Rectilinear motions

Types of movement

Agendas

01 Rectilinear motion

02 Uniform rectilinear motion

03 Regularly varying rectilinear motion

04 Rectilinear motion with variable acceleration.

Types of movement

Straight

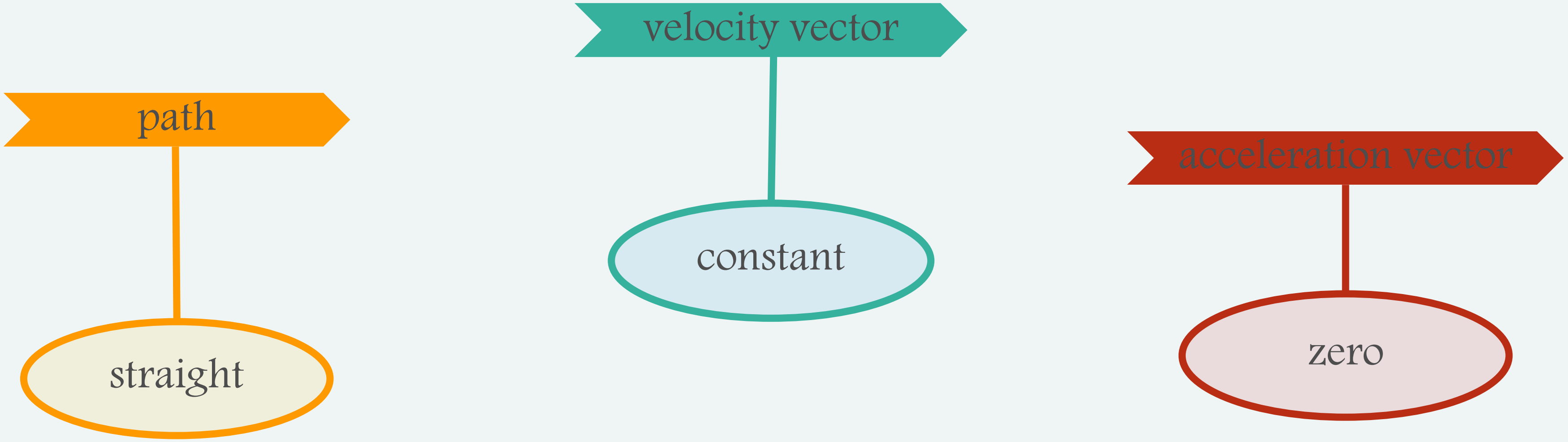
If we consider the path as a criterion for classifying movement

curved movement (sinusoidal circle)

rectilinear motion :

We call a motion rectilinear if it changes along a straight line.

Uniform rectilinear motion (Mouvement rectiligne uniforme)



Time equation

- we define the initial conditions

$$t = t_0 \,, \quad x = x_0$$

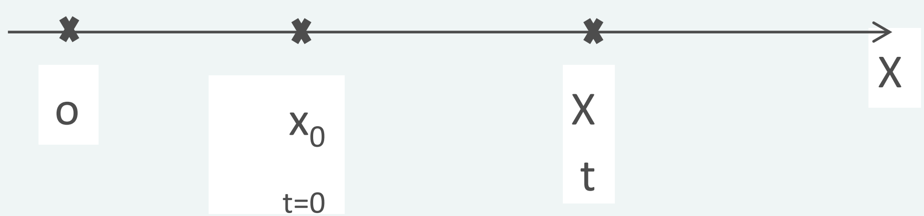
we have $v = \dot{x} = \frac{dx}{dt} = v_0 \Longrightarrow dx = v_0 dt = \int_{x_0}^{x} dx = \int_{t_0}^{t} v_0 dt$

$$x\Big|_{x_0}^{x} = v_0 t\Big|_{t_0}^{t} \Rightarrow x - x_0 = v_0(t - t_0)$$

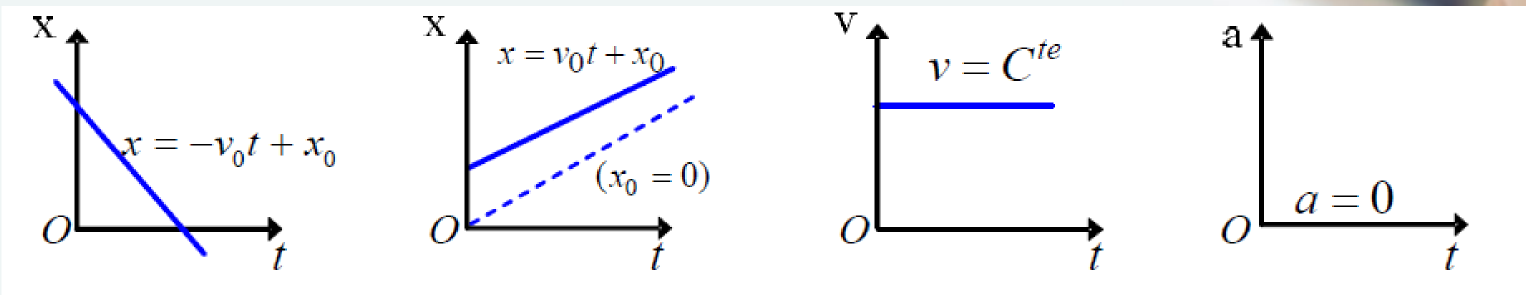
$$x = v_0(t - t_0) + x_0$$

in order to $t = 0s$

$$x = v_0 t + x_0$$



Movement diagrams (Diagram du Mouvement):



