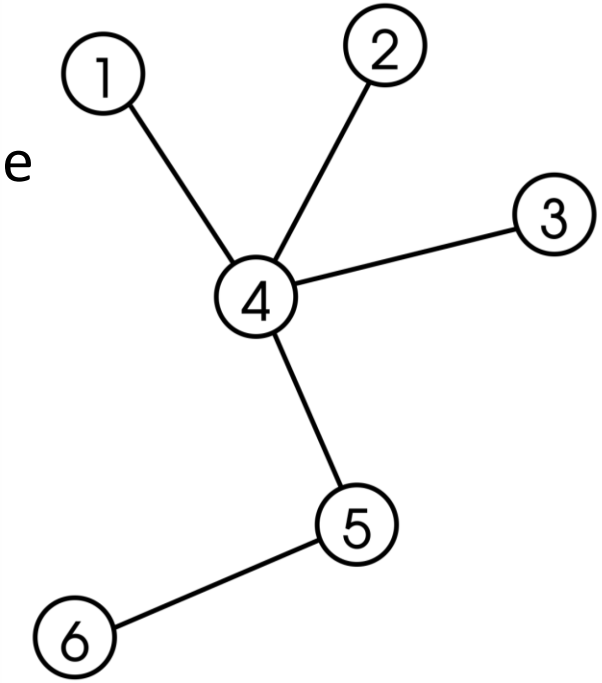
Part 3: Search for Optimal Weight Trees

Search for Optimal Weight Trees

- **Search for Optimal Weight Trees** problem focuses on connecting nodes with the minimum total cost, often via well-known algorithms like **Prim's** and **Kruskal's** for the Minimum Spanning Tree (**MST**).
- In networking, these formulations enable loop-free topologies (e.g., Spanning Tree Protocol), efficient multicast routing, and energy-saving data aggregation in Wireless Sensor Networks (WSNs).
- By minimizing redundant links, optimal weight trees improve reliability, reduce costs, and streamline communication across various systems.

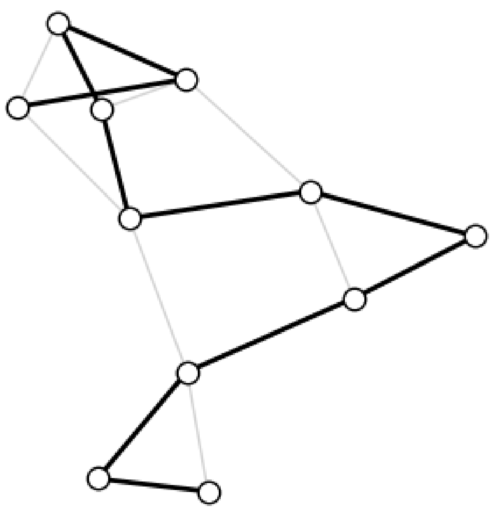
Trees

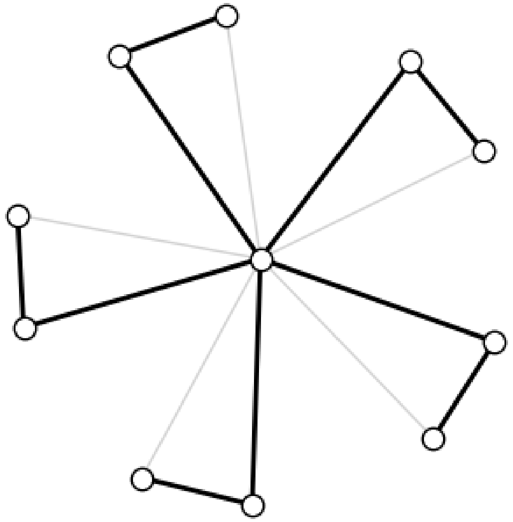
- The concept of a **tree** is probably the most important in graph theory, especially for those interested in applications of graphs.
- A **tree** is a connected graph without any circuits (acyclic)
- Properties:
  - One and only one path between every pair of vertices in a tree
  - A tree with n vertices has n – 1 edges
  - A tree is minimally connected
  - A tree is maximally acyclic
  - A tree (n>=2) have at least two pendant vertices
  - A tree with a specific vertex (root) is named rooted tree

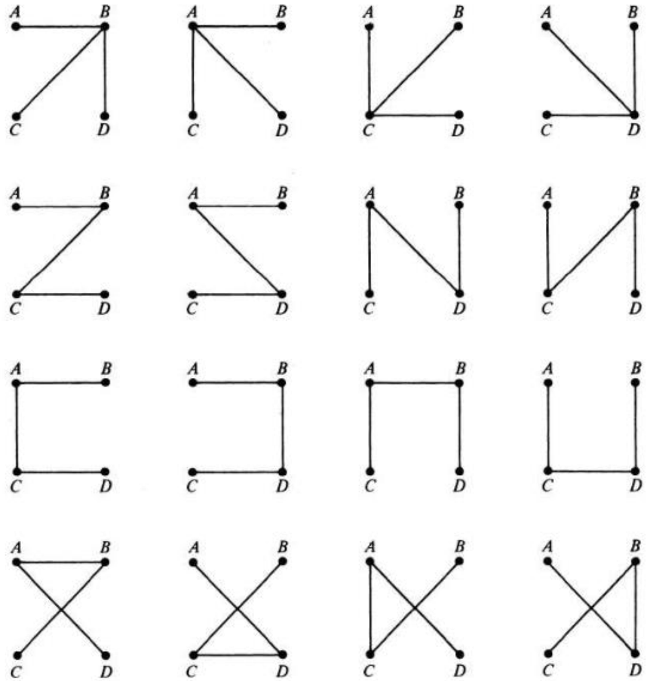


Related Problems – Spanning tree

- A tree T is said to be a **spanning tree (ST)** of a connected graph G if T is a subgraph of G and $V_T = V_G$ (partial graph).
  - Number of ST for n ($n \geq 2$) vertices: n^{n-2}
  - Number of ST of a graph G: $< n^{n-2}$
  - Find all possible ST of a graph (cyclic interchange)

