any frame of reference in uniform rectilinear translation with respect to a Galilean frame of reference is itself Galilean.*

The earth's reference frame is not reallyGalilean because of its movement. But we consider it to be a Galilean reference because we carry out studies with low times.

الإطار المرجعي للأرض ليس غاليليًا حقًا بسبب حركته. لكننا نعتبره معلما غاليليًا لأننا نجري دراسات في أوقات قصيرة جدا

Example on a non-Galilean frame of reference: an object placed in a truck in uniform rectilinear motion. The body remains immobile in relation to the truck as long as the latter's movement maintains its uniform rectilinear character. When the truck executes a movement in a turn, the body would slide. Indeed, the reference linked to the truck is animated by a curvilinear movement and the principle of inertia is no longer applicable (the object would not maintain its state of rest in relation to the truck).

جسم موضوع في شاحنة تتحرك بحركة مستقيمة منتظمة. يظل الجسم ثابتًا بالنسبة للشاحنة طالما أن حركة الأخيرة تحافظ على طابعها المستقيم المنتظم. عندما تقوم الشاحنة بحركة دوران، ينزلق الجسم. في الواقع، يتم تحريك المرجع المرتبط بالشاحنة من خلال حركة منحنية ولم يعد مبدأالعطالة قابلاً للتطبيق (لن يحافظ الجسم على حالة سكونه بالنسبة للشاحنة)

Note: The principle of inertia can then be stated as follows: "A free particle moves with a constant quantity of movement in a Galilean frame of reference. »

$$\vec{P} = \overrightarrow{cst} \Rightarrow \vec{F} = \frac{d\vec{P}}{dt} = \vec{0}$$

This is another formulation of the principle of inertia.

II.4.2. 2nd Newton's Law: 'Fundamental Relation of Dynamics (FRD)'

The resultant of the forces exerted on a body is the derivative of the momentum:

$$\sum \overrightarrow{F_{ext}} = \frac{d\vec{P}}{dt} = \frac{d(m\vec{v})}{dt}$$

If the mass of the system is constant then:

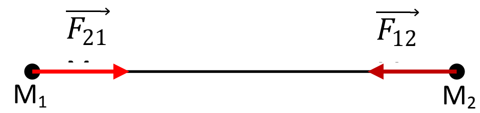
$$\sum \overrightarrow{F_{ext}} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

II.4.3. 3rd Newton's Law: 'Principle of reciprocal actions'

Given that M_1 and M_2 two material points, $\overrightarrow{F_{12}}$ and $\overrightarrow{F_{21}}$the reciprocal interaction forces, applied by M_1 on M_2 and that applied by M_2 on M_1, respectively. The principle of reciprocal actions, also called the principle of action $\overrightarrow{F_{12}}$ and the reaction $\overrightarrow{F_{21}}$, states that:

- These two actions (forces) are exerted simultaneously and are of the same nature
- These two forces are opposite $\overrightarrow{F_{12}} = -\overrightarrow{F_{12}}$ and equal in moduli $\|\overrightarrow{F_{12}}\| = \|\overrightarrow{F_{12}}\|$.
- $\overrightarrow{F_{12}}$ and $\overrightarrow{F_{21}}$are belong to the same segment $[M_1M_2]$:

  $\overrightarrow{F_{12}} \wedge \overrightarrow{M_1M_2} = \vec{0}\ ,\ \overrightarrow{F_{21}} \wedge \overrightarrow{M_1M_2} = \vec{0}$



II.5. Classification of forces

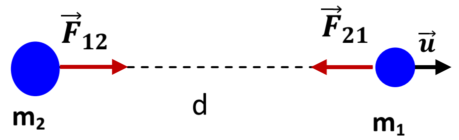
II.5.1. Forces at a distance

The body which exerts the force is not in contact with the one on which it acts. There are 3 kinds of forces at a distance:

a) Gravitational forces: this is the action of one mass (body) on another. These two bodies attract each other mutually with two opposing forces (according to Newton's 3rd law): $\overrightarrow{F_{12}} = -\overrightarrow{F_{21}}$

It's the **Law of universal gravitation** which explains attraction between two bodies of respective masses m_1 and m_2, separated by distance d.

$$F_{12} = F_{21} = F_g = G\frac{m_1 m_2}{d^2}$$



These forces are attractive.

