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Chapter 2: Numbering systems

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Lessons Objectives

- Definition
- System of base b (base 2, 8, 10 and 16)
- Conversions
- Fractional numbers conversion
- Binary addition and subtraction

Definition

Definition

- The number system is the mathematical notation for representing numbers.
- It defines how numbers are expressed and written using a consistent set of digits or symbols.
- It defines how numbers are manipulated using arithmetic operations like addition, subtraction, etc.
- Number systems differ based on their base, that is also called radix.

Definition

- The base of a number system refers to the total number of unique digits that are actually used in the given number system to represent numbers.
- **Example**:
- ✓ Decimal number system or base 10: has ten digits.
- ✓ These digits would be 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
- ✓ With range : [0, base-1] = [0, 9].

System of base b

System of base b

- Several number systems are commonly used in different fields, which are :

Number System	Digits Used	Base
Decimal Number System	(Total 10 digits) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9	10
Binary Number System	(Total 2 digits) 0, 1	2
Octal Number System	(Total 8 digits) 0, 1, 2, 3, 4, 5, 6, 7	8
Hexadecimal Number System	(Total 16 digits) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F	16

- A number expressed in base 8 could be presented as follows : $(756)_8$

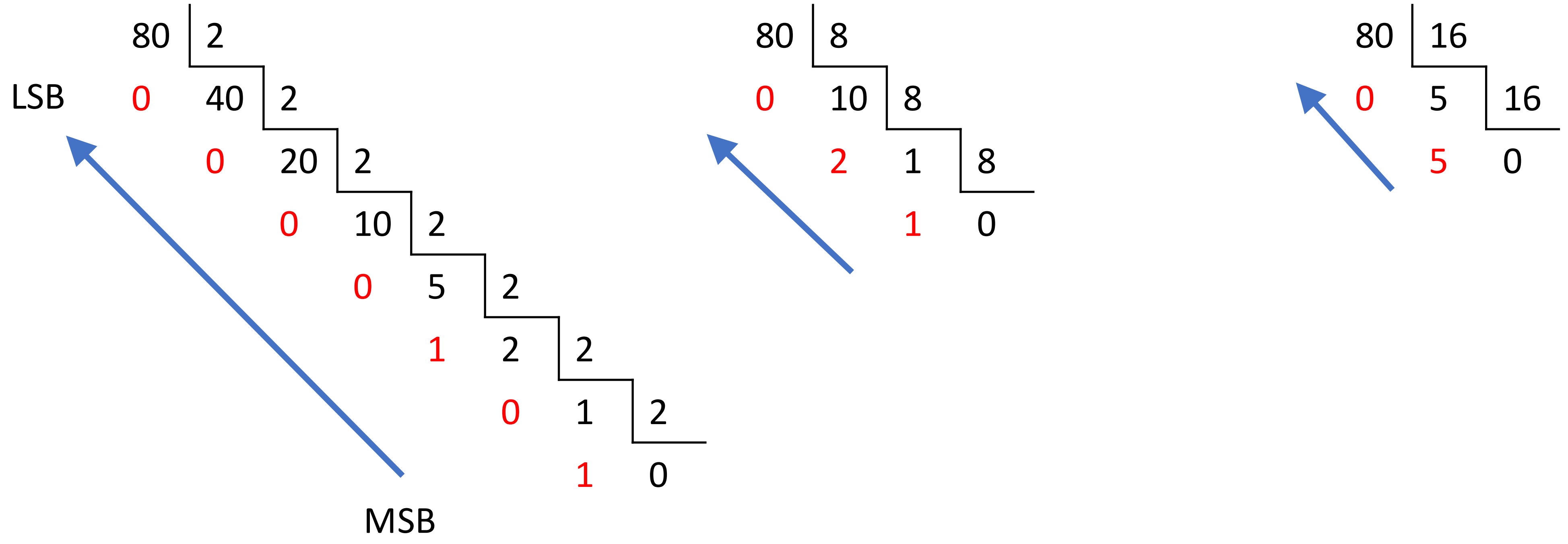
Conversions

Conversion (decimal to base b)

- **Use the division method**:
  - ✓ Divide the dicimal number by the base that we want to convert to.
  - ✓ Repeat the division using the quotient from previous division, and divide it by the base.
  - ✓ Continue until the quotient becomes zero.
  - ✓ Read the remainders in reverse order to obtain the number in the new base (i.e. Most Significant Bit (MSB) to Least Significant Bit (LSB)) .