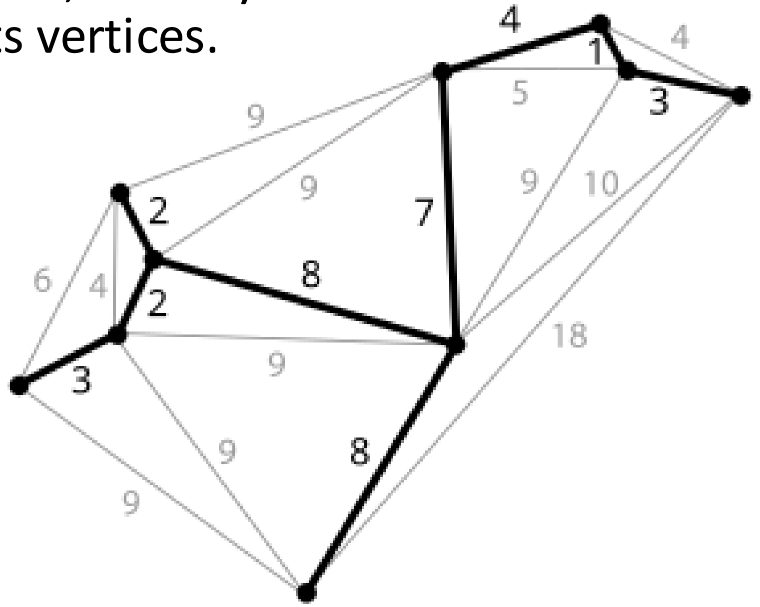


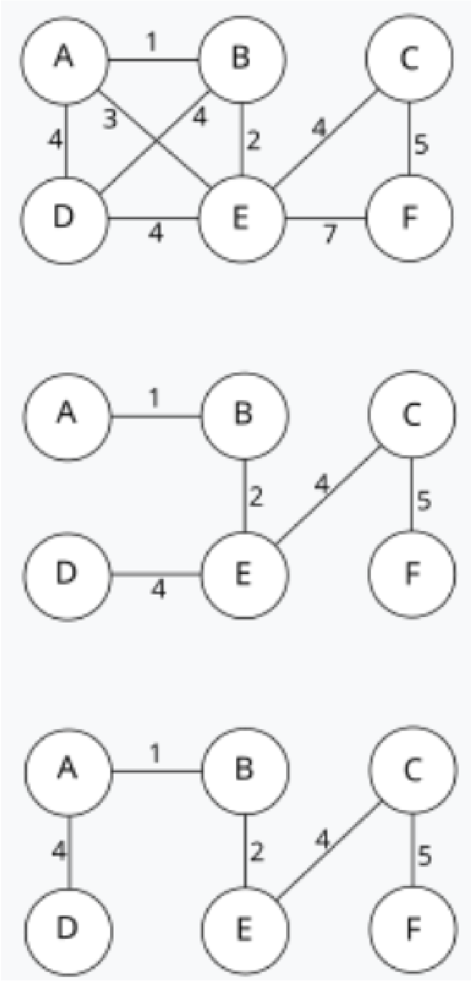




Related Problems – Optimal spanning tree

- The **weight** of a ST is defined as the sum of the weights of all the its branches
- **Minimal spanning tree** (MST) is one with the smallest weight.
  - Find all MST.
  - We can also need to find **Maximal** ST.
  - In some situations, we may be interested in an MST with a maximum degree constraint for its vertices.



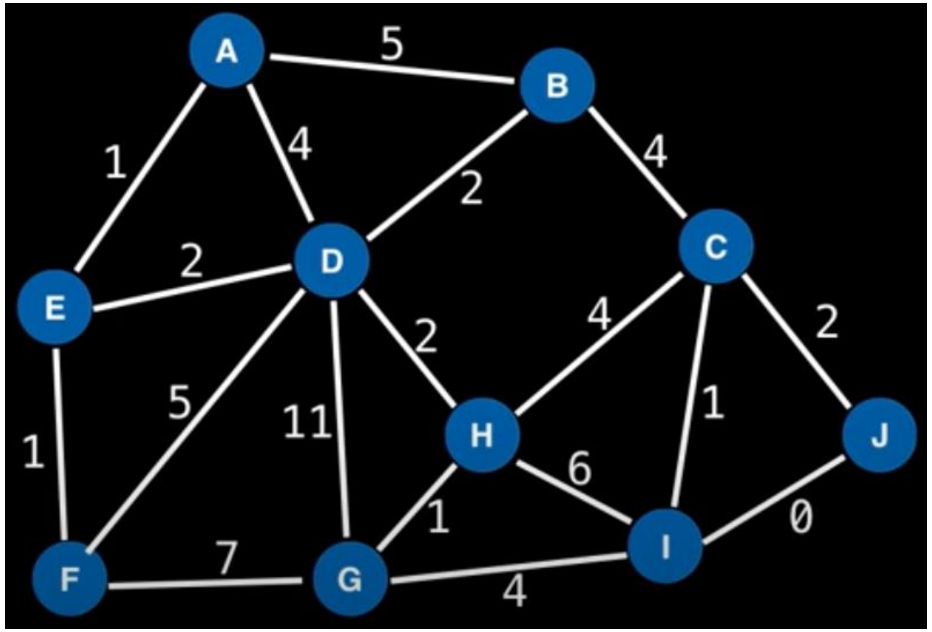


Kruskal’s Algorithm

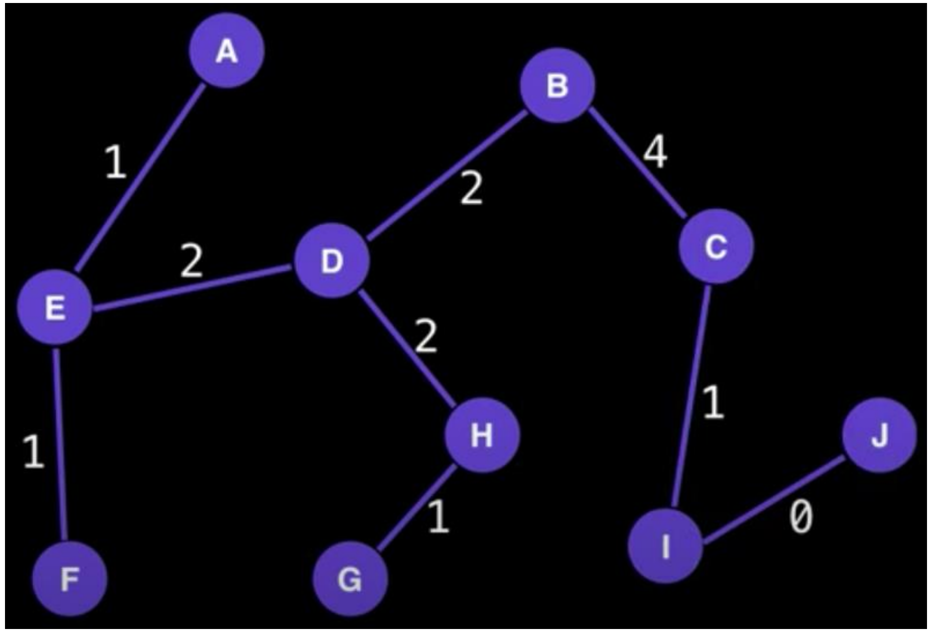
$n = \text{order}(G)$
$m = \text{size}(G)$
Let $a_1, a_2, \ldots, a_m$ be the the sorted edges
$F = \emptyset, k = 1$
while $k \le m$ **and** $\text{card}(F) < n - 1$
    if adding a_k does not create a cycle
        $F = F \cup \{a_k\}$
    $k = k + 1$

