Badji Mokhtar University -Annaba
Computer Sciences
Electronic Department

Mechanics of the material point

Chapter one part two

Chapter 1

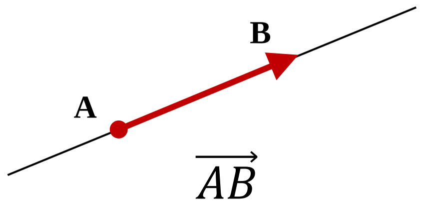
Part two

Mathematical reminders

Vector Calculus

Definition

A vector is represented graphyically by a line segment AB, where A is the chosen origin and B is the end point.



A vector is defined by:

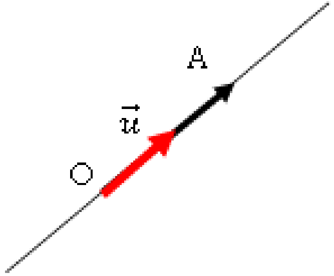
- It’s direction A$\rightarrow B$.
- It’s Magnitude AB

Chapter 1

The Unit Vector

the unit vector of vector $\vec{A}$: $\overrightarrow{U_A} = \frac{\vec{A}}{|\vec{A}|}$

the vector $\overrightarrow{U_A}$ is unitary when it's magnitude $\|\overrightarrow{U_A}\| = 1$.

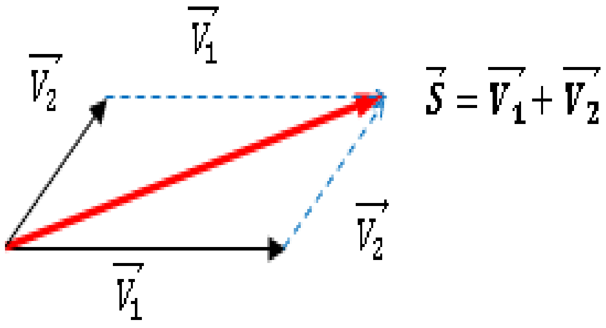


Vector addition

$$\vec{S} = \vec{V}_1 + \vec{V}_2$$

$$\vec{V}_1 = x_1\vec{\imath} + y_1\vec{\jmath} \quad , \quad \vec{V}_2 = x_2\vec{\imath} + y_2\vec{\jmath}$$

$$\vec{S} = (x_1+x_2)\vec{\imath} + (y_1 + y_2)\vec{\jmath}$$



- Commutative : $\vec{S} = \overrightarrow{V_1} + \overrightarrow{V_2} = \overrightarrow{V_2} + \overrightarrow{V_1}$.
- Associative : $\left(\overrightarrow{V_1} + \overrightarrow{V_2}\right) + \overrightarrow{V_3} = \overrightarrow{V_1} + \left(\overrightarrow{V_2} + \overrightarrow{V_3}\right)$.
- Distributive : $(a+b).\overrightarrow{V_1} = a.\overrightarrow{V_1} + b.\overrightarrow{V_1}$; $a.(\overrightarrow{V_1}+\overrightarrow{V_2}) = a.\overrightarrow{V_1}+a.\overrightarrow{V_2}$

Example: $\vec{A} = -5\,\vec{\imath} + 7\vec{\jmath} + 6\vec{k}$; $\vec{B} = 6\vec{\imath} + 4\vec{\jmath} + 3\vec{k}$

calculate $(\vec{A}+\vec{B})$

Solution:

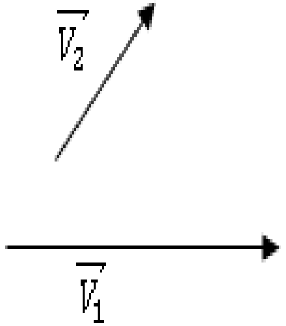
$\vec{A}+\vec{B} = (-5+6)\,\vec{\imath} + (7+4)\,\vec{\jmath} + (6+3)\,\vec{k}$

