University of Badji-Mokhtar Annaba
Faculty of Technologie
Computer Science Department Annaba 2024-2025
Mathematical Analysis 1

Serie 1: the Field of real numbers

Exercise 1: Show that:
1• $\left(\sqrt{x}-\sqrt{y}\right)\left(\sqrt{x}+\sqrt{y}\right)\in\mathbb{Q}$
such that $(x,y)\in(\mathbb{Q},\mathbb{Q})$ and $\left(\sqrt{x},\sqrt{y}\right)\notin(\mathbb{Q},\mathbb{Q})$
2• $\sqrt{3}\notin\mathbb{Q}$.
3• $0,336433643364\in\mathbb{Q}$.

Exercise 2: Show the following properties :
1)$\forall x\in\mathbb{R},\forall n\in\mathbb{N}: E(x+n)=E(x)+n$
2)$\forall x\in\mathbb{R},\forall n\in\mathbb{N}^*: E(\dfrac{E(nx)}{n})=E(x)$.
3)For $x\in\mathbb{R}$,we define $F(x)=E[10x]$
using the integer part, show that $F(x)=F(y)\Longrightarrow|x-y|<\frac{1}{10}$

Exercise 3: Prove for every $a,b\in\mathbb{R}$.that
1) $|a|+|b|\leq|a+b|+|a-b|$
2) $||a|-|b||\leq|a-b|$

Exercise 4:
Find the Sup, Inf, Max, Min (if they exist) of the given set
1)The set $A=\{[-3,1[\cap\mathbb{Q}.\}$
2)The set of all nonnegative integers.
3)The set $B=\{x\in\mathbb{R}:0<x\leq 3\}$.
4)The set $C=\{x\in\mathbb{R}:x^2-2x-3<0\}$

Exercise 5:
A and B two parts bounded in $\mathbb{R}$,show that:
1) $\sup(A\cup B)=\max(\sup A,\sup B)$
2) $\inf(A\cup B)=\min(\inf A,\inf B)$
We consider the set
$C=\left\{1+(-1)^{n+1}+(-\frac{1}{2})^n, n\in\mathbb{N}\right\}$
Determines $\sup(C)$ and $\inf(C)$.

Exercise 6: Using the characterization of Supremum and infimum find sup,inf, max and min (if they exist) of the given set
$A=\left\{\dfrac{n-1}{n}/\quad n\in\mathbb{N}^*\right\}$, $B=\left\{\dfrac{1}{n}+\dfrac{1}{n^2}/\quad n\in\mathbb{N}^*\right\}$,

Additional exercises

Exercise 1:

Let $(a,b) \in \mathbb{Q}^+ \times \mathbb{Q}^+$ such that $\sqrt{ab} \notin \mathbb{Q}$

Prove that $\sqrt{a} + 3\sqrt{b} \notin \mathbb{Q}$ (irrational).

Exercise 2:

1) Show the following properties :

$$\begin{cases} \forall x \in \mathbb{Z}, \quad E(x) + E(-x) = 0 \\ \forall x \in \mathbb{R}\backslash\mathbb{Z}, \ E(x) + E(-x) = -1. \end{cases}$$

2) $\forall x \in \mathbb{R} : \ E(\frac{x}{2}) + E(\frac{x+1}{2}) = E(x).$

Exercise 3:

1) Prove the following inequalitie:

- $\forall x, y \in \mathbb{R}, \ \ ||x| - |y|| \leq |x + y|$
- $\forall\,(x,y) \in \mathbb{R}^2 : 1 + |xy - 1| \leq (1 + |x - 1|)\,(1 + |y - 1|)$

Exercise 4:

Put the following sets in the form of an interval of $\mathbb{R}$ or a union of intervals.

. $A1 = \{x \in R, x^3 \leq 1\}$. $\quad A2 = \left\{x \in R, -1 \leq \dfrac{2x}{x^2+1} < 1\right\}$

$A3 = \{x \in R, \dfrac{2}{|x-1|} > 1\}$.

Exercise 5:

For any set A define the set $-A = \{\ y \in R\ /\exists x \in A$ such that. $y = -x\ \}$.

Prove that if $A \subset R$ is non-empty and bounded then

$$\sup(-A) = -\inf(A)$$

Exercise 6:

Let $\quad E_1 = \{x \in \mathbb{R}, e^x < \frac{1}{2}\}, \ \ E_2 = .\{E(\frac{1}{n}), n \in \mathbb{N}^*\}$

Are E_1, E_2,bounded below? bounded above?, do they admit Inf, Min? Sup, Max?.

Exercise 7:

Using the characterization of Supremum and infimum find sup,inf, max and min (if they exist) of the given set:

$$A = \{\frac{n-2}{n+1}; n \in \mathbb{N}^*\}, \quad B = \{x^2 + 1; x \in]1, 2]\}$$