Introduction to kinematics

Basics

Agendas

01 Definitions

02 Position Vector

03 Time formulas

04 Trajectory formula

05 Average velocity vector

06 Instantaneous velocity vector

07 Average acceleration vector

Definitions

Kinematics: it's the study of motion without references of forces

Position: The location of an object

Distance: is the trajectory of the kinematic point

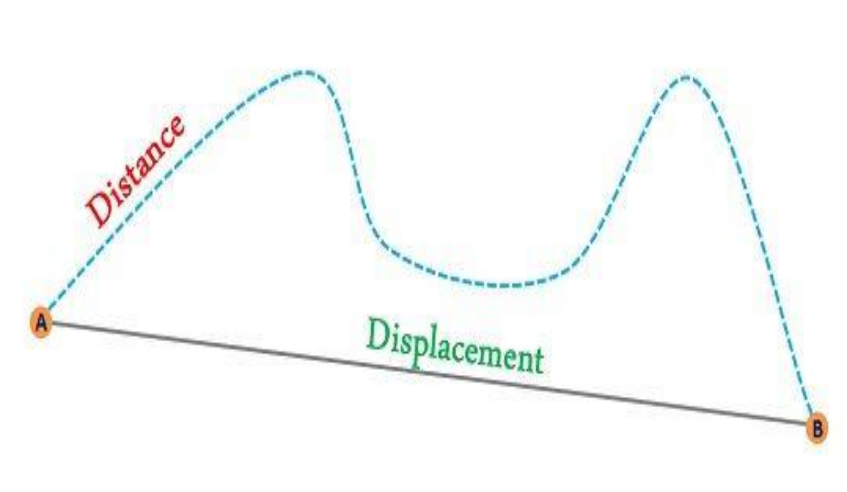
Displacement: The shortest distance between the starting and finishing point (linear distance)

Velocity: change Of position per unit of time

Acceleration: change of velocity per unit of time

Distance vs Displacement

- Lets have a body moving from point A to point B as it's shown in the figure below :
- The distance is the trajectory token by the body
- The displacement is the linear distance between the two points
- The distance is a scalar
- The displacement is a vector



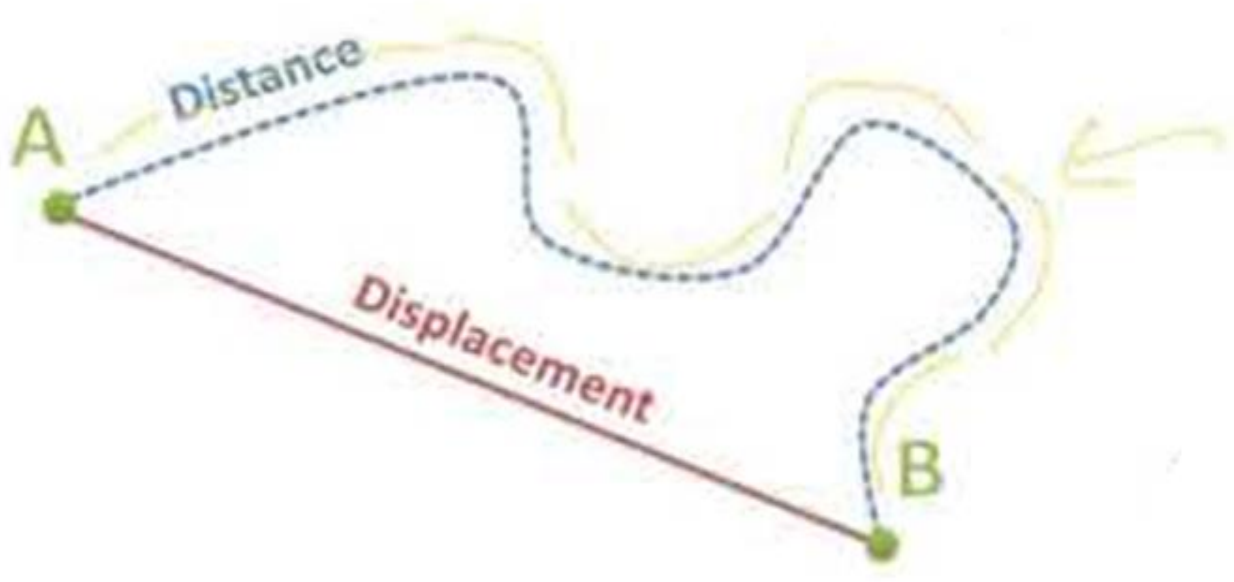
Distance vs Displacement

Distance

- Scalar quantity
- Units: metres (m), Kilometers(Km)
- Totale distance Traveled
- Used to find speed
- Symbol: d

