5. Tutorial exercises - Second Series - Directed graphs

Exercise 1

In a chess game, a player makes the following moves using the knight:

$$B1 \rightarrow A3 \rightarrow B5 \rightarrow C3 \rightarrow A4 \rightarrow C3 \rightarrow D5$$

Model these moves by using a directed graph

Exercise 2

Consider the relation R defined on the first ten natural numbers.

$$x \; R \; y \leftrightarrow x \text{ divides } y$$

1. Use a directed graph to model this relation.
2. Explain how to answer the following questions using your graph:
   - What are the even numbers?
   - What are the prime numbers?

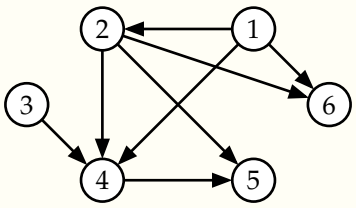
Exercise 3

Assume we have two buckets with a capacity of 4 liters and 3 liters, respectively. We wish to put exactly 2 liters of water in the first bucket.

Describe how to solve this problem using a directed graph.

Exercise 4

Consider the following graph.

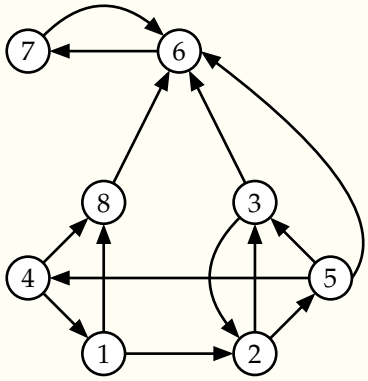


1. Find the indegrees and the outdegrees of each vertex.
2. What is the degree of this graph?
3. Give its adjacency matrix and its adjacency lists.
4. Assume $X = \{1, 2, 5\}$ and $V = \{(1, 4), (2, 5), (4, 5)\}$. What is the subgraph induced by X? What is the partial graph induced by V?

Exercise 5

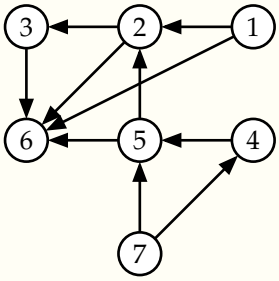
Assume G is the graph given below. Answer the following questions:

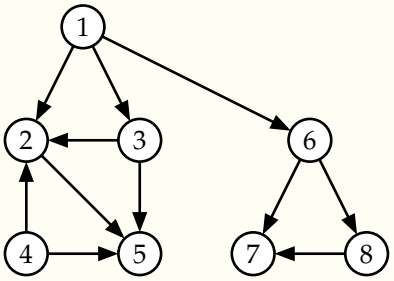
1. Is G strongly connected? Justify your answer.
2. Find its strongly connected components by applying the marking algorithm.
3. Give the reduced graph of G.



Exercise 6

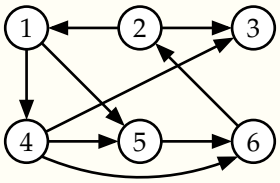
Using topological sorting, determine the levels of the following graphs:





Exercise 7

Consider the graph G given by:



Prove that G includes a circuit and then give one of its circuits.