4. Tutorial exercises - First Series - Undirected graphs

Exercise 1

Consider a group of 10 people. The friendship relations between the pairs of people are shown in the following table.

1	2	3	4	5	6	7	8	9	10
3,6,7	6,8	1,6,7	5,10	4,10	1,2,3,7	1,3,6	2	-	4,5

1. Give a graph G that models this situation.
2. Give a pendant vertex and an isolated vertex in G.
3. Check whether at least two persons have the same number of friends. Could you explain that?

Exercise 2

A non-increasing sequence of integers is said to be graphical if there exists a graph with degrees that correspond to this series.

- Are the following series graphical $(3, 3, 2, 1, 0)$ and $(3, 3, 2, 2, 0)$? Justify your answers.
- If so, provide a simple graphs that correspond to the series.

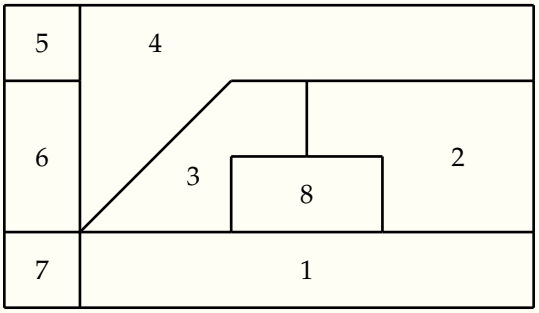
Exercise 3

Three countries have to send two spies to a conference. Each agent should spy on every foreign spy (but not on his own colleague).

1. Use a graph to represent this situation.
2. Give the degree of each vertex. Deduce the size of the graph.

Exercise 4

Eight countries along with their borders are depicted by the following figure:



When two countries share a border, they are considered as neighbors. Use a simple graph to model this situation.

Exercise 5

A company would like to organize four conferences involving seven department managers. Each manager can attend in many conferences, as shown in the table below:

Participant	Conferences	Participant	Conferences
R1	C1,C2,C3	R5	C1,C3
R2	C2,C4	R6	C1,C3
R3	C2,C4	R7	C2,C4
R4	C1,C2		

Give the graph G of the conferences that cannot be held simultaneously. Deduce the graph of the conferences that can be held at the same time.

Exercise 6

Six palyers compete in a chess tournament. Each player has to confront the other players.
1. Build a graph that corresponds to all possible duels.
2. What kind of graph do you get?

Exercise 7

In a workshop, 5 workers can perform 1 to 4 tasks according to the following table:

Worker	Tasks
1	1,2
2	2,4
3	2,3
4	2,3
5	3,4

1. Use a bipartite graph to represent all possible ways to assign workers to the tasks.
2. Give a graph in which any worker can perform all of the tasks.

Exercise 8

Consider the class of undirected simple graphs with all vertices having a degree of three (known as 3-regular graphs).
1. Give 3-regular graphs with four, five, six, and seven vertices.
2. What conclusions can you make?
3. Find the size of a 3-regular graph with n vertices. Give a conclusion.

Exercise 9

In an archipelago, bridges bind the islands together. Each island has an even number of bridges. Furthermore, any island may be reached from any other one. One day, there was maintenance on one of the bridges, but we found that traveling between all islands is still feasible.

Use the graph concepts to model this situation, then explain the last observation.

Exercise 10

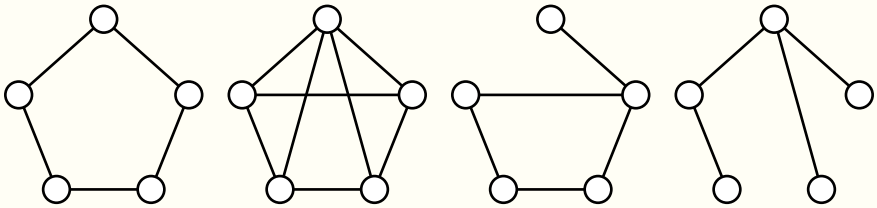
- Give the planar graphs of K_4 and $K_{2,3}$. In each situation, check for the planarity formula.
- Check the planarity of the graph in exercise 4 then build its dual graph.

