Solution Examen Theorie du signal E2N2

Solution Exo. 1

5 pts

$$s_1(t) = \begin{cases} 2 & si\ -\frac{1}{2} \le t+1 \le \frac{1}{2} \longrightarrow -\frac{3}{2} \le t \le -\frac{1}{2} \\ -1 & si\ -\frac{1}{2} \le \frac{t-1.5}{3} < \frac{1}{2} \longrightarrow 0 \le t \le 3 \\ 0 & sinon \end{cases}$$

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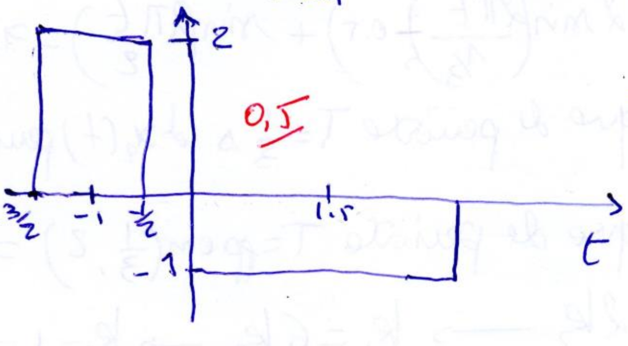


calcul de l'energie :

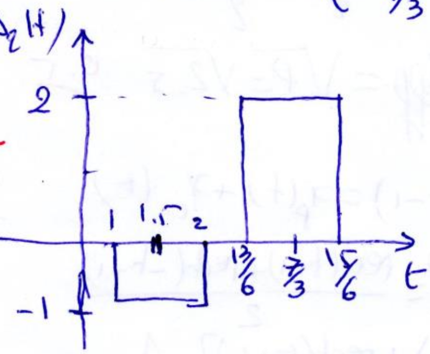
$E_1 = 2^2 \times 1 + (-1)^2 \times 3$

$= 4+3 = 7$ Joule

0.5

$$s_2(t) = s_1(-3t+6) = 2\,\text{rect}\left((-3t+6)+1\right) - \text{rect}\left(\frac{(-3t+6)-1.5}{3}\right)$$

$$= 2\,\text{rect}\left(\frac{t-7/3}{1/3}\right) - \text{rect}\,(t-1.5) \quad 1$$



Calcul de l'energie

$E_2 = (-1)^2 \times 1 + (2)^2 \times \frac{1}{3}$

$= 1 + \frac{4}{3} = \frac{7}{3}$ Joule

0.5

1 c. Conclusion : l'energie depend du facteur d'echelle (dans ce cas $\alpha = 3$) et ne depend pas de la translation du signal

$$E_2 = \frac{E_1}{\alpha} = \frac{1}{3} E_1 = \frac{7}{3} \text{ joule}$$

Solution
Exo. 2 :

(4 pts)

$$x(t) = 2\sin(6\pi t - 0.5) + \cos\left(\pi t + \frac{\pi}{2}\right)$$
$$= 2\sin(6\pi t - 0.5) + \sin(\pi t)$$
$$= 2\sin\left(\frac{2\pi t}{1/3} - 0.5\right) + \sin\left(2\pi \frac{t}{2}\right) = x_1(t) + x_2(t)$$

1 $x_1(t)$ périodique de période $T_1 = \frac{1}{3}$ s et $x_2(t)$ périodique $T_2 = 2$ s

donc $x(t)$ est périodique de période $T = \text{ppcm}\left(\frac{1}{3}, 2\right) = 2$ s

$$T = k_1 \frac{1}{3} = 2k_2 \longrightarrow k_1 = 6k_2 \longrightarrow k_2 = 1 \longrightarrow k_1 = 6$$
$$= 6 \cdot \frac{1}{3} = 2 \times 1 = \boxed{2\,s} \quad 1{,}5$$

Puissance du signal $= \frac{2^2}{2} + \frac{1^2}{2} = 2 + 0.5 = 2.5$ W 1

valeur efficace $x_{eff} = \sqrt{P} = \sqrt{2.5}$ 0.5

Solution
Exo 3

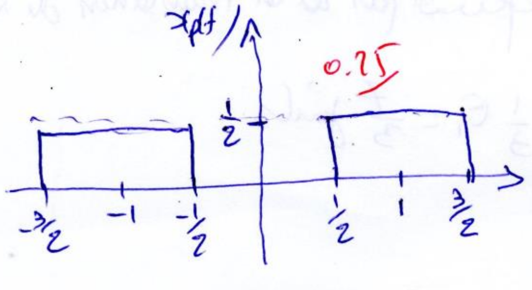
(4 pts)

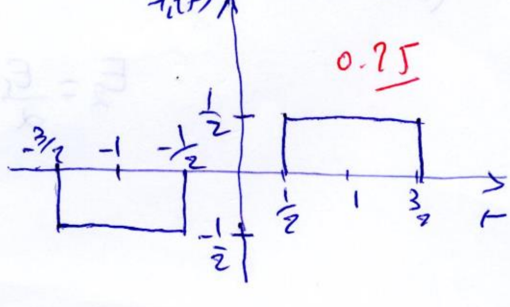
$$x(t) = \text{rect}(t-1) = x_p(t) + x_i(t)$$