1. **Initialisation**: F is the set of tree edges, its initial value is set to Ø
2. **Sorting**: sort all graph edges by weight increasing order.
3. **Iterating** cycle checking and add edges if no cycle accurs

Use example of Kruskal’s Algorithm



I to J = 0
A to E = 1
C to I = 1
E to F = 1
G to H = 1
B to D = 2
C to J = 2
D to E = 2
D to H = 2
A to D = 4
B to C = 4
C to H = 4
G to I = 4
A to B = 5
D to F = 5
H to I = 6
F to G = 7
D to G = 11



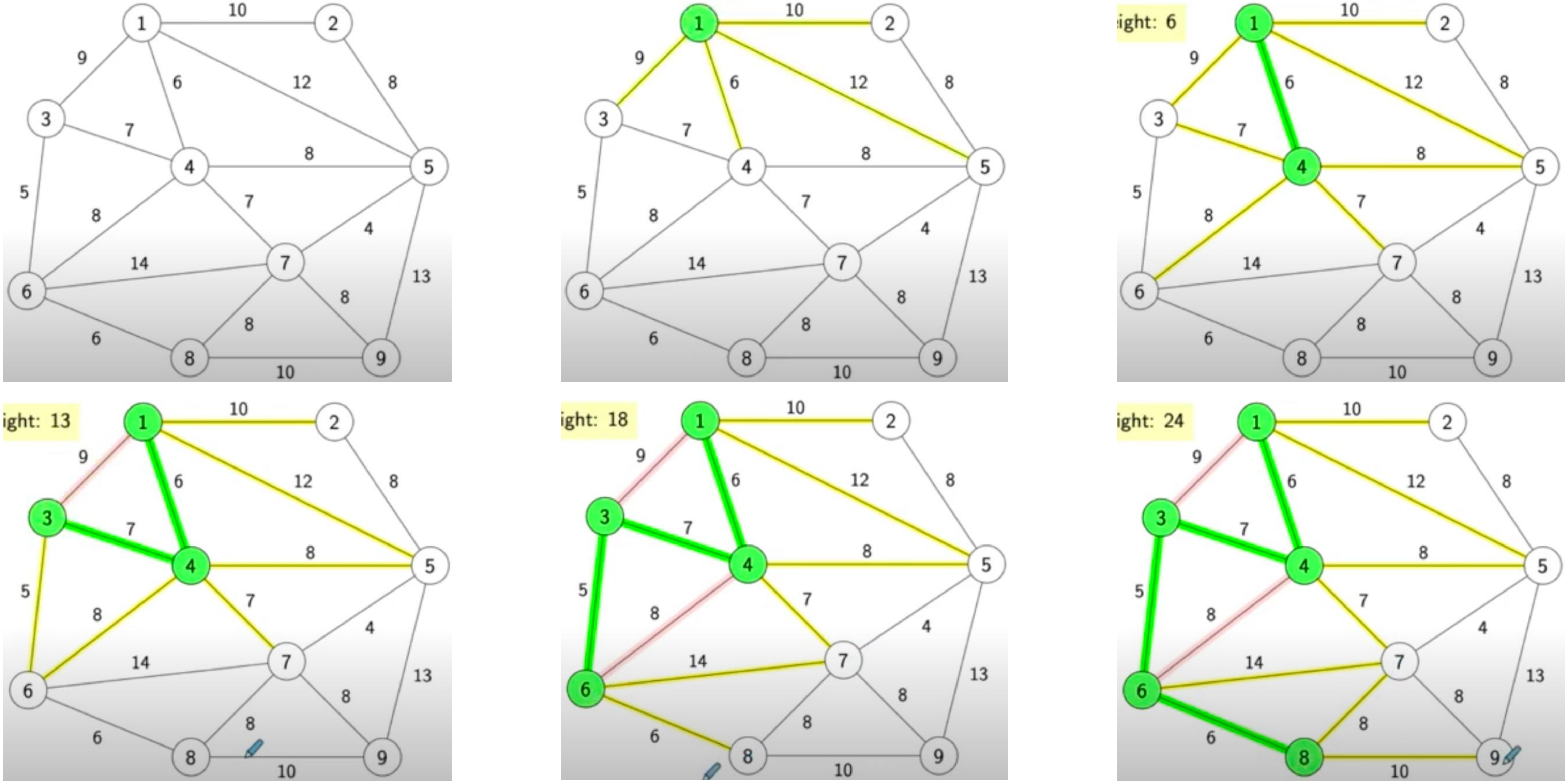
Prim's Algorithm

Choose an arbitrary starting vertex s
$V_T = \{s\}$
$E_T = \emptyset$

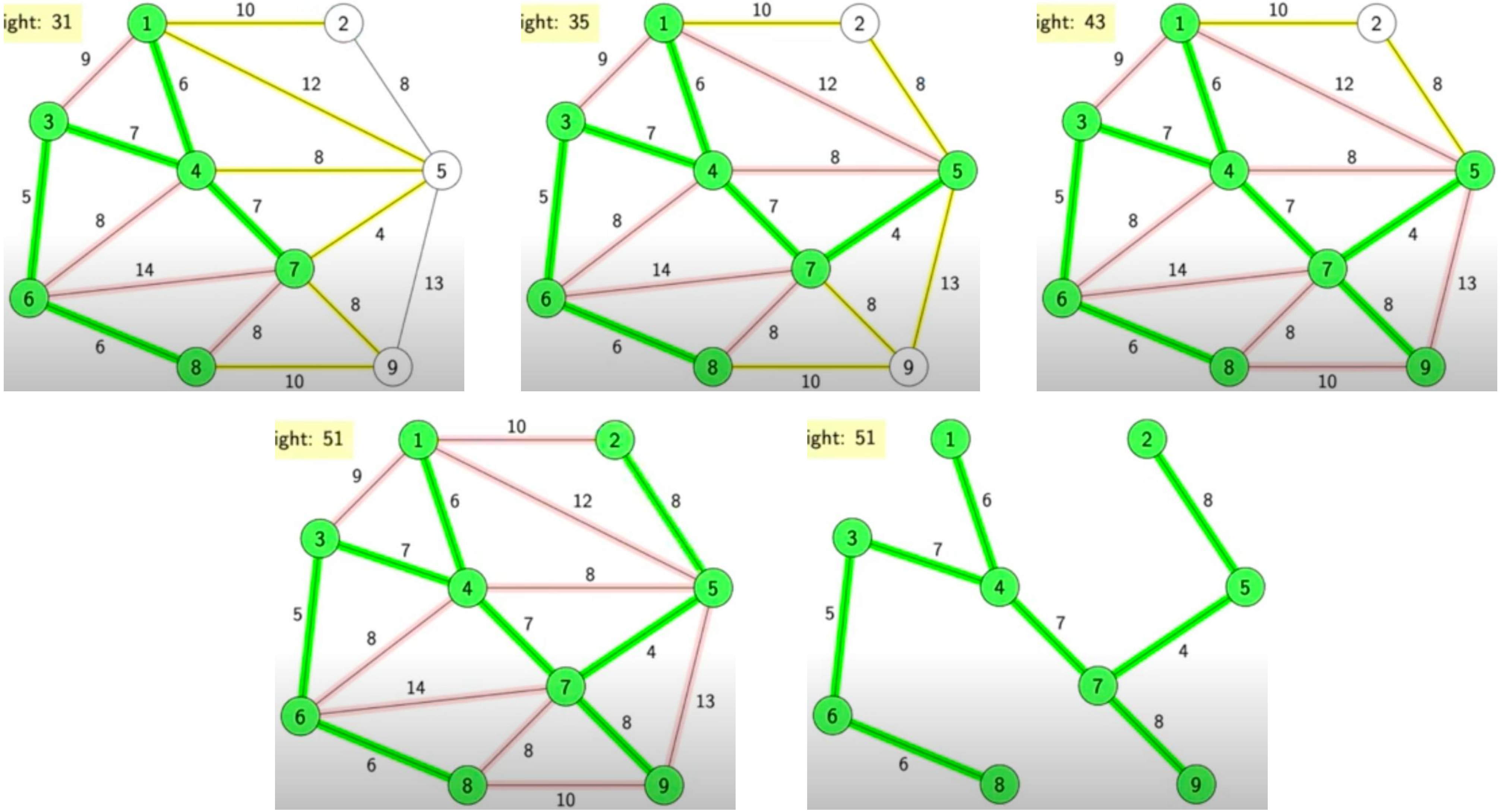
While $V_T \neq V$
    Select the edge (u,v) with the min weight, $u \in V_T$ and $v \notin V_T$
    Add v to V_T and (u,v) to E_T

1. **Choose** one arbitrary vertex s, $V_T = \{s\}$
2. **Select** the edge e from cocycle(V_T) with min cost that not form a cycle, v is its incident ($v \notin V_T$), $V_T = V_T + \{v\}$
3. **Iterating** until $V_T = V$

Use example of Prim’s Algorithm



Use example of Prim’s Algorithm