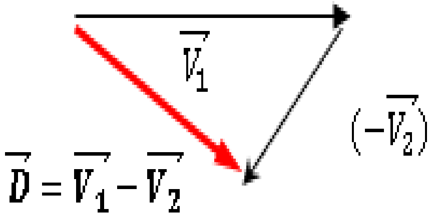
$= 1\,\vec{\imath} + 11\,\vec{\jmath} + 9\,\vec{k}$

Chapter 1

Vector substraction

$$\vec{D} = \vec{V_1} - \vec{V_2} = \vec{V_1} + (-\vec{V_2})$$





$$\vec{D} = \vec{V_1} - \vec{V_2}$$

$$\vec{V_1} = x_1\vec{i} + y_1\vec{j} \quad , \quad \vec{V_2} = x_2\vec{i} + y_2\vec{j}$$

$$\vec{D} = (x_1 - x_2)\vec{i} + (y_1 - y_2)\vec{j}$$

$$* \ \vec{V_1} - \vec{V_2} \neq \vec{V_2} - \vec{V_1}$$

Example: $\vec{A} = -5\,\vec{i} + 7\vec{j} + 6\vec{k}$; $\vec{B} = 6\vec{i} + 4\vec{j} + 3\vec{k}$

calculate $(\vec{A} - \vec{B}\,)$

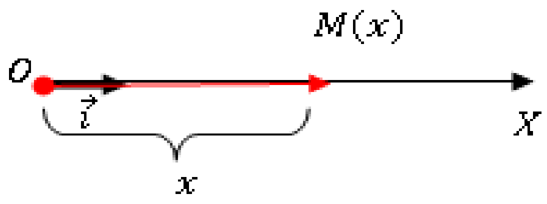
Solution:

$\vec{A} - \vec{B}$ = (-5-6) $\vec{i}$ +(7-4) $\vec{j}$ +(6-3) $\vec{k}$ =-11 $\vec{i}$ +3 $\vec{j}$ + 3 $\vec{k}$

Components of a vector

1- Lineare reference

$$\overrightarrow{OM} = x\vec{i}$$



Chapter 1

2- Orthogonal reference

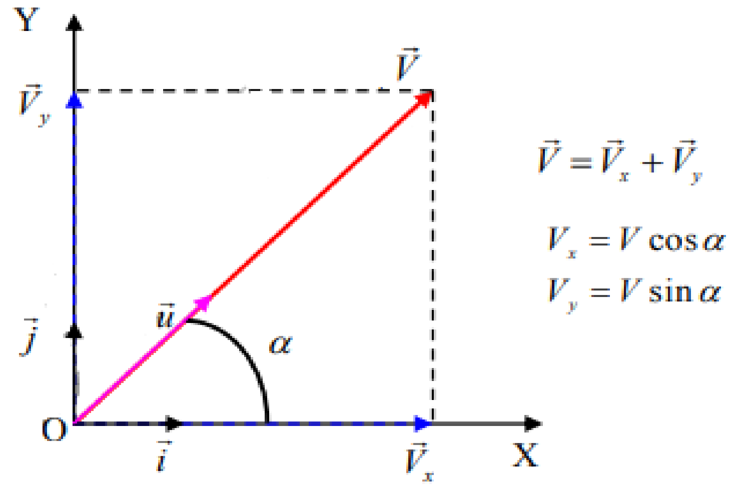
$\vec{i}$ and $\vec{j}$ the unit vector respectively in the directions of two axes (ox) and (oy).

We can write:

$$\overrightarrow{V_x} = V_x\vec{i}$$

$$\overrightarrow{V_y} = V_y\vec{j} \qquad \vec{V} = \overrightarrow{V_x} + \overrightarrow{V_y}$$