$x_p(t)$: partie paire
$x_i(t)$: partie impaire

$$x_p(t) = \frac{x(t) + x(-t)}{2} = \frac{\text{rect}(t-1) + \text{rect}(-t-1)}{2}$$
$$= \frac{1}{2}\left[\text{rect}(t-1) + \text{rect}(t+1)\right] \quad 1$$

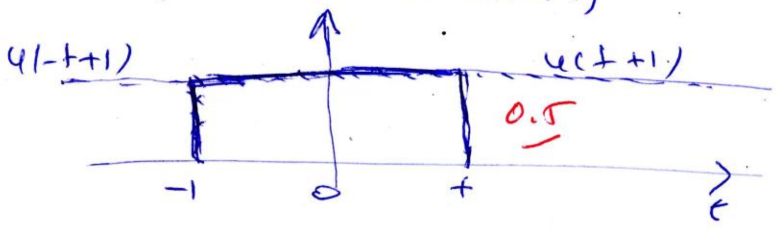
$$x_i(t) = \frac{x(t) - x(-t)}{2} = \frac{\text{rect}(t-1) - \text{rect}(-t-1)}{2} = \frac{1}{2}\left[\text{rect}(t-1) - \text{rect}(t+1)\right] \quad 1$$





suite exo. 3 solution

$$y(t) = u(-t+1) \times u(t+1)$$



$y(t) = rect\left(\frac{t}{2}\right) = rect\left(-\frac{t}{2}\right)$ paire 1

$y(t)$ n'a pas de composante impaire $y_i(t) = 0 \ \forall t$ 1

$y(t) = y_p(t)$

solution exo. 4

7pts

définition de la transformée de Fourier

$$x(t) \xrightarrow{TF} X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi f t} dt \quad 1$$

si $x(t) = \delta(t-2)$ alors $X(f) = \int_{-\infty}^{+\infty} \delta(t-2) e^{-j2\pi f t} dt = e^{-j2\pi f t}\Big|_{t=2} = e^{-j4\pi f}$

2.5

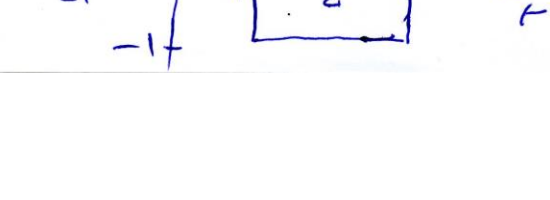
def de la convolution :

$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau) h(t-\tau) d\tau \quad 1$$

$x(t) = rect\left(\frac{t}{2}\right)$ et $h(t) = \delta(t) - \delta(t-2)$

$\rightarrow y(t) = x(t) * h(t) = x(t) * [\delta(t) - \delta(t-2)] = x(t) - x(t-2)$

$= rect\left(\frac{t}{2}\right) - rect\left(\frac{t-2}{2}\right)$ 1.5



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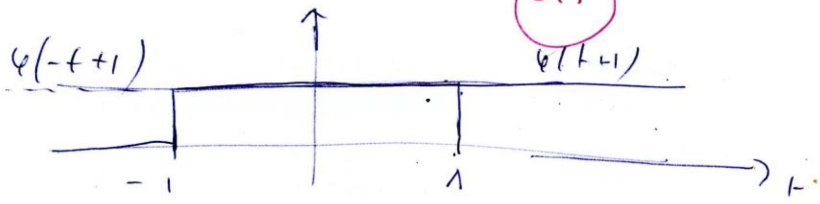
$y(t) = u(-t+1) \times u(t+1)$ 0.5



$y(t) = rect\left(\frac{t}{2}\right) = rect\left(-\frac{t}{2}\right)$ fonction paire 1

$y(t)$ n'a pas de composante impaire 1

$y_i(t) = 0 \;\forall t \quad y_p(t) = y(t).$

7pts 4.) $X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt$ 1

si $x(t) = \delta(t-2)$ alors 2.5

$$X(f) = \int_{-\infty}^{+\infty} \delta(t-2)\, e^{-j2\pi f t}\, dt = e^{-j2\pi f t}\Big|_{t=2} = \boxed{e^{-j4\pi f}}$$

<u>Convolution</u>:

1 $y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$

$x(t) = rect\left(\frac{t}{1}\right)$ et $h(t) = \delta(t) - \delta(t-2)$

$\rightarrow y(t) = x(t) * h(t) = x(t) * \delta(t) - x(t) * \delta(t-2)$

$= x(t) - x(t-2) = rect\left(\frac{t}{1}\right) - rect\left(\frac{t-2}{2}\right)$

1.5 $= rect\left(\frac{t}{1}\right) - rect\left(\frac{t-2}{2}\right)$