



Physics 1: Series 2

Vector calculus

Exercise 1

Let the vectors in space be represented in an orthonormal coordinate system $R(OXYZ)$,

$$\vec{V}_1 = 2\vec{i} - 3\vec{j} + \vec{k} \quad \text{and} \quad \vec{V}_2 = -\vec{i} + 2\vec{j} + \vec{k}$$

1. Represent these vectors in the reference $R(OXYZ)$.
2. Calculate $\vec{S} = \vec{V}_1 + \vec{V}_2$ and the modules: $\|\vec{V}_1\|$, $\|\vec{V}_2\|$ and $\|\vec{S}\|$.
3. Calculate the scalar product of \vec{V}_1 and \vec{V}_2 and deduce the angle between them.
4. Determine the unit vector carried by the vector \vec{V}_2 . Deduce the direction cosines of \vec{V}_2 .
5. Determine the unit vector perpendicular to the plane (\vec{V}_1, \vec{V}_2)

Exercise 2

Consider the points $A(1,0,-1)$, $B(-1,2,1)$, $C(2,1,3)$ and $D(0,1,0)$ in the frame $(OXYZ)$.

- 1- Determine the components and magnitudes of the vectors \vec{AB} , \vec{AC} and \vec{AD} .
- 2- Determine the projection and the vector projection of \vec{AB} on \vec{AC} .
- 3- Calculate the surface area of triangle ABC and the volume constituted by \vec{AB} , \vec{AC} and \vec{AD} .

Exercise 3

a. Given the two vectors $\vec{A} = \begin{pmatrix} 1 \\ \alpha \\ \beta \end{pmatrix}$, $\vec{B} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$.

Find the values of α and β such that \vec{B} is parallel to \vec{A} .

- b. Determine the value of the number a for which the vectors $\vec{V}_1 = 2\vec{i} + a\vec{j} + \vec{k}$ and $\vec{V}_2 = 4\vec{i} - 2\vec{j} - 2\vec{k}$ are perpendicular.

Exercise 4

In the frame $R(O, \vec{i}, \vec{j}, \vec{k})$ we give the sliding vector $\vec{V} = \vec{i} + 2\vec{j} + 3\vec{k}$ and which passes through the point $A(3, 4, 2)$.

1. Calculate the moment of the vector \vec{V} relative to the origin O , then relative to the axes OX and OY .
2. Calculate the moment of vector \vec{V} relative to point $B(3, 6, 0)$
3. Consider the (Δ) axis of unit vector $\vec{u}(-1/\sqrt{2}, 1/2, 1/2)$ and passing through B , calculate the moment of \vec{V} relative to (Δ) .