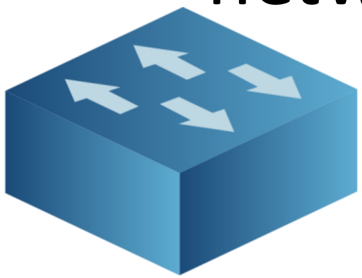


Applications

1. Spanning Tree Protocol (STP) in Ethernet and other switched networks
2. Data aggregation in Wireless Sensor Networks (WSN)

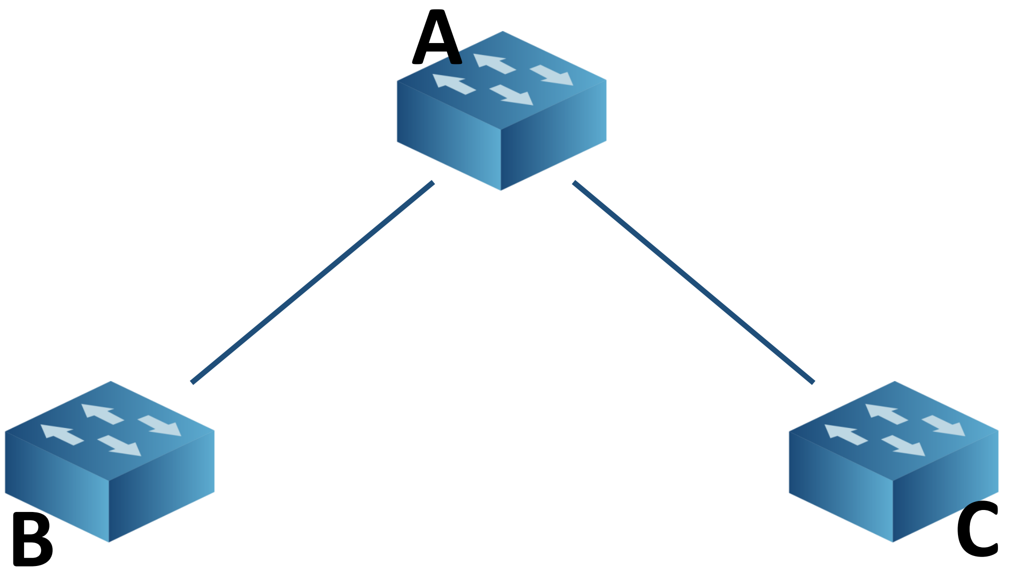
Spanning Tree Protocol

- **Switched networks** (**SN**) are communication systems where data is directed through dedicated devices called switches, rather than being **broadcast** to every device on the network.
- **Spanning Tree Protocol** (**STP**) is employed in SNs to prevent data loops, ensuring a stable and efficient network topology.
- Most modern **LANs** (Local Area Networks), such as **Ethernet** networks found in offices, are built on SN architectures.



Spanning Tree Protocol

- **STP** is a network protocol used in Ethernet networks to create a **loop-free** topology by selectively disabling **redundant** paths, preventing **broadcast storms** and ensuring efficient data transmission.
- Why STP ?



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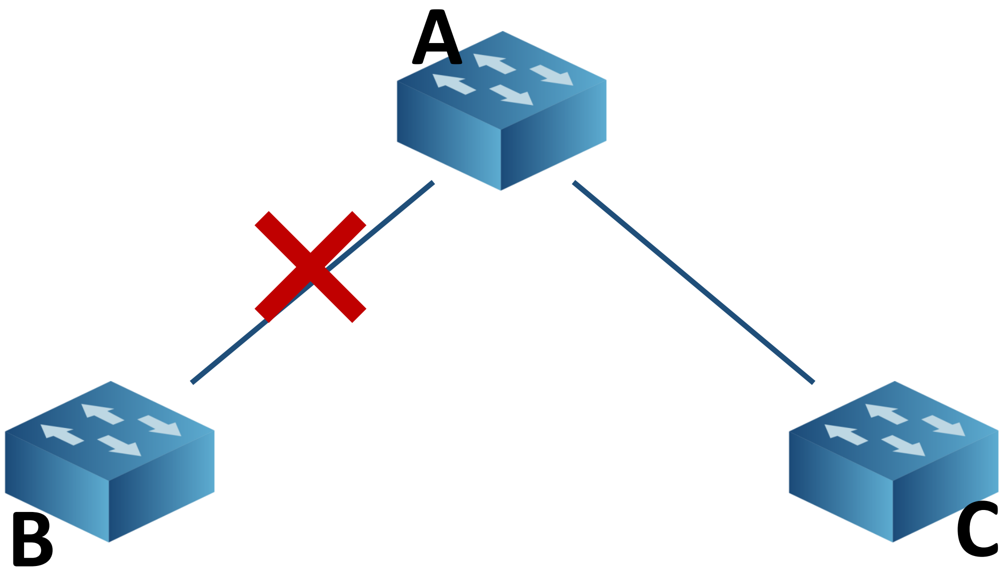