Displacement

- Vector quantity
- Units: metres (m), Kilometers(Km)
- $\Delta d = d_{final} - d_{initial}$
- Shortest distance between 2points
- Used to find velocity
- Symbol: d



Δ:means change

Velocity vs Speed

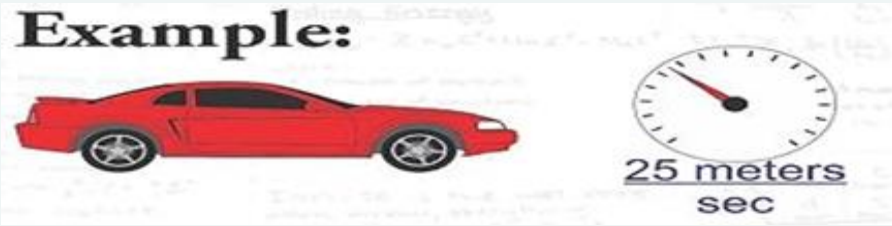
Velocity

Velocity is the vector quantity that signifies The magnitude of the rate of change of position And also the direction of an object's movement

vs

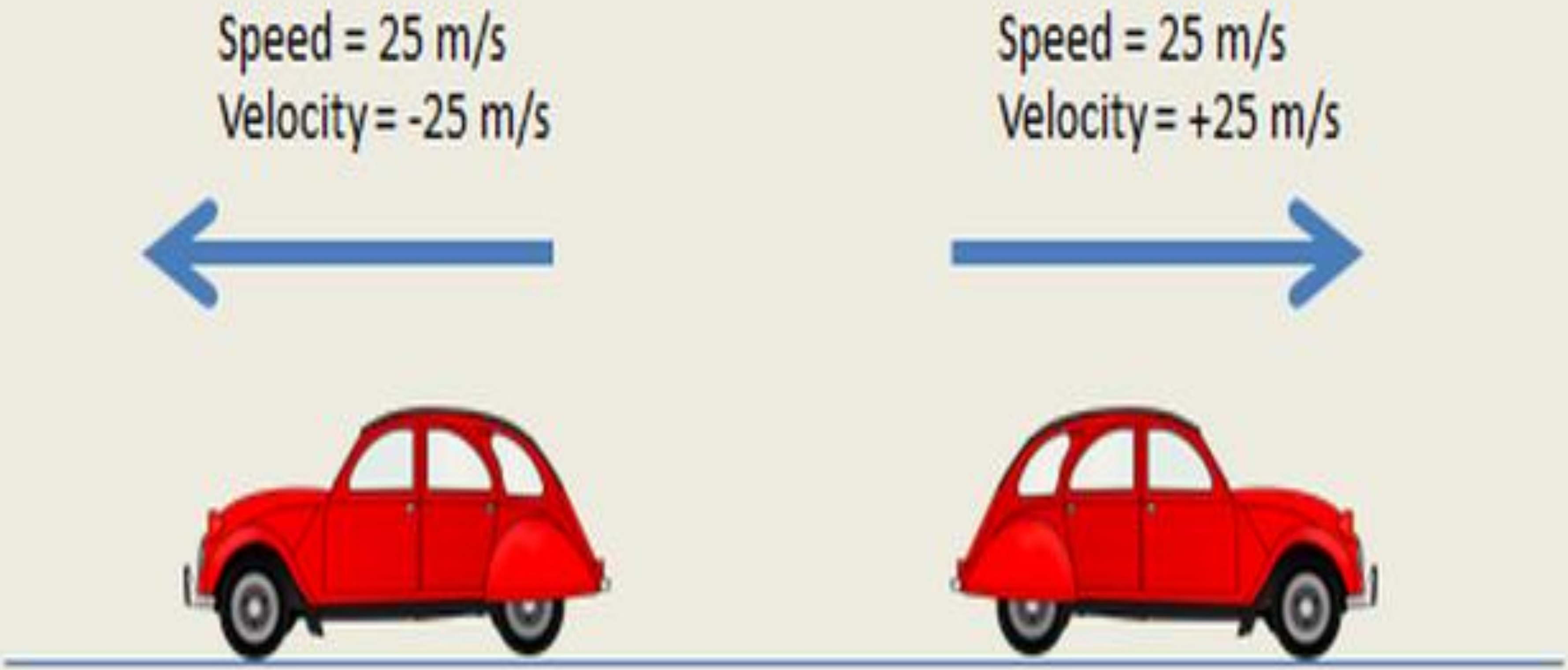
Speed

Speed is the scalar quantity that signifies only the magnitude of the rate of an object's movement



Simply, speed is the distance travelled per second …

Velocity is the distance travelled per second in a specific direction ..