G: the gravitational constant, G = 6.67.10-11 $[m^3/kg.s^2]$

Near the earth, the force of gravitation is what keeps objects on the ground.

b) Weight of a mass: Consider a point mass m, in gravitational interaction with Earth. The latter acts on the mass with a force that we called the weight of the mass. Newton's second law allows us to define this weight: $\vec{P} = m\vec{g}$

$\vec{g}$ is the acceleration of gravity (terrestrial acceleration), $g=9.80\ m/s^2$.

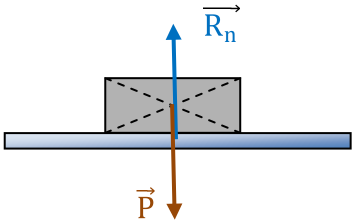
c) Electric forces: They are exerted between two bodies carrying electrical charges. They can be both attractive or repulsive.

d) Magnetic forces: They are exerted between magnets or between the latter and certain materials (particularly iron). Both can be attractive or repulsive.

II.5.2. Contact forces

There must be contact between the two objects for a contact force to arise.

a) Reaction of a support (Solid-solid contact)



The force acting on an object placed on a horizontal support is called the support reaction, $\overrightarrow{R_n}$. Represents the result of all actions performed on the contact surface. The object being in equilibrium

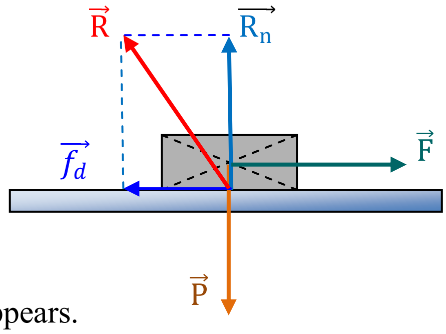
$\sum \overrightarrow{F_{ext}} = \vec{P} + \overrightarrow{R_n} = \vec{0} \Longrightarrow \vec{P} = -\overrightarrow{R_n}$

b) Friction force

Friction force is the force that opposes the movement of the body. There are two friction forces: solid and fluid.

- **Solid–Solid friction:** dynamic friction force (the body is moving):

When the solid moves under the action of an external force $\overrightarrow{F_e}$, the intensity f_d of the friction force is proportional to that of the reaction normal to the support $\overrightarrow{R_n}$.



$\overrightarrow{f_d} = \mu_d \overrightarrow{R_n}$

μ_d:the dynamic friction coefficient معامل الاحتكاك الحركي

Note:static friction force (the body is fixed) $\overrightarrow{f_s} = \mu_s \overrightarrow{R_n}$

μ_s: the static friction coefficientمعامل الاحتكاك السكوني

- **Friction forces in fluids**

When a solid body moves in a fluid (gas or liquid), a friction force appears.

It is calculated by the formula:

$$\overrightarrow{f_f} = -k\eta\vec{v}$$

k is the coefficient which depends on the shape of the solid body and η is the viscosity coefficient.

c) Tension forces:

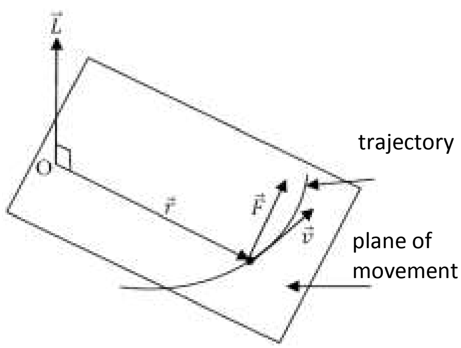
II.6. Cinematic moment (Angular momentum)

II.6.1. Angular momentum of a material point

We call angular momentum noted $(\overrightarrow{L_O})$ of point M rotating around a point O, the moment of its quantity of movement $\vec{P} = m\vec{v}$:

$\overrightarrow{L_O} = \overrightarrow{OM} \wedge \vec{P} = \vec{r} \wedge m\vec{v}$

The unit of angular momentum: Kg.m^2.s^{-1}



The angular momentum is a vector perpendicular to the plane containing the vectors $\vec{r}$and $\vec{P}$.