$$\vec{V} = V_x\vec{i} + V_y\vec{j}$$



$$\vec{V} = \text{V } \cos\alpha\vec{i} + V\sin\alpha\vec{j}$$

$$\vec{V} = \text{V}(\cos\alpha\ \vec{i} + \overrightarrow{\sin\alpha\vec{j})}$$

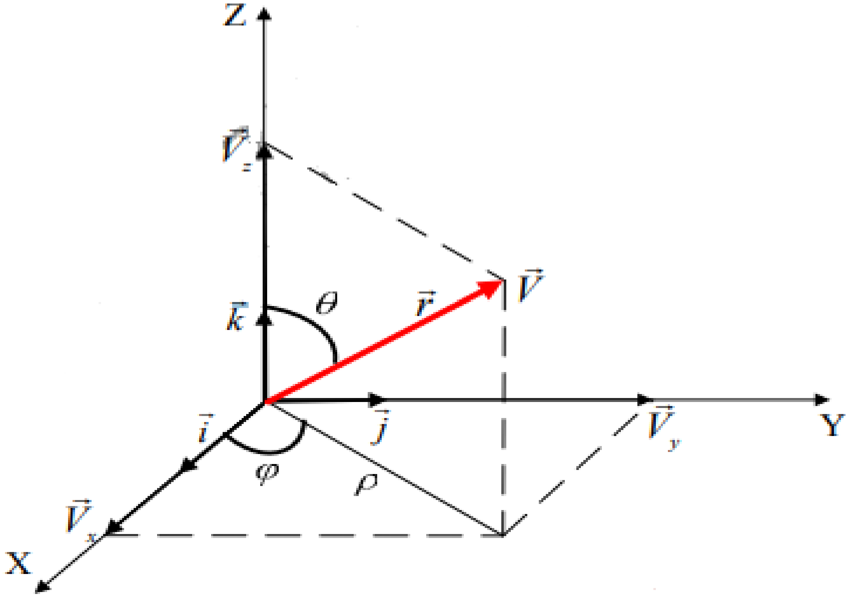
$$\vec{V} = \text{V}\ \vec{u}$$

$$\vec{u} = \cos\alpha\ \vec{i} + \sin\alpha\vec{j}$$

Chapter 1

3- Orthonorme reference

In the reference R(o,$\vec{i},\vec{j},\vec{k}$) (orthonormal base)



$$\vec{V} = \vec{V_x} + \vec{V_y} + \vec{V_z} \rightarrow \vec{V} = V_x\vec{i} + V_y\vec{j} + V_z\vec{k}$$

$$\cos\theta = \frac{V_z}{r} \rightarrow V_z = \mathrm{r}\cos\theta$$

$$\sin\theta = \frac{\rho}{r} \rightarrow \rho = \mathrm{r}\, sin\,\theta$$

$$\cos\varphi = \frac{V_x}{\rho} \rightarrow V_x = \rho\,\cos\varphi \rightarrow V_x = \mathrm{r}\sin\theta\cos\varphi$$

$$\sin\varphi = \frac{V_y}{\rho} \rightarrow V_y = \rho\, sin\,\varphi \rightarrow V_y = \mathrm{r}\sin\theta sin\varphi$$

$$\vec{V} = \mathrm{r}\sin\theta\cos\varphi\,\vec{i} + \mathrm{r}\sin\theta sin\varphi\vec{j} + \mathrm{r}\cos\theta\,\vec{k}$$

$$\vec{V} = \mathrm{r}\,\vec{u}$$

$$\vec{u} = \sin\theta\cos\varphi\,\vec{i} + \sin\theta sin\varphi\vec{j} + \cos\theta\,\vec{k}$$

The magnitude of a vector

we have a vector $\vec{A}$ $\quad \vec{A} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$

the magnitude of a vector $\vec{A}$: $\quad \|\vec{A}\| = \sqrt{x^2 + y^2 + z^2} \geq 0$

Example: calculate the magnitude of $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$

$\overrightarrow{V_1} = 3\,\vec{\imath} - 4\vec{\jmath} + 4\vec{k}$

$\overrightarrow{V_2} = 2\,\vec{\imath} + 3\vec{\jmath} - 4\vec{k}$

Solution:

$$V_1 = \|\overrightarrow{V_1}\| = \sqrt{3^2 + (-4)^2 + 4^2} = \sqrt{41}$$

$$V_2 = \|\overrightarrow{V_2}\| = \sqrt{2^2 + 3^2 + (-4)^2} = \sqrt{29}$$

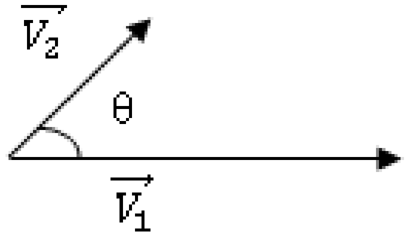
Chapter 1

Scalar product

1- Geometrical form:

We have: $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$

$$\overrightarrow{V_1}.\overrightarrow{V_2} = \left\|\overrightarrow{V_1}\right\|.\left\|\overrightarrow{V_2}\right\|.cos\theta$$



2- Analytical form:

We have:

$$\overrightarrow{V_1} = x_1\vec{i} + y_1\vec{j} + z_1\vec{k}$$
$$\overrightarrow{V_2} = x_2\vec{i} + y_2\vec{j} + z_2\vec{k}$$
$$\overrightarrow{V_1}.\overrightarrow{V_2} = x_1x_2 + y_1y_2 + z_1z_2$$

Scalar product

Example:

$$\overrightarrow{V_1} = 4\vec{\imath} + 5\vec{\jmath} + 3\vec{k}\ ,\ \overrightarrow{V_2} = -2\vec{\imath} + 6\vec{\jmath}$$

Calculate the scalar product $\overrightarrow{V_1}.\overrightarrow{V_2}$

Solution:

$$\overrightarrow{V_1}.\overrightarrow{V_2} = x_1x_2 + y_1y_2 + z_1z_2$$
$$\overrightarrow{V_1}.\overrightarrow{V_2} = (4\times(-2))+((+5)\times 6) + 3\times 0$$
$$\overrightarrow{V_1}.\overrightarrow{V_2} = 30.$$

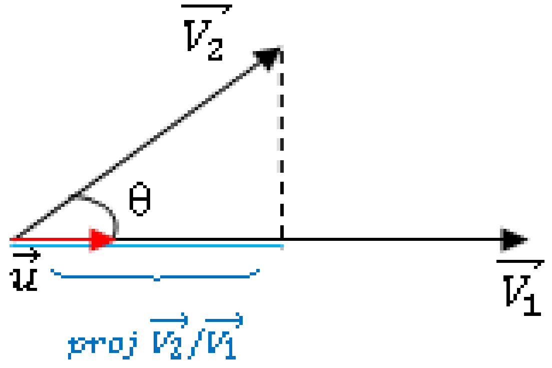
Properties of Scalar product

- $\overrightarrow{V_1}.\overrightarrow{V_2} = \left\|\overrightarrow{V_1}\right\|.\left\|\overrightarrow{V_2}\right\|.cos\theta = \left\|\overrightarrow{V_2}\right\|.\left\|\overrightarrow{V_1}\right\|.\cos(-\theta) \Rightarrow \overrightarrow{V_1}.\overrightarrow{V_2} = \overrightarrow{V_2}.\overrightarrow{V_1}$
- $\overrightarrow{V_1}.\left(\overrightarrow{V_2}+\overrightarrow{V_3}\right) = \overrightarrow{V_1}.\overrightarrow{V_2}+\overrightarrow{V_1}.\overrightarrow{V_3}$
- $\left(\overrightarrow{V_1} \pm \overrightarrow{V_2}\right)^2 = {V_1}^2 + {V_2}^2 \pm 2V_1V_2cos\theta$
- $\overrightarrow{V_1} \perp \overrightarrow{V_2} \Rightarrow \overrightarrow{V_1}.\overrightarrow{V_2} = 0$

Projecting a vector

The projection of the vector $\overrightarrow{V_2}$ onto $\overrightarrow{V_1}$ is given by the following relation:

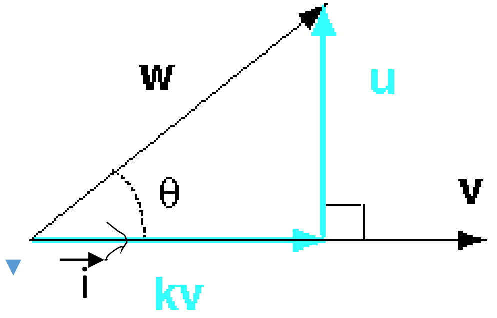
$$proj\ \overrightarrow{V_2}/\overrightarrow{V_1} = \left\|\overrightarrow{V_2}\right\|.cos\theta$$