Speed = 25 m/s
Velocity = -25 m/s
Speed = 25 m/s
Velocity = +25 m/s

Position Vector

Here the position of a material point M at time t is located in a R mark by its coordinates M(x,y,z), and the vector position is expressed by the formula below:

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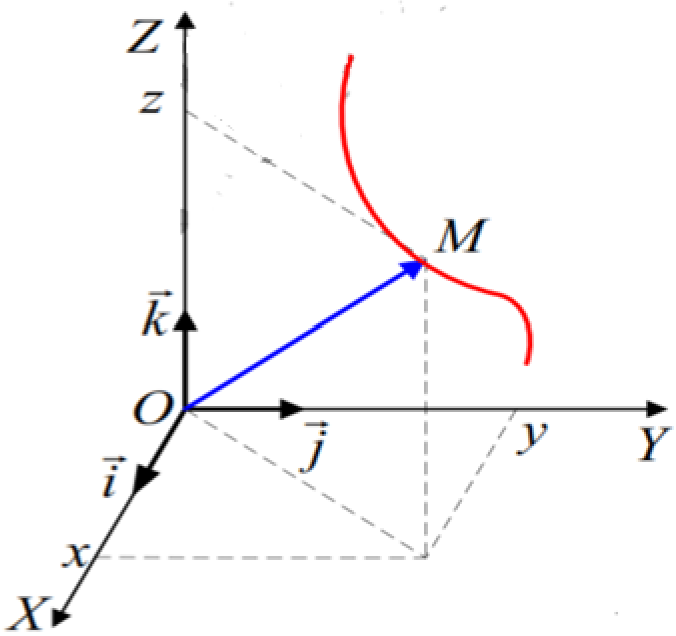


$$\overrightarrow{OM} = \vec{r} = x.\vec{i} + y.\vec{j} + z.\vec{k}$$

Time formulas

- We established that a body in landmark is located by it's coordinates M(x,y,z)
- If the body isn't moving (at rest) then it's coordinates will be constants
- But if the body is moving then it's coordinates varies in function of time and they are expressed by: $x(t)$, $y(t)$, $z(t)$
- We call those functions <u>**time equations**</u>

Trajectory Equation

- Let's have a body M moving in a landmark $R(0;\vec{i},\vec{j},\vec{k})$
- We get the trajectory formula of a M(x,y,z) by elimination time between it's coordinates
- Example : The time formulas of a moving kinematic

point M are x=3t, y=t^2 + 3t

What's it's trajectory formula?

Solution :

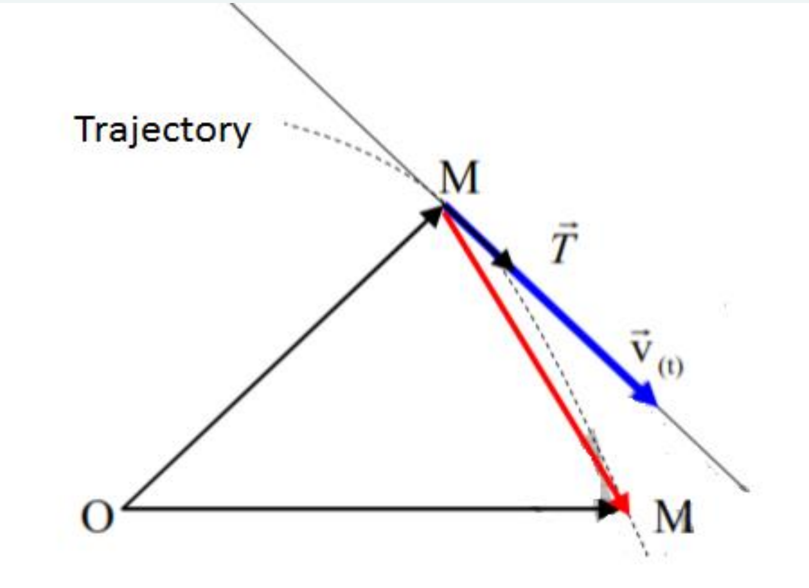
$t = \frac{x}{3}$,

$y = \frac{x^2}{9} + 3$ wish is it's trajectory formula

Average Velocity vector

- Average velocity $= \dfrac{Displacement}{change\ in\ time}$
- $\overrightarrow{\Delta v} = \dfrac{\overrightarrow{\Delta x}}{\Delta t} = \dfrac{\overrightarrow{MM\prime}}{t-t\prime}$
- X: is the position of the body at time t
- X': is the position of the body at time t'