Conversion (decimal to base b)

- **Example**: Convert $(80)_{10}$ to binary (base 2), octal (base 8), and hex (base16).



$$(80)_{10} = (1010000)_2 = (120)_8 = (50)_{16}$$

Conversion (decimal to base b)

- **Example**: Convert $(95)_{10}$ to binary (base 2), octal (base 8), and hex (base16).

$$(95)_{10} = (?)_2 = (?)_8 = (?)_{16}$$

Conversion (decimal to base b)

- **Example**: Convert $(95)_{10}$ to binary (base 2), octal (base 8), and hex (base16).

$$(95)_{10} = (1011111)_2 = (137)_8 = (5F)_{16}$$

Conversion (base b to decimal)

- **Use the multiplication method**:
  - ✓ Identify the value of each digit.
  - ✓ Multiply each digit with reducing the power of the base number (starting from 0 on the right).
  - ✓ Sum all the products to get the decimal value.

Conversion (base b to decimal)

- **Examples**:

✓ Convert $(1101001)_2$ into a decimal number:

$(1101001)_2$ = $(1 * 2^6) + (1 * 2^5) + (0 * 2^4) + (1 * 2^3) + (0 * 2^2) + (0 * 2^1) + (1 * 2^0)$

= 64+32+0+8+0+0+1 = $(105)_{10}$

✓ Convert $(2024)_8$ into a decimal number:

$(2024)_8$ = $(2 * 8^3) + (0 * 8^2) + (2 * 8^1) + (4 * 8^0)$= 1024+0+16+4= $(1044)_{10}$

✓ Convert $(5B9)_{16}$ into a decimal number:

$(5B9)_{16}$ = $(5 * 16^2) + (B * 16^1) + (9 * 16^0)$= 1280+176+9= $(1465)_{10}$

Conversion (base b to decimal)

- **Examples**:

- ✓ Convert $(11001011)_2$, $(7620)_8$, and $(AF8)_{16}$ into a decimal number:

Conversion (base b to decimal)

- **Examples**:

- ✓ Convert $(11001011)_2$, $(7620)_8$, and $(AF8)_{16}$ into a decimal number:
- ✓ $(11001011)_2 = (203)_{10}$
- ✓ $(7620)_8$ = $(3984)_{10}$
- ✓ $(AF8)_{16}$= $(2808)_{10}$

Conversion (base p to base q)

1. Covert base p to Decimal then Decimal to base q:

Example: $(725)_8$ = $(?)_{16}$:

✓ **Octal to Decimal** : $(725)_8 = (469)_{10}$ (Using the multiplication method)

✓ **Decimal to Hexadecimal** : $(469)_{10}$= $(1D5)_{16}$ (Using the division method)

2. Covert base p to Binary then Binary to base q (when p and q are powers of 2):

Example: $(725)_8$ = $(?)_{16}$:

✓ **Octal to Binary**: 7 =111 ; 2= 010; 5= 101 $\longrightarrow$ 111010101

✓ **Binary to Hexadecimal:** group 4 bits from left**: 0001 1101 0101 =** $(1D5)_{16}$

Fractional numbers conversion

Fractional numbers conversion

➢ **Conversion base b to decimal:**

✓ The fractional number can be presented as:

$(N)_B = a_n a_{n-1}\ a_{n-2}\ \ldots..a_1\ a_0,\ b_1\ b_2\ \ldots.\ b_{m-2}\ b_{m-1}\ b_m$