❒ If the movement is circular with radius r, we will have

$\vec{r} \perp \vec{v}$and $v = \omega r$

$$\overrightarrow{L_O} = \vec{r} \wedge m\vec{v} \Longrightarrow L_O = r.mv.sin\frac{\pi}{2} = rmv = mr^2\omega$$

$$\overrightarrow{L_O} = mr^2\vec{\omega}$$

❒ For a curvilinear plane movement, we use polar coordinates, with pole O:

$$\vec{v} = \dot{r}\overrightarrow{u_r} + r\dot{\theta}\overrightarrow{u_\theta}$$

$$\overrightarrow{L_O} = \vec{r} \wedge m\vec{v} = m\vec{r} \wedge \vec{v} = m\vec{r} \wedge \left(\dot{r}\overrightarrow{u_r} + r\dot{\theta}\overrightarrow{u_\theta}\right) = mr^2\dot{\theta}\vec{k} \Longrightarrow L_O = mr^2\dot{\theta}$$

II.6.2. Angular momentum theorem

At a fixed point O of a Galilean frame of reference, the derivative with respect to time of the angular momentum of a material point is equal to the sum of the moments of all the forces applied to it.

$$\frac{d\overrightarrow{L_O}}{dt} = \overrightarrow{\mathcal{M}}_{/O}\left(\sum \overrightarrow{F_{ext}}\right)$$

Proof:

$$\overrightarrow{L_O} = \vec{r} \wedge \vec{P} = \vec{r} \wedge m\vec{v} \Longrightarrow \frac{d\overrightarrow{L_O}}{dt} = \frac{d\vec{r}}{dt} \wedge m\vec{v} + \vec{r} \wedge \frac{dm\vec{v}}{dt} = \vec{v} \wedge m\vec{v} + \vec{r} \wedge \frac{md\vec{v}}{dt} = \vec{r} \wedge m\vec{a}$$

$$= \vec{r} \wedge \vec{F} = \overrightarrow{\mathcal{M}}_{/O}(\vec{F})$$

II.6.3. Conservation of angular momentum – central forces

a. Definition of a central force

We call "Central force" any force acting on a material point and having the following properties:

* It is carried by the line joining the material point to a fixed point O (center of force).

* Its module depends only on the distance "r" to the point O:

$\vec{F} = f(r)\overrightarrow{u_r}$ *and* $\overrightarrow{OM} = r\overrightarrow{u_r}$.

b. Conservation of angular momentum

The derivative of angular momentum vanishes (=0) if:

a. The particle is isolated $\sum \overrightarrow{F_{ext}} = \vec{0}$: which means that the angular momentum of a free particle is constant $\frac{d\overrightarrow{L_O}}{dt} = \vec{0} \Longrightarrow L_O = cte$

b. If the force $\vec{F}$is central: $\vec{F}$ is parallel to $\vec{r}$, so the angular momentum relative to the center of forces is constant.

$$\frac{d\overrightarrow{L_O}}{dt} = \overrightarrow{\mathcal{M}}_{/O}\left(\sum \overrightarrow{F_{ext}}\right) = \vec{r} \wedge \vec{F} = \vec{0}(\vec{r}//\vec{F}) \Longrightarrow \overrightarrow{L_O} = cte\left(\overrightarrow{L_O}\text{isconserved}\right)$$

The opposite is true;if the angular momentum is constant then the **force is central.**

II.6.4. Inertia forces or pseudo forces: a non-Galilean frame reference

Let R be a Galilean frame of reference and R' a non-Galilean frame of reference. R' is mobile relative to R. The law of composition of accelerations gives:

$\overrightarrow{a_a} = \overrightarrow{a_r} + \overrightarrow{a_e} + \overrightarrow{a_c}$

The fundamental principle of dynamics in a Galilean frame R is written:

$$\sum \overrightarrow{F_{ext}} = m\overrightarrow{a_a} = m\frac{d\overrightarrow{v_a}}{dt}$$

$\overrightarrow{a_a}$and$\overrightarrow{v_a}$ are the absolute acceleration and speed.

In the non-Galilean (relative) frame R', the fundamental principle of dynamics is:

$\overrightarrow{a_r} = \overrightarrow{a_a} - \overrightarrow{a_e} - \overrightarrow{a_c} \Rightarrow m\overrightarrow{a_r} = m\overrightarrow{a_a} - m\overrightarrow{a_e} - m\overrightarrow{a_c} \Rightarrow m\overrightarrow{a_r} = \sum \overrightarrow{F_{ext}} + \overrightarrow{F_e} + \overrightarrow{F_c}$

with $\overrightarrow{F_e} = -m\overrightarrow{a_e}$ (force of inertia of Entrainment),$\overrightarrow{F_c} = -m\overrightarrow{a_c}$ (force of Coriolis inertia) are pseudo forces or forces of 'inertia. Therefore, the law of dynamics can be applied in a non-Galilean frame of reference provided that the Entrainmentinertia force and the Coriolis inertia force are added.