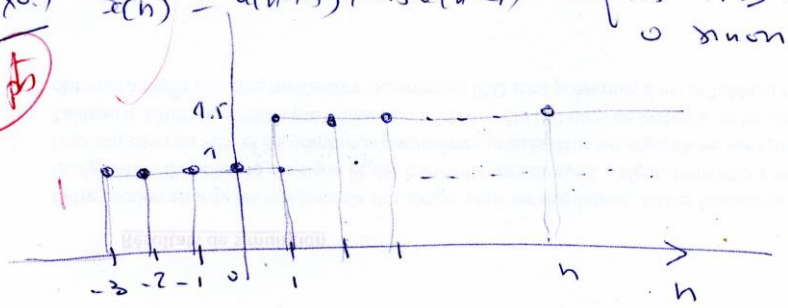
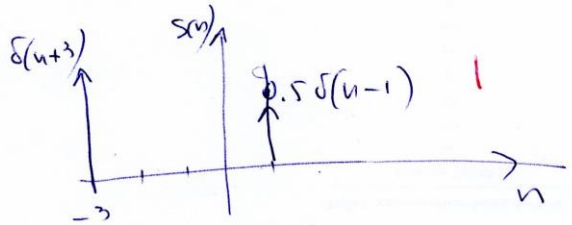
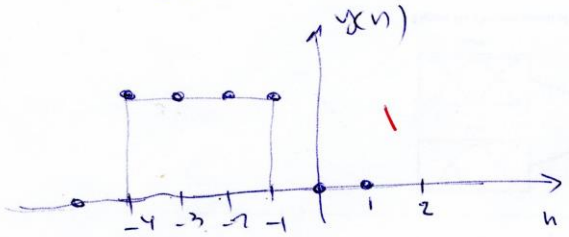
Solution

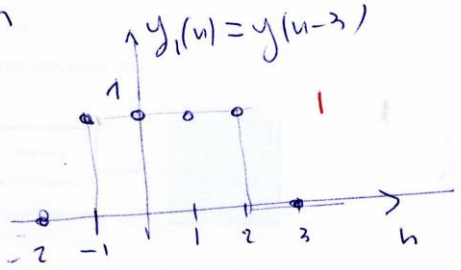
Exo 1

$x(n) = u(n+3) + 0.5u(n-1) = \begin{cases} 1 & -3 \le n \le 0 \\ 1.5 & n \ge 1 \\ 0 & \text{sinon } (n < -3) \end{cases}$









Exo 2

$y(n) = \frac{1}{3}(x(n) + x(n-1) + x(n-2))$

$Y(j\Omega) = \frac{1}{3}\left(X(j\Omega) + e^{-j\Omega}X(j\Omega) + e^{-j2\Omega}X(j\Omega)\right)$ 0.5

$H(j\Omega) = \frac{Y(j\Omega)}{X(j\Omega)} = \frac{1}{3}\left(1 + e^{-j\Omega} + e^{-j2\Omega}\right)$ 0.5

$= \frac{1}{3}e^{-j\Omega}\left(e^{j\Omega} + 1 + e^{-j\Omega}\right)$ 0.5

$= \frac{1}{3}e^{-j\Omega}\left(2\cos\Omega + 1\right)$ 0.5

$= |H(j\Omega)| \angle H(j\Omega) \longrightarrow |H(j\Omega)| = \frac{1}{3}(1 + 2\cos\Omega)$

$\angle H(j\Omega) = -\Omega$ 1

Exo3 TFD: $\{2, 4, -1, 6\} = x(n)$

4pts $N = 4$ 0.5

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{3} x(n) e^{-j\frac{\pi}{2}kn}$$ 0.5

$X(0) = 2 + 4 - 1 + 6 = 11$ 0.5

$X(1) = 2 + 4e^{-j\frac{\pi}{2}} - e^{-j\frac{\pi}{2}\times 2} + 6e^{j\frac{3\pi}{2}}$

$= 2 - 4j + 1 + 6(-j) = 3 - 10j$ 1

$X(2) = x(0) + x(1)e^{-j\pi} + x(2)e^{-j2\pi} + x(3)e^{-j3\pi}$

$= 2 - 4j - 1 + 6(-1) = -5 - 4j$ 1

$X(3) = 3 + 10j$ 0.5

Exo.4

a.) $x(n) = 4\left(\frac{1}{3}\right)^n u(n) - \left(\frac{2}{3}\right)^n u(n)$

$$X(z) = 4\frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{1-\frac{2}{3}z^{-1}} = 4\frac{z}{z-\frac{1}{3}} - \frac{z}{z-\frac{2}{3}}$$ 1

b.) $h(n) = 3\left(\frac{2}{3}\right)^n u(n) \longrightarrow H(z) = 3\frac{z}{z-\frac{2}{3}}$ 0.25