Chapter 1

Projecting a vector

Let's have this figure here



$\mathrm{Cos}(\theta) = \dfrac{||\overrightarrow{Kv}||}{||\overrightarrow{W}||}$ and also $\mathrm{Cos}(\theta) = \dfrac{\overrightarrow{W}\,.\,\overrightarrow{V}}{||\overrightarrow{W}||.||\overrightarrow{V}||}$ (proven formula)

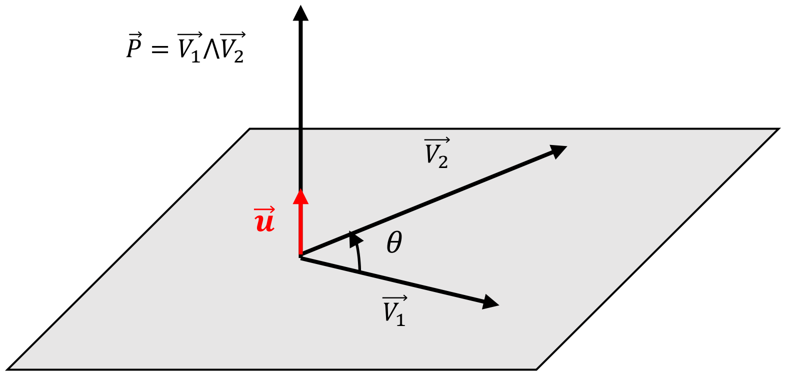
$$\frac{\overrightarrow{W}\,.\,\overrightarrow{V}}{||\overrightarrow{W}||.||\overrightarrow{V}\,||} = \frac{||\overrightarrow{Kv}||}{||\overrightarrow{W}\,||} \Rightarrow ||\overrightarrow{Kv}|| = \frac{\overrightarrow{W}\,.\,\overrightarrow{V}}{||\overrightarrow{V}||}$$

$\overrightarrow{Kv} = ||\overrightarrow{Kv}||.\,\overrightarrow{i}$ and $\overrightarrow{i} = \dfrac{\overrightarrow{V}}{||\overrightarrow{V}||}$ (here $\overrightarrow{V}$ and $\overrightarrow{Kv}$ have the same unit vector)

$$\overrightarrow{Kv} = \frac{\overrightarrow{W}\,.\,\overrightarrow{V}}{||\overrightarrow{V}||}\,.\,\frac{\overrightarrow{V}}{||\overrightarrow{V}||} = \frac{\overrightarrow{W}.\overrightarrow{V}.\overrightarrow{V}}{||\overrightarrow{V}||^2} \Rightarrow \overrightarrow{\mathrm{proj}(\overrightarrow{W},\overrightarrow{V})} = \frac{\overrightarrow{W}.\overrightarrow{V}.\overrightarrow{V}}{||\overrightarrow{V}||^2}$$

Chapter 1

Cross product



1- Geometrical form:

The cross product of two vectors $\vec{V_1}$ and $\vec{V_2}$

Is another **vector** $\vec{P}$ perpendicular to the plane which carries these two vectors.

$$\vec{P} = \vec{V_1} \wedge \vec{V_2} = \|\vec{V_1}\| . \|\vec{V_2}\| . sin\theta . \vec{u}$$

Where $\vec{u}$ is the unit vector perpendecular to the plane formed by $\vec{V_1}$ and $\vec{V_2}$

Cross product

Properties of cross product:

- The magnitude of cross product: $\|\vec{P}\| = \|\vec{V_1}\wedge\vec{V_2}\| = \|\vec{V_1}\|.\|\vec{V_2}\|.|sin\theta|$
- Anticommutative : $\vec{V_1}\wedge\vec{V_2} = -(\vec{V_2}\wedge\vec{V_1})$
- Distributive : : $\vec{V_1}\wedge(\vec{V_2} \pm \vec{V_3}) = \vec{V_1}\wedge\vec{V_2} \pm \vec{V_1}\wedge\vec{V_3}$
- $\vec{V_1}\ //\ \vec{V_2}$ then $\|\vec{V_1}\wedge\vec{V_2}\| = 0$
- $\vec{V_1} \perp \vec{V_2}$ then $\|\vec{V_1}\wedge\vec{V_2}\| = \|\vec{V_1}\|.\|\vec{V_2}\|$
- $\vec{\imath}\wedge\vec{\imath} = \vec{\jmath}\wedge\vec{\jmath} = \vec{k}\wedge\vec{k} = \vec{0}$ and $\vec{\imath}\wedge\vec{\jmath} = \vec{k},\ \vec{\jmath}\wedge\vec{k} = \vec{\imath},\ \vec{k}\wedge\vec{\imath} = \vec{\jmath}$

2- Analytical form:

we have $\vec{V_1}\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ and $\vec{V_2}\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$, with the matrix method

Chapter 1

$$\vec{V_1} \wedge \vec{V_2} = \begin{vmatrix} \vec{i} & -\vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1 z_2 - z_1 y_2).\vec{i} - (x_1 z_2 - z_1 x_2).\vec{j} + (x_1 y_2 - y_1 x_2).\vec{k}$$

Example:

$$\vec{A} = \vec{i} + 5\vec{j} + 8\vec{k}\ ;\ \vec{B} = 3\vec{i} + 2\vec{j} + 4\vec{k}$$

Calculate the vector product $\vec{A} \wedge \vec{B}$

$$\vec{A} \wedge \vec{B} = \begin{vmatrix} \vec{i} & -\vec{j} & \vec{k} \\ 1 & 5 & 8 \\ 3 & 2 & 4 \end{vmatrix} = (5 \times 4 - 2 \times 8)\vec{i} - (1 \times 4 - 3 \times 8)\,\vec{j} + (1 \times 2 - 3 \times 5)\vec{k}$$

$$\vec{A} \wedge \vec{B} = \overrightarrow{4i} + 20\vec{j} - 13\vec{k}$$

Chapter 1

Magnitude of Cross product

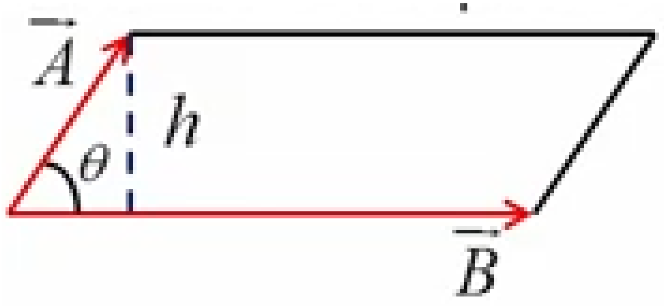
The magnitude of the vector product of two vectors
$\vec{A}$ and $\vec{B}$ represents the surface of a constructed parallelogram on its two vectors:

$S = h \, . \, |\vec{B}|$

$h = |\vec{A}|.\sin\theta$

$S = |\vec{A}|.|\vec{B}|.\sin\theta$

$S = |\vec{A} \wedge \vec{B}|$



Mixed product

the mixed product of three vectors $\overrightarrow{V_1}$; $\overrightarrow{V_2}$ and $\overrightarrow{V_3}$ is the scalar quantity defined by:

$$\overrightarrow{V_1}.\left(\overrightarrow{V_2} \wedge \overrightarrow{V_3}\right) = \begin{vmatrix} x_1 & -y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

$$= (y_2 z_3 - z_2 y_3)x_1 - (x_2 z_3 - z_2 x_3)y_1 + (x_2 y_3 - y_2 x_3)z_1$$

- $\overrightarrow{V_1}.\left(\overrightarrow{V_2} \wedge \overrightarrow{V_3}\right) = \overrightarrow{V_3}.\left(\overrightarrow{V_1} \wedge \overrightarrow{V_2}\right) = \overrightarrow{V_2}.\left(\overrightarrow{V_3} \wedge \overrightarrow{V_1}\right)$