Exercise 11

Let V1, V2, and V3 be three residences that need to be piped into a water supply service (W), an electricity factory (E), and a gas factory (G). Is it possible to connect these pipes without crossing them, except at the ends?

Exercise 12

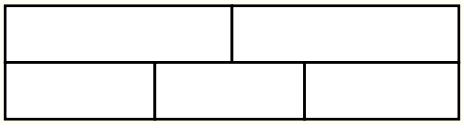
Determine which of the following graphs are Hamiltonian and/or Eulerian.



G_1 G_2 G_3 G_4

Exercise 13

Consider the following scheme.



Is it possible to draw a curve that only passes once over each of the scheme's 16 segments without lifting the pen?

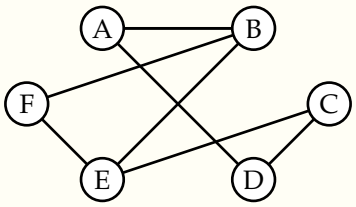
Exercise 14

A Gray code is an integer binary code where each integer differs by one bit from its successor. In this case, the binary representation does not match to its value.

1. Build a graph with vertices representing all possible three-bit codes, with each vertex differing by one bit from its adjacent vertices.
2. If such a graph contained a Hamiltonian cycle, what would that mean?
3. What is the nature of the resulting graph?

Exercise 15

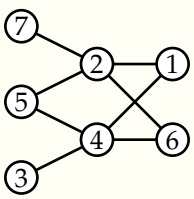
Six people are invited to a wedding dinner. The graph below shows those people's mood incompatibility. An edge connecting two persons indicates they cannot sit near to one other.



Propose a seating arrangement in which no one will sit next to someone who is incompatible with him (we assume that the table is circular).

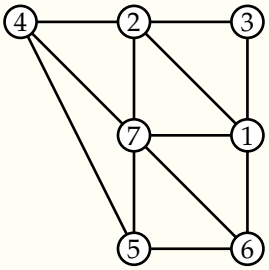
Exercise 16

Give the adjacency matrix and the incidence matrix of the following graph:



Exercise 17

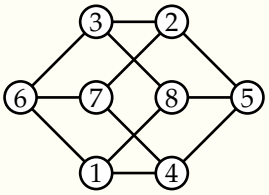
Consider the following graph:



1. Give the upper and lower bounds for the chromatic number.
2. Use Welsh and Powell's algorithm to colorize this graph.
3. Is the result consistent with the nature of the graph?

Exercise 18

Consider this graph:



1. Give the upper and lower bounds for the chromatic number.
2. Prove that the graph is bipartite. Deduce its chromatic number.
3. Since all vertices have the same degree, use Welsh and Powell's algorithm in the same order as the vertices' names. What results do you get? Comment.

Exercise 19

A zoo contains six species of fish. Unfortunately, certain species cannot be kept in the same aquarium, either because they would attack each other or because they require specific conditions (temperature, etc.). The table below highlights the incompatibilities between them:

Fish species	A	B	C	D	E	F
Incompatibility	B, C	A, C, E	A, B, D, E	C, F	B, C, F	D,E

1. Give a graph to model this situation.
2. What is the minimal number of aquariums required to keep all of the species?

Exercise 20

Assume $T = \{t_1, t_2, t_3, t_4, t_5\}$ a set of 5 tasks and $M = \{m_1, m_2, m_3, m_4, m_5\}$ a set of 5 machines. As illustrated by this table, every task requires the use of many machines.

t_1	t_2	t_3	t_4	t_5
m_1, m_3, m_5	m_1, m_2, m_4	m_2, m_3, m_5	m_2, m_4	m_5

The required time for performing a task using a given machine is always the same. But if two tasks are using the same machine, they cannot be performed simultaneously.

1. Use a graph to model this problem.
2. What are the tasks that cannot be performed simultaneously.
3. What is the shortest possible time required to complete all tasks?