$x(n) = \left(\frac{1}{2}\right)^n u(n) \longrightarrow X(z) = \frac{z}{z-\frac{1}{2}}$ 0.25

$\longrightarrow Y(z) = X(z)H(z) = 3\frac{z}{z-\frac{2}{3}} \cdot \frac{z}{z-\frac{1}{2}}$ 0.5

$\frac{Y(z)}{z} = 3\frac{z}{(z-\frac{2}{3})(z-\frac{1}{2})} = \frac{A}{z-\frac{2}{3}} + \frac{B}{z-\frac{1}{2}}$ 0.5

$A = \left.3\frac{z}{z-\frac{1}{2}}\right|_{z=\frac{2}{3}} = 3\cdot\frac{2}{3}\cdot\frac{1}{\frac{2}{3}-\frac{1}{2}} = 2\times\frac{1}{\frac{1}{6}} = 12$ 0.5

$$B = \left.\frac{3z}{z-\frac{2}{3}}\right|_{z=\frac{1}{2}} = 3\cdot\frac{1}{2}\cdot\frac{1}{\frac{1}{2}-\frac{2}{3}} = 3\cdot\frac{1}{2}\cdot\frac{1}{\frac{-1}{2\times 3}} = -9 \quad 0.5$$

$$\mathcal{Z}^{-1}Y(z) = \frac{12}{z-\frac{2}{3}} - \frac{9}{z-\frac{1}{2}} \longrightarrow Y(z) = 12\frac{z}{z-\frac{2}{3}} - 9\frac{z}{z-\frac{1}{2}}$$

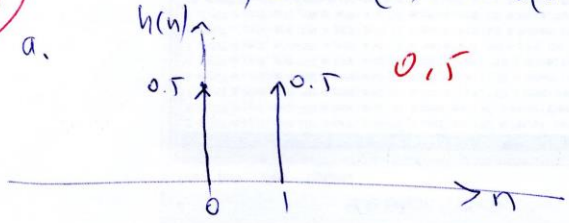
1 $\boxed{y(n) = 12\left(\frac{2}{3}\right)^n u(n) - 9\left(\frac{1}{2}\right)^n u(n)}$

4pts

Exo 1

$$h(n) = 0.5\delta(n) + 0.5\delta(n-1)$$

a.



0,5

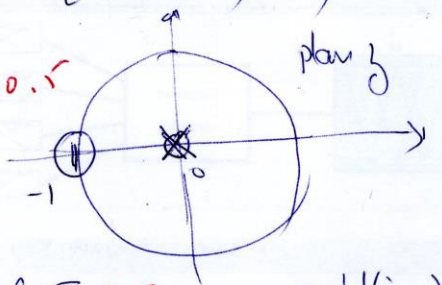
$$y(n) = x(n) * h(n) = x(n) * (0.5\delta(n) + 0.5\delta(n-1))$$
$$= 0.5\left(x(n)*\delta(n) + x(n)*\delta(n-1)\right)$$

1 $\boxed{y(n) = \frac{1}{2}\left(x(n) + x(n-1)\right)}$

$$Y(z) = \frac{1}{2}\left(1+z^{-1}\right)X(z) \longrightarrow H(z) = \frac{1}{2}\left(1+z^{-1}\right) = \frac{1}{2}\left(\frac{z+1}{z}\right)$$

1 zéro $z=-1$ 0,5

1 pole $z=0$ 0,5



0,5

f. filtre passe-bas 0,5

d. filtre RIF 0.5

e. $H(j\Omega) = TFTD(h(n)) = \frac{1}{2}\left(1+e^{-j\Omega}\right)$ 0.5

$|H(j\Omega)| = \left|\frac{1}{2}(1+\cos\Omega)\right| e^{\frac{-j\Omega}{2}} = e^{\frac{-j\Omega}{2}}\left(\frac{e^{\frac{j\Omega}{2}}+e^{\frac{-j\Omega}{2}}}{2}\right)$ 0.5