Double vector product

Double vector product of three vectors $\vec{A}$; $\vec{B}$ and $\vec{C}$ is defined by a vector $\vec{D}$:

$$\vec{D} = \vec{A} \wedge (\vec{B} \wedge \vec{C}) = (\vec{A}.\vec{C}).\vec{B} - (\vec{A}.\vec{B}).\vec{C}.$$

$$\vec{D} = (\vec{A} \wedge \vec{B}) \wedge \vec{C} = -(\vec{C}.\vec{B}).\vec{A} + (\overrightarrow{C}.\vec{A}).\vec{B}$$

Example:

$\vec{a} = 1\vec{i} + 1\vec{j} + 3\vec{k}$ / $\vec{b} = 2\vec{i} - 3\vec{j} + 1\vec{k}$ / $\vec{c} = 1\vec{i} - 1\vec{j} + 2\vec{k}$

calculate the double vector product $\vec{D} = \vec{A} \wedge (\vec{B} \wedge \vec{C})$?

Solution:

$(\vec{A}.\vec{C}).\vec{B}$ =(1*1-1*1+3*2) . $\vec{B}$ =$6(2\vec{i} - 3\vec{j} + 1\vec{k}) = 12\vec{i} - 18\vec{j} + 6\vec{k}$

$(\vec{A}.\vec{B}).\vec{C}$ =(1*2-1*3+3*1). $\vec{C} = 2(1\vec{i} - 1\vec{j} + 2\vec{k}) = 2\vec{i} - 2\vec{j} + 4\vec{k}$

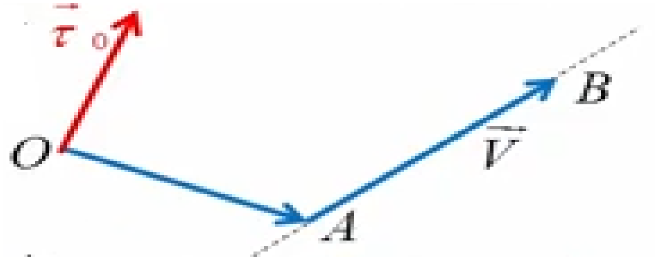
$\vec{D} = 12\vec{i} - 18\vec{j} + 6\vec{k}$ - $(2\vec{i} - 2\vec{j} + 4\vec{k}) = 10\vec{i} - 16\vec{j} + 2\vec{k}$

Chapter 1

Moment vector $\vec{V}$ with respect to a point O :

The moment of a vector $\overrightarrow{V_1}$ which passes through point A, by contribution to a point O is defined by the vector $\overrightarrow{M}$ such that:

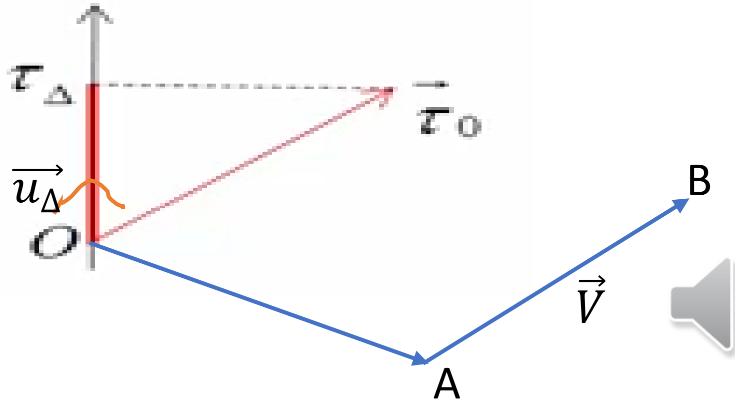
$$\overrightarrow{\mathcal{M}}_{\vec{V}/O} = \overrightarrow{OA} \wedge \vec{V}$$



Moment vector $\vec{V}$ with respect to axis (Δ):

$$\mathcal{M}_{\vec{V}/(\Delta)} = \overrightarrow{\mathcal{M}}_{\vec{V}/O} = \left(\overrightarrow{OA} \wedge \vec{V}\right).\overrightarrow{u_\Delta}$$

$\overrightarrow{u_\Delta}$: the unit vector de l'axis (Δ).