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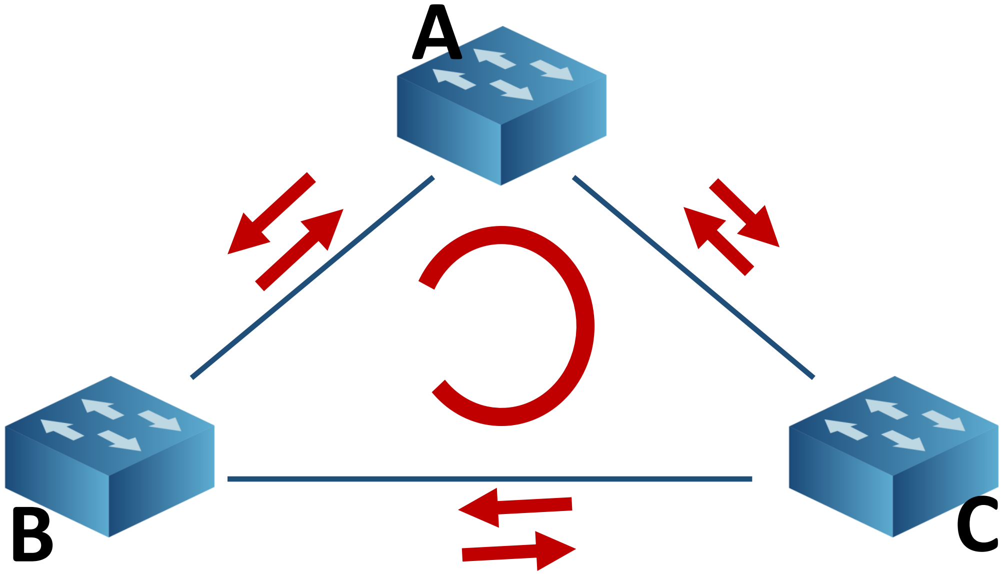
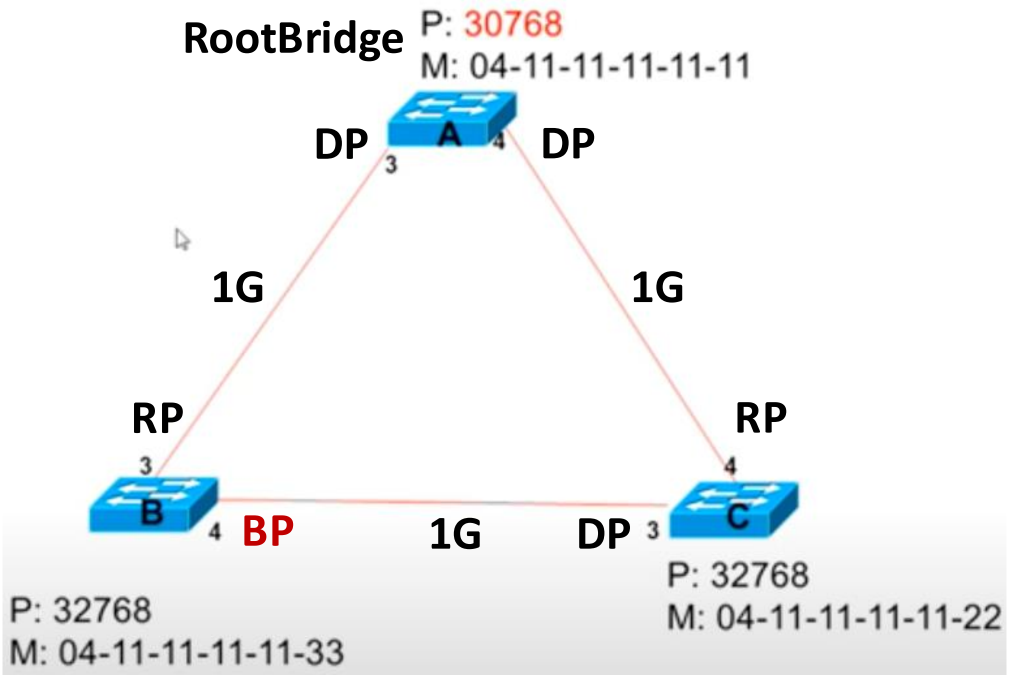


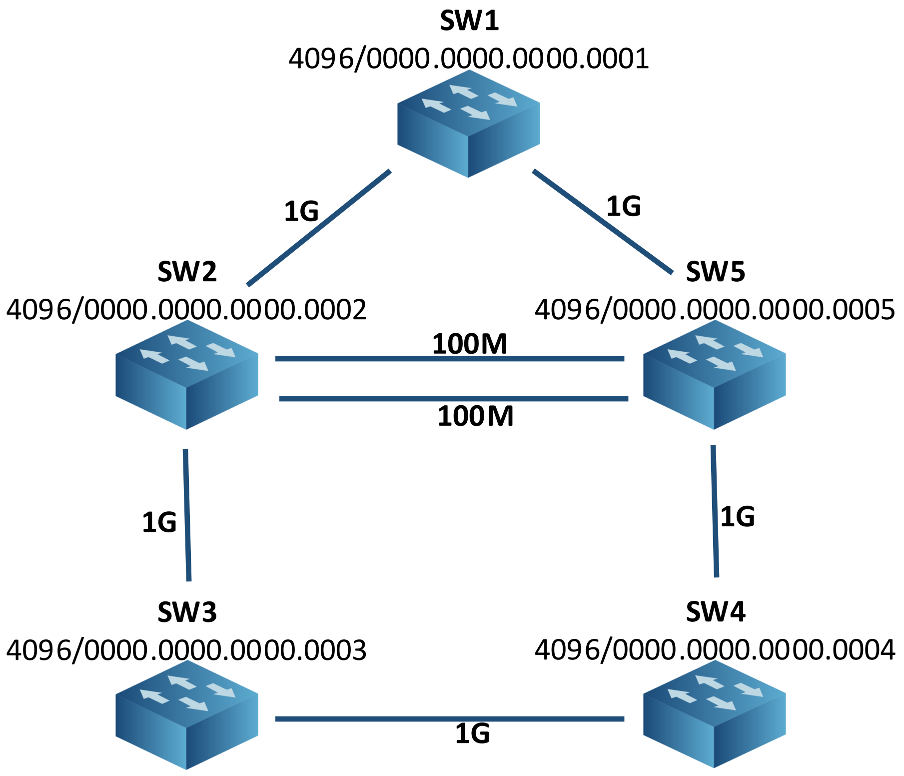
Spanning Tree Protocol (STP)