✓ The decimal value can be calculated as:

$a_n B^n + a_{n-1}\ B^{n-1} + \cdots + a_1 B^1 + a_0 + b_1 B^{-1} + b_2 B^{-2} + \cdots + b_m B^{-m}$

✓ **Example:**

$126.125 = 1 * 10^2 + 2 * 10^1 + 6 * 10^0 + 1 * 10^{-1} + 2 * 10^{-2} + 5 * 10^{-3}$

$= 100+20+6+0.1+0.02+0.005 =$

Fractional numbers conversion

- **Example:**

✓ Convert $(1100.1011)_2$ into a decimal number:

$(1100.1011)_2$ = $1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 0 * 2^0 + 1 * 2^{-1} + 0 * 2^{-2} + 1 * 2^{-3} +$ $1 * 2^{-4}$ = 8+4+0+0+0.5+0+0.125+0.0625 = $(12.6875)_{10}$

✓ Convert $(250.56)_8$ into a decimal number:

$(250.56)_2 = 2 * 8^2 + 5 * 8^1 + 0 * 2^0 + 5 * 8^{-1} + 6 * 8^{-2} =$ 128+ 40+ 0+ 0.625+ 0.09375 = 168.71875

Fractional numbers conversion

- **Conversion decimal to base b:**
- **For the real part:** we use the same division method for integers.
- **For the fractional part:**
  - ✓ Multiply the fractional part by base B.
  - ✓ Extract the integer part of the product as the first digit.
  - ✓ Repeat the process by removing the integer part and multiply the remaining fractional part by base B, then extract the next integer part.
  - ✓ Repeat this until the fractional part becomes 0.
  - ✓ Concatenate of the integer parts obtained in the order to their calculations.

Fractional numbers conversion

- **Example:**

✓ Convert $(14.6875)_{10}$ into a binary number, octal number, and hexa number :

✓ **For the real part :** 14= $(1110)_2$

14	2			
0	7	2		
	1	3	2	
		1	1	2
			1	0

✓ **For the fractional part:**

0.6875 * 2 = 1 + 0.375

0.375 * 2 = 0 + 0.75

0.75 * 2 = 1 + 0.5

0.5 * 2 = 1

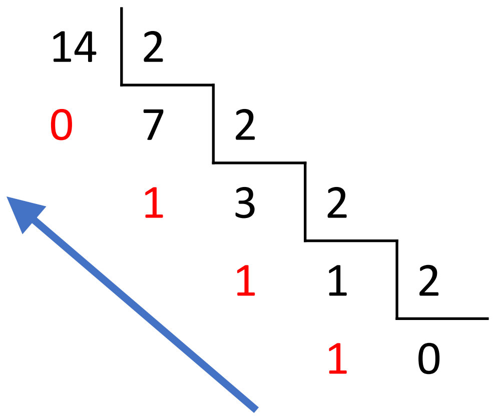
$$(14.6875)_{10} = (1110.1011)_2 = (?)_8 = (?)_{16}$$

Fractional numbers conversion

- **Example:**

✓ Convert $(14.6875)_{10}$ into a binary number, octal number, and hexa number :

✓ **For the real part :** 14= $(1110)_2$



✓ **For the fractional part:**

0.6875 * 2 = 1 + 0.375

0.375 * 2 = 0 + 0.75

0.75 * 2 = 1 + 0.5

0.5 * 2 = 1

$$(14.6875)_{10} = (1110.1011)_2 = (16.54)_8 = (E.B)_{16}$$

Binary addition and subtraction

Binary addition

- Use same basics as adding decimal numbers
- Start from the rightmost bit
- Follow the binary addition rules:
  - 0 + 0 = 0
  - 1 + 0 = 1
  - 1 + 1 = 10 (which is 0, carry 1)
  - 1 + 1 + 1 = 11 (which is 1, carry 1)