Chapter 1

Derivative of a vector

Let a vector $\vec{V}$ depend on time (t):

$$\vec{V}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

The derivative of the vector $\vec{V}$ with respect to time is defined as follows:

$$\frac{d\vec{V}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

Chapter 1

- $\frac{d}{dt}(\vec{A}+\vec{B}) = \frac{d\vec{A}}{dt} + \frac{d\vec{B}}{dt}$
- $\frac{d}{dt}(f.\vec{A}) = \frac{df}{dt} + f.\frac{d\vec{A}}{dt}$
- $\frac{d}{dt}(\vec{A}.\vec{B}) = \frac{d\vec{A}}{dt}.\vec{B} + \overrightarrow{A.}\frac{d\vec{B}}{dt}$
- $\frac{d}{dt}(\vec{A}\wedge\vec{B}) = \frac{d\vec{A}}{dt}\wedge\vec{B} + \overrightarrow{A\wedge}\frac{d\vec{B}}{dt}$

Chapter 1

Vector analysis

- Operator « nabla »

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{\imath} + \frac{\partial}{\partial y}\vec{\jmath} + \frac{\partial}{\partial z}\vec{k}$$

- Operator « gradient »

if f (x,y,z) is a scalar function

$$\overrightarrow{grad}\, f = \vec{\nabla} f = \left(\frac{\partial f}{\partial x}\right)\vec{\imath} + \left(\frac{\partial f}{\partial y}\right)\vec{\jmath} + \left(\frac{\partial f}{\partial z}\right)\vec{k}$$

- Operator « divergence »

Let a vector $\vec{V} = V_x\vec{\imath} + V_y\vec{\jmath} + V_z\vec{k}$

$$div\vec{V} = \vec{\nabla}.\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

Chapter 1

Vector analysis

❑ Operator « curl »

a vector function $\vec{V} = V_x\vec{i} + V_y\vec{j} + V_z\vec{k}$

$$\overrightarrow{rot}(\vec{V}) = \vec{\nabla} \wedge \vec{V} = \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\vec{i} - \left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right)\vec{j} + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\vec{k}$$

Explanation:

$$\overrightarrow{rot}(\vec{V}) = \begin{vmatrix} +\vec{i} & -\vec{j} & +\vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix} = A+B+C$$

Chapter 1

Vector analysis

$$A = \begin{vmatrix} +\vec{i} & & \\ & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ & V_y & V_z \end{vmatrix} = \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\vec{i}$$

$$B = \begin{vmatrix} & -\vec{j} & \\ \frac{\partial}{\partial x} & & \frac{\partial}{\partial z} \\ V_x & & V_z \end{vmatrix} = -\left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right)\vec{j}$$

$$B = \begin{vmatrix} & & +\vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \\ V_x & V_y & \end{vmatrix} = \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\vec{k}$$

Chapter 1

Vector analysis

$$\overrightarrow{curl}(\vec{V}) = \begin{vmatrix} +\vec{i} & -\vec{j} & +\vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix} = \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\vec{i} - \left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right)\vec{j} + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\vec{k}$$

Example: Calculate the curl of vector :

$\vec{V}$ = $2xy\,\vec{i} + 3yz^2\,\vec{j} + 9xy^3\vec{k}$

Solution:

$\overrightarrow{curl}(\vec{V}) = (27x\,y^2 - 6yz)\,\vec{i} - (9y^3 - 0)\,\vec{j} + (0 - 2x)\vec{k}$

$\overrightarrow{curl}(\vec{V}) = (27x\,y^2 - 6yz)\,\vec{i} - 9y^3\vec{j} - 2x\,\vec{k}$

Chapter 1

Vector analysis

❑ Operator « laplacien »

the laplacien of a scalar function is given by the following relation: $$\overrightarrow{\nabla}^2.(f) = \vec{\nabla}.\vec{\nabla}(f) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

the laplacien of a vector function is given by the following relation: $$\vec{\nabla}^2.(\vec{V}) = \vec{\nabla}.\vec{\nabla}(\vec{V}) = \frac{\partial^2 V_x}{\partial x^2}\vec{i} + \frac{\partial^2 V_y}{\partial y^2}\vec{j} + \frac{\partial^2 V_z}{\partial z^2}\vec{k}$$