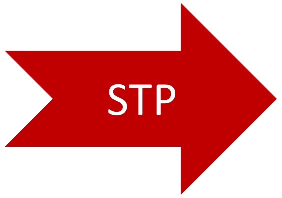
- Algorithm:
  1. Elect **RootBridge (RB),** viewing switch id (switch priority + MAC @),
  2. Elect **RootPort (RP)** per switch, viewing the smallest **cost** to the RootBridge
  3. Elect **DesignedPort (DP)** per segment, using cost & switch id if equal
  4. Make remaining ports as **BlockedPorts (BP)**

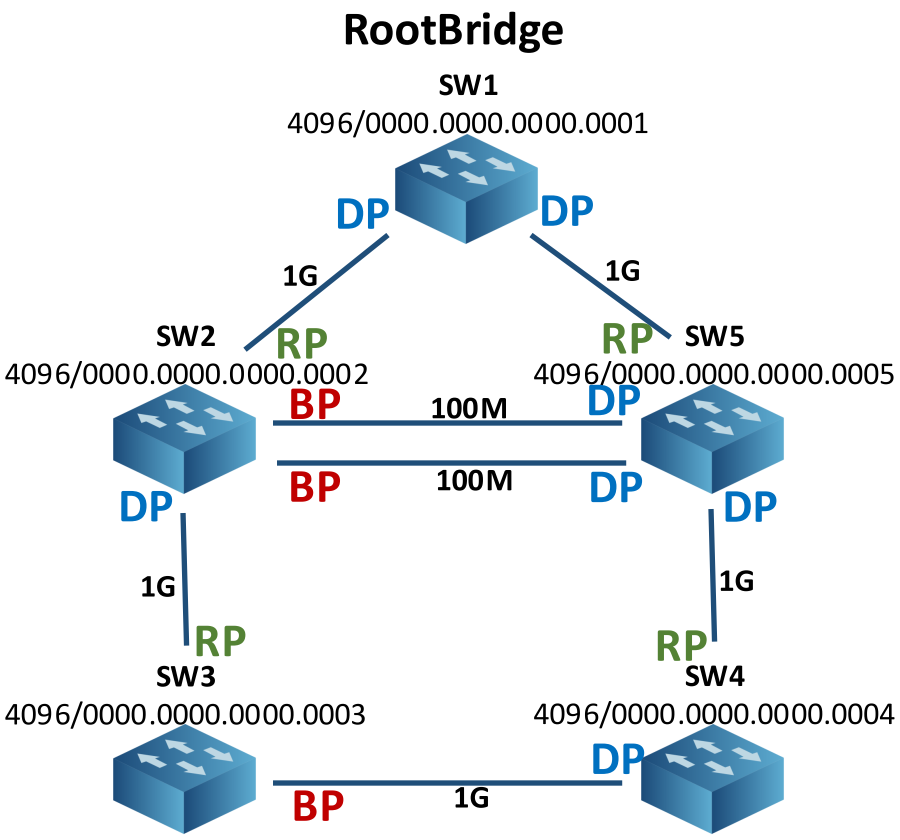
Speed	Cost
10 M	100
100 M	19
1 G	4
10 G	2



Example







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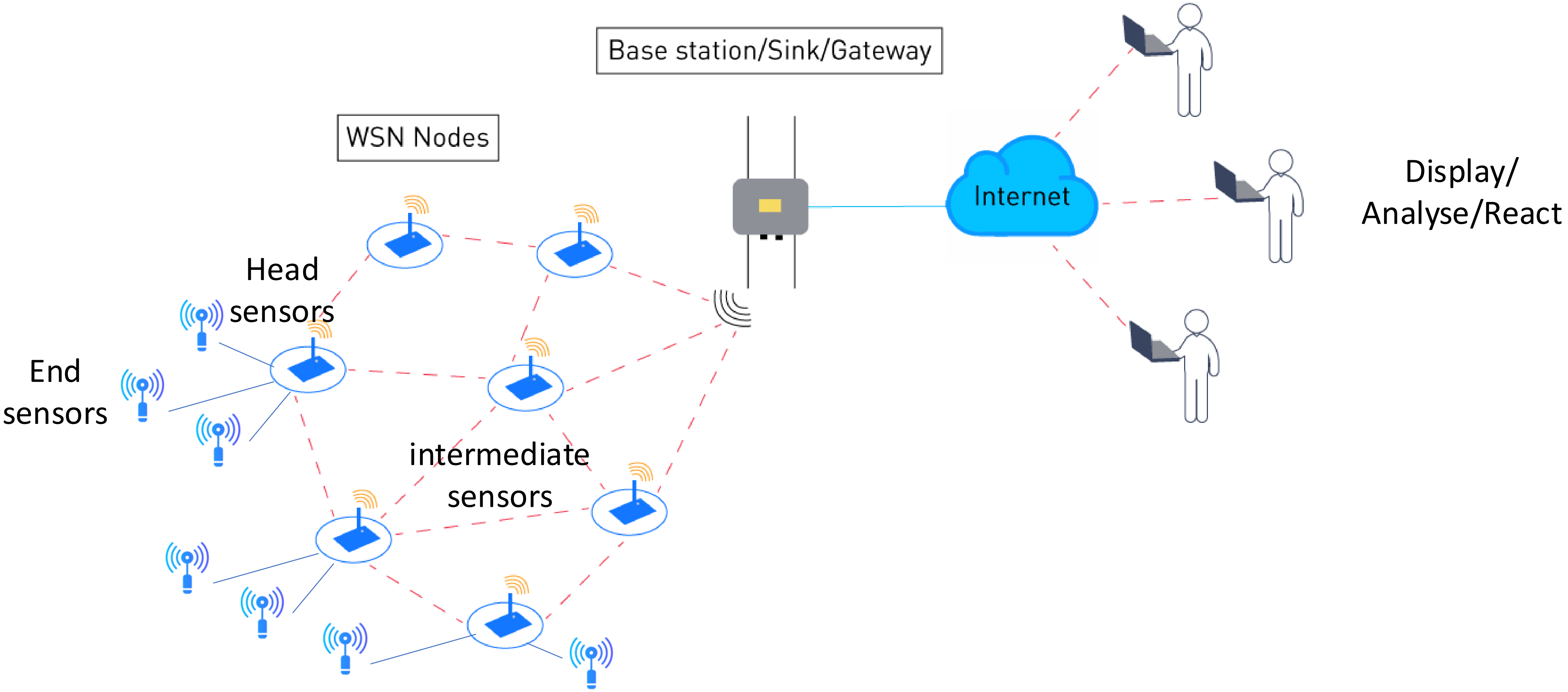
Wireless Sensor Networks (**WSN**)

A Wireless Sensor Network (WSN) is a network of spatially distributed, autonomous **sensor** nodes that monitor physical or environmental conditions and communicate their data **wirelessly** to a central system or **sink** for processing and analysis.