- $\Delta v = \dfrac{|\Delta x|}{\Delta t} = \dfrac{|MM'|}{t-t\prime}$



Instantaneous velocity vector

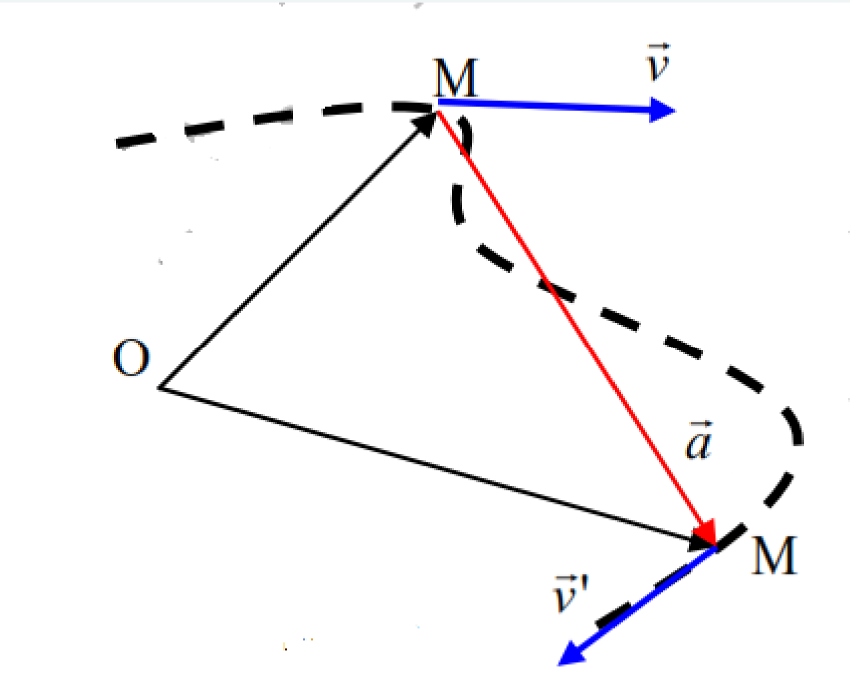
- The instantaneous velocity at a time t where the body is at a point M is the tangent of trajectory at the point M
- Wish mean that the Instantaneous velocity vector is the derivative of position vector with respect to time
- The module of the Instantaneous velocity vector is given by :
- $\vec{v}_t = \lim\limits_{t \to t'} \frac{\overrightarrow{OM'} - \overrightarrow{OM}}{t' - t} = \lim\limits_{t \to t'} \frac{\Delta \overrightarrow{OM}}{\Delta t} = \frac{d\overrightarrow{OM}}{dt}$

$$\overrightarrow{OM} = \vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k} \Rightarrow \vec{v} = \frac{d\overrightarrow{OM}}{dt} = \dot{x}\,\vec{i} + \dot{y}\,\vec{j} + \dot{z}\,\vec{k}$$

- The magnitude of the Instantaneous velocity vector is given by : $V = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$

Average acceleration vector

- Average Acceleration = $\frac{Average\ Velocity}{change\ in\ time}$
- $\overrightarrow{\Delta a} = \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{\overrightarrow{V'} - \overrightarrow{V}}{t' - t}$



Instantaneous acceleration vector

- The instantaneous velocity vector is the derivative of the velocity vector with respect to time
    - $\overrightarrow{OM} = \vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k} =>$

$$\vec{v} = \frac{d\overrightarrow{OM}}{dt} = \dot{x}\,\vec{i} + \dot{y}\,\vec{j} + \dot{z}\,\vec{k} => \vec{a} = \ddot{x}\,\vec{i} + \ddot{y}\,\vec{j} + \ddot{z}\,\vec{k}$$

- $\vec{v} = \frac{d\overrightarrow{OM}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k} => \vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\vec{i} + \frac{dv_y}{dt}\vec{j} + \frac{dv_z}{dt}\vec{k}$
- The magnitude of the instantaneous acceleration vector is given by:

- a= $\sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}$

Example :

A moving kinematic point M, has it's time formulas given by

$$M = \begin{cases} x = 2t - 5 \\ y = 2t^2 \\ z = \frac{1}{2}t \end{cases}$$

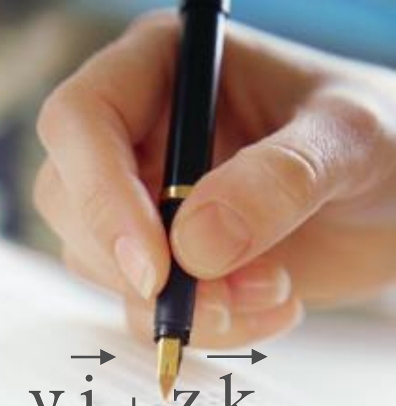
1- Write it's vector position

2- Write it's instantaneous velocity vector

3- Write it's instantaneous acceleration vector

4- Write it's average velocity vector between $t_1 = 1s$, and $t_2 = 3s$

5-Write it's average acceleration vector between $t_1 = 1s$, and $t_2 = 3s$



- Solution 1:

- 1- The vector position is given by : $\overrightarrow{OM} = x.\vec{i} + y.\vec{j} + z.\vec{k}$

thus $\overrightarrow{OM} = (2t-5).\vec{i} + 2t^2.\vec{j} + \frac{1}{2}t.\vec{k}$

- 2 – The instantaneous velocity vector is given by:

$V = \dot{x}.\vec{i} + \dot{y}.\vec{j} + \dot{z}.\vec{K}$

so $V = 2.\vec{i} + 4t.\vec{j} + 0.\vec{k}$

$$\vec{V}\begin{cases} Vx = 2 \\ Vy = 4t \\ Vz = \frac{1}{2} \end{cases}$$

3) The instantaneous acceleration vector is given by :

$\vec{a} = \ddot{x}.\vec{i} + \ddot{y}.\vec{j} + \ddot{z}\ .\vec{K}$ or $\vec{a} = \dot{v}_x.\vec{i} + \dot{v}_y.\vec{j} + \dot{v}_z.\vec{K}$

$\vec{a} = 0.\vec{i} + 4\ .\vec{j} + 0.\vec{k}$

$$\vec{a}\begin{cases} a_x = 0 \\ a_y = 4 \\ a_z = 0 \end{cases}$$

4) To calculate the average velocity first let's calculate it's positions at both t_1 = 1s (OM1), and $t_2 = 3$s (OM2)

- $\overrightarrow{OM1} = -3 \,.\, \vec{i} + 2 \,.\, \vec{j} + \frac{1}{2} .\, \vec{k}$
- $\overrightarrow{OM2} = 1 .\, \vec{i} + 18 \,.\, \vec{j} + \frac{3}{2} \,.\, \vec{K}$

Here $\Delta t = t_2 - t_1 = 2$s

$$\overrightarrow{\Delta v} = \frac{\overrightarrow{OM2} - \overrightarrow{OM1}}{t_2 - t_1} = \frac{4.\vec{i} + 16.\vec{j} + 1.\vec{k}}{2} = 2.\, \vec{i} + 8.\, \vec{j} + \frac{1}{2}.\, \vec{k}$$

$$\overrightarrow{\Delta v} = \begin{cases} \Delta v_x = 2 \\ \Delta v_y = 8 \\ \Delta v_z = \frac{1}{2} \end{cases}$$

5) To calculate the average acceleration first let's calculate it's instantaneous velocities at both

t_1 = 1s (V1), and t_2 = 3s (V2)

- $\overrightarrow{V1} = 2.\overrightarrow{i} + 4.\overrightarrow{j} + \frac{1}{2}.\overrightarrow{k}$
- $\overrightarrow{V2} = 2.\overrightarrow{i} + 12.\overrightarrow{j} + \frac{1}{2}.\overrightarrow{k}$
- $\overrightarrow{\Delta a} = \frac{\overrightarrow{V2} - \overrightarrow{V1}}{t_2 - t_1} = \frac{0.\vec{i} + 8.\vec{j} + 0.\vec{k}}{2} = 0.\overrightarrow{i} + 4.\overrightarrow{j} + 0.\overrightarrow{k}$
- $\overrightarrow{\Delta a} \begin{cases} \Delta a_x = 0 \\ \Delta a_y = 4 \\ \Delta a_z = 0 \end{cases}$