Binary addition

- Example: **1011 + 1101 = ?**

$$
\begin{array}{ccccc}
 & \color{red}{1} & \color{red}{1} & \color{red}{1} & \\
 & 1 & 0 & 1 & 1 \\
+ & 1 & 1 & 0 & 1 \\
\hline
\color{blue}{1} & \color{blue}{1} & \color{blue}{0} & \color{blue}{0} & \color{blue}{0}
\end{array}
$$

Binary addition

- **Example: 1011 + 1101 = 11000 (11+13=24)**

$$\begin{array}{cccccc}
 & 1 & & 1 & 1 & \\
 & 1 & 0 & 1 & 1 \\
+ & 1 & 1 & 0 & 1 \\
\hline
1 & 1 & 0 & 0 & 0
\end{array}$$

- **10111 + 10011 = ?**

Binary addition

- Example: 1011 + 1101 = 11000 (11+13=24)

$$
\begin{array}{cccccc}
 & 1 & 1 & 1 & & \\
 & & 1 & 0 & 1 & 1 \\
+ & & 1 & 1 & 0 & 1 \\
\hline
 & 1 & 1 & 0 & 0 & 0
\end{array}
$$

- 10111 + 10011 = 101010

Binary subtraction

- Use same basics as subtracting decimal numbers
- Start from the rightmost bit
- Follow the binary subtraction rules:
  - ✓ 0 - 0 = 0
  - ✓ 1 - 0 = 1
  - ✓ 1 - 1 = 0
  - ✓ 0 - 1 = 1 and borrow 1 from the next bit

Binary subtraction

- Example: 1101 - 1010 = ?

$$\begin{array}{ccccc} & & & {}^{1} & \\ & 1 & 1 & 0 & 1 \\ - & 1 & {}^{1}0 & 1 & 0 \\ \hline & 0 & 0 & 1 & 1 \end{array}$$

Binary subtraction

- Example: 1101 - 1010 = 0011 (13-10=3)

$$
\begin{array}{cccccc}
 & & & ^{1} & & \\
 & 1 & 1 & 0 & 1 \\
- & 1 & ^{1}0 & 1 & 0 \\
\hline
 & 0 & 0 & 1 & 1
\end{array}
$$

Binary subtraction

- **Example: 1100 - 111= ?**

Binary subtraction

- Example: 1100 - 111= **0101**

Binary Multiplication and Division

Binary Multiplication

- Use same basics as multiply decimal numbers
- Start from the rightmost bit
- Follow the following rules:
  - ✓ 0 * 0 = 0
  - ✓ 0 * 1 = 0
  - ✓ 1 * 0 = 0
  - ✓ 1 * 1 = 1

Binary Multiplication

➢ **Example: 1010 * 101 = 0011**

			1	0	1	0
	*			1	0	1
			1	0	1	0
	1					
	0	0	0	0	.	
1	0	1	0	.	.	
1	1	0	0	1	0	

	10
*	5
	50

Binary Division

- Use same basics as decimal division
- If the first digit of the dividend is smaller than the dividsor, place 0 in the quotient, and bring down the next digit.
- If the first digit of the dividend is equal or greater than the dividsor, place 1 in the quotient, and subtract the divisor from the dividend and write the remainder.
- Bring down the next bit from dividend
- Repeat these steps untill all bits have been brought down.

Binary Division

- Example: 1011 / 10 =

1	0	1	1	10	
0					0 1 0 1
1	0				
1	0				
0	0	1			
	0	0			
0	0	1	1		
		1	0		
0	0	0	1		

Binary Division

- Example: 1011 / 10 =

Quotient = 101, Remainder = 1

1	0	1	1		10
0					0 1 0 1
1	0				
1	0				
0	0	1			
	0	0			
0	0	1	1		
		1	0		
0	0	0	1		

Binary Division

- **Example: 11000 / 100 =?**

Binary Division

- Example: 11000 / 100 =110
- Quotient = 110, Remainder = 0

The end