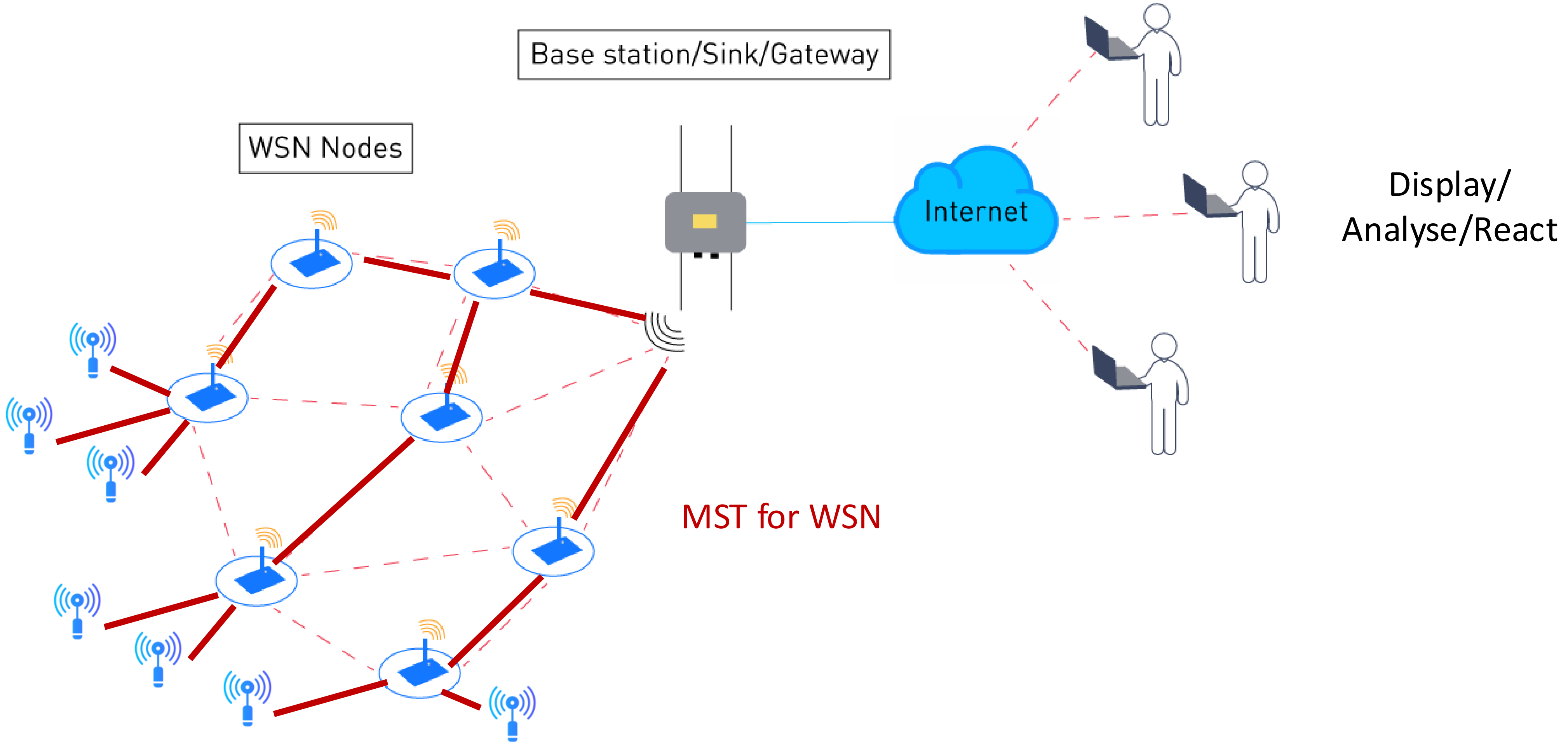
Structure of WSN



Efficient Routing

- By constructing an MST over the sensor nodes, a tree structure is formed that connects all nodes with the minimal total transmission cost.
- This tree eliminates redundant paths and cycles, ensuring that data packets follow the most energy-efficient routes toward the base station.
- In practice, each node is treated as a vertex, and the weight of each edge is often defined based on factors like the distance or estimated energy cost for communication.
- Algorithms such as **Prim’s** or **Kruskal’s** are then used to build the MST, providing a robust structure for routing.

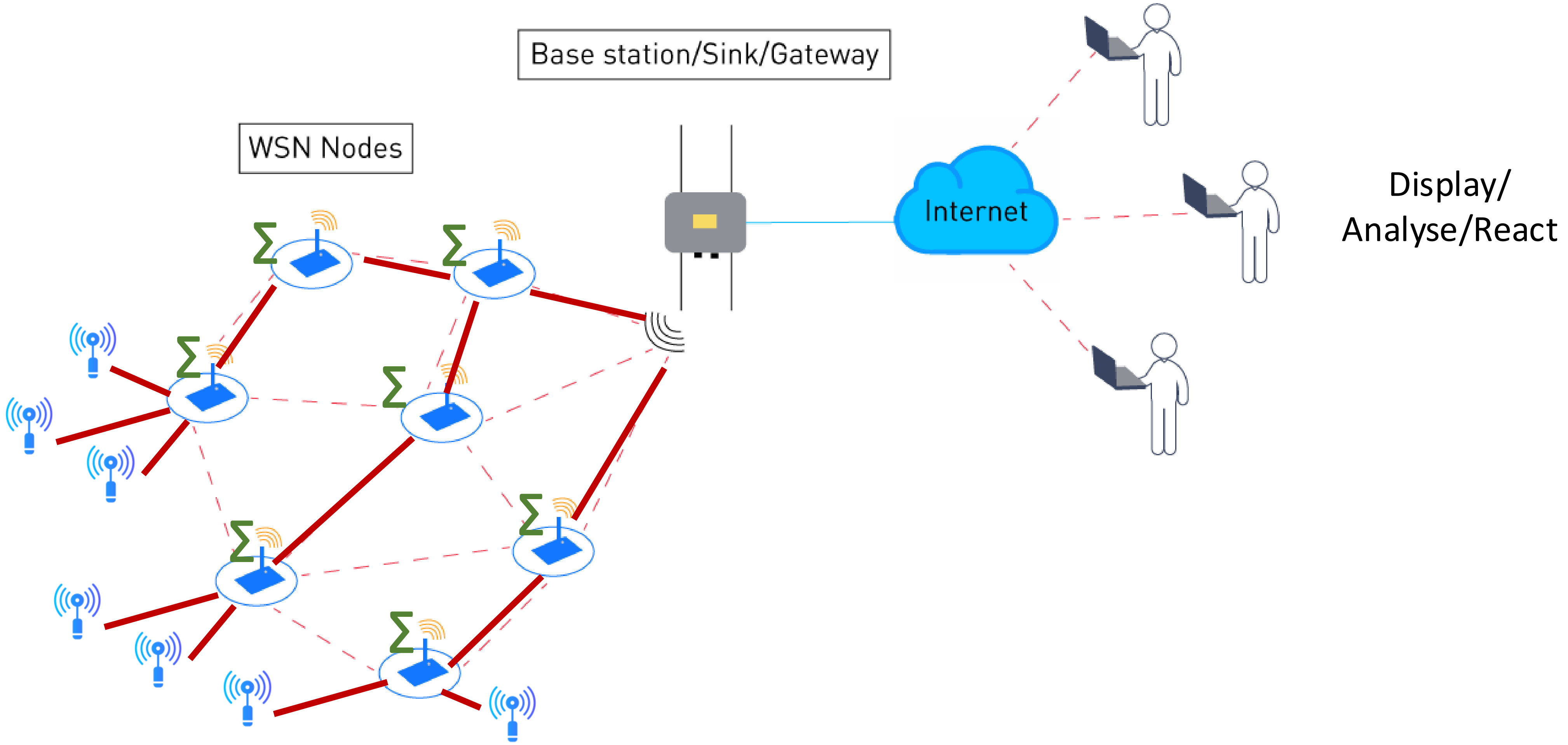
Structure of WSN



Tree-Based Data Aggregation

- Once the MST is established, it naturally lends itself to hierarchical data aggregation. Sensor nodes send their raw data up the tree structure to intermediate nodes, which aggregate (or fuse) the data before forwarding it further.
- This aggregation process reduces the total number of transmissions, thereby saving energy and reducing network congestion.
- The tree structure ensures that each piece of data is transmitted along a single, well-defined path, which simplifies the aggregation process and avoids duplicate transmissions

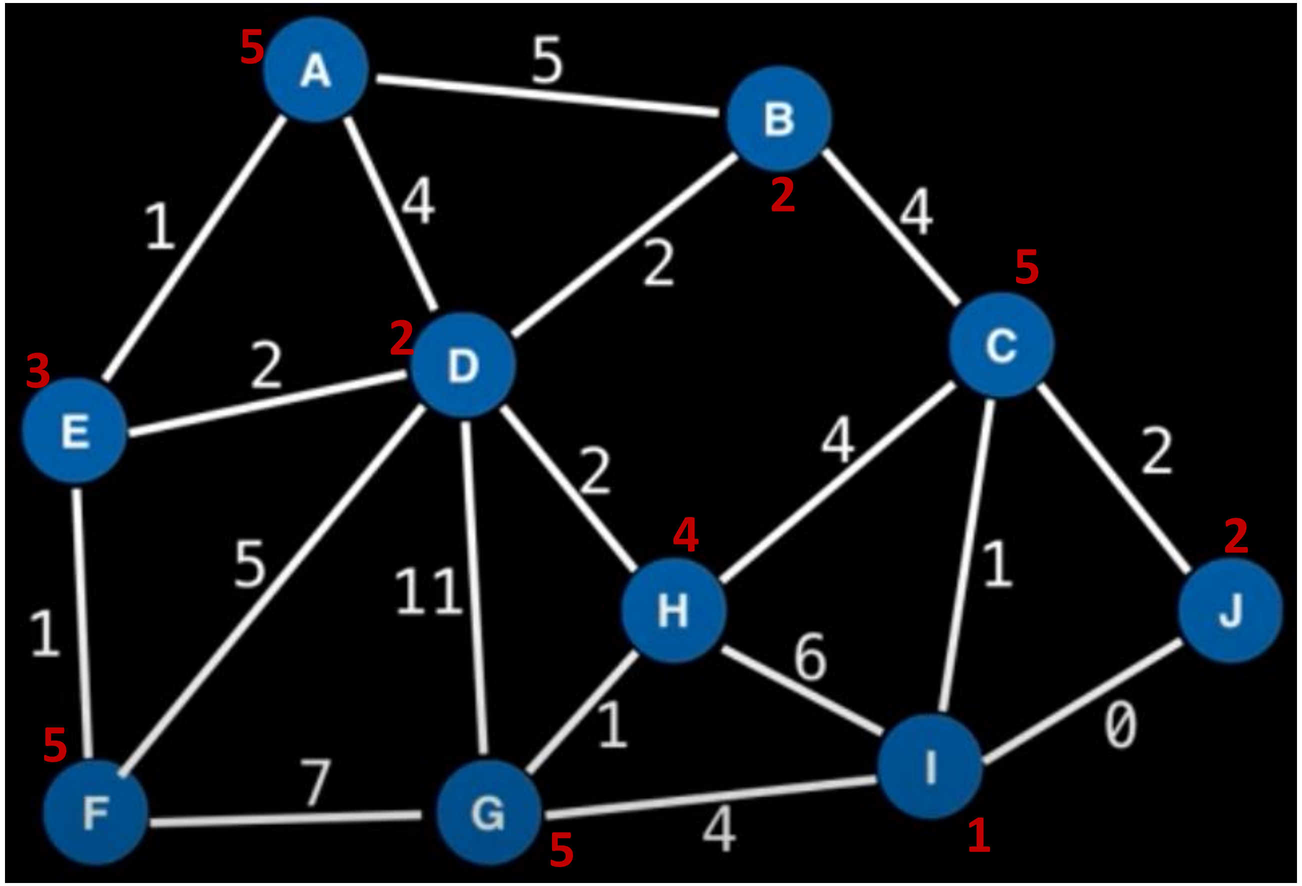
Structure of WSN



MST Building and Adapting

- Algorithm:
  - **Collecting information**: each node measure energy and distance
  - **MST construction**:
    1. A node initiate the process
    2. Exchange cost information with neighbors
    3. Add a new node that has the min cost edge without forming loop
    4. Repeat until all nodes are collected
  - **Recomputation**: The network can periodically re-run the MST construction using updated weights, ensuring that the routing structure remains energy-efficient.

Example



Cost = Distance/Energy