Example: The time equations of motion of a material point are $x=2t$; $y=2t+4$; $z=0$ (all units are in the international system). Show that the movement is rectilinear and uniform.

Answer : 1)Let us first look for the equation of the trajectory.

From the time equations we have :

$t = \frac{x}{2}$ $\Rightarrow$ $y = x+4$ equation of a line

therefore the movement is rectilinear.

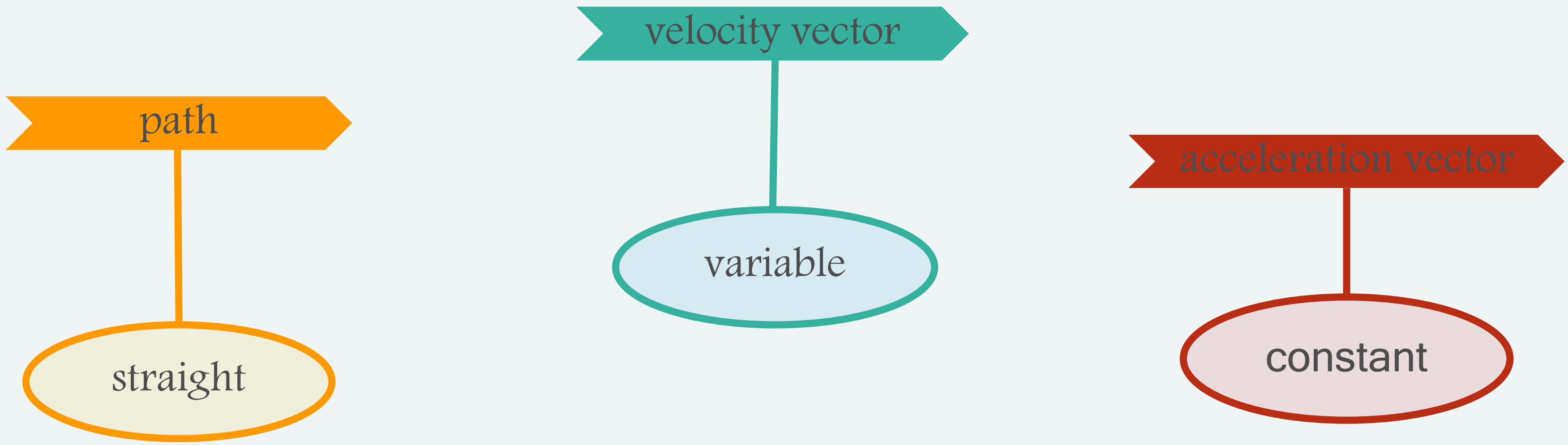
2) Let's calculate the velocity

$\vec{v} = 2\vec{i} + 2\vec{j}$ $\Rightarrow$ $v = \sqrt{2^2 + 2^2} = \sqrt{8}$ $\Rightarrow$ $v = 2{,}83 ms^{-1}$

The velocity is constant so the motion is uniform

Regularly varying rectilinear motion

The motion of a straight material point is uniformly variable



Algebraic speed: Taking into account the initial conditions $v=v_0$ and $t=0$ Based on the previous definitions

$$a = \frac{dv}{dt} \Rightarrow dv = adt \Rightarrow \int_{v_0}^{v} dv = \int_{t_0}^{t} adt \Rightarrow v\Big|_{v_0}^{v} = at\Big|_{t_0}^{t}$$

$$v - v_0 = a(t - t_0) \quad \Rightarrow v = a(t - t_0) + v_0$$

$$v = at + v_0$$

- **Time equation of motion:** If we take in $x=x_0, t=0$ And starting tangent.
- $v = \frac{dx}{dt} = at + v_0 \Rightarrow dx = (at + v_0)dt \Rightarrow$

$$\int_{x_0}^{x} dx = \int_{0}^{t} (at + v_0)dt$$

$$x - x_0 = \int_{0}^{t} at.\,dt + \int_{0}^{t} v_0 dt = \frac{1}{2}at^2 + v_0 t$$

- When $x=x_0$ and $t=0s$

$$x = \frac{1}{2}at^2 + v_0 t + x_0$$

- **The relationship is independent of time:**

$$a = \frac{dv}{dt} \ldots\ldots\ldots \textcircled{1}$$

$$v = \frac{dx}{dt} \Rightarrow dt = \frac{1}{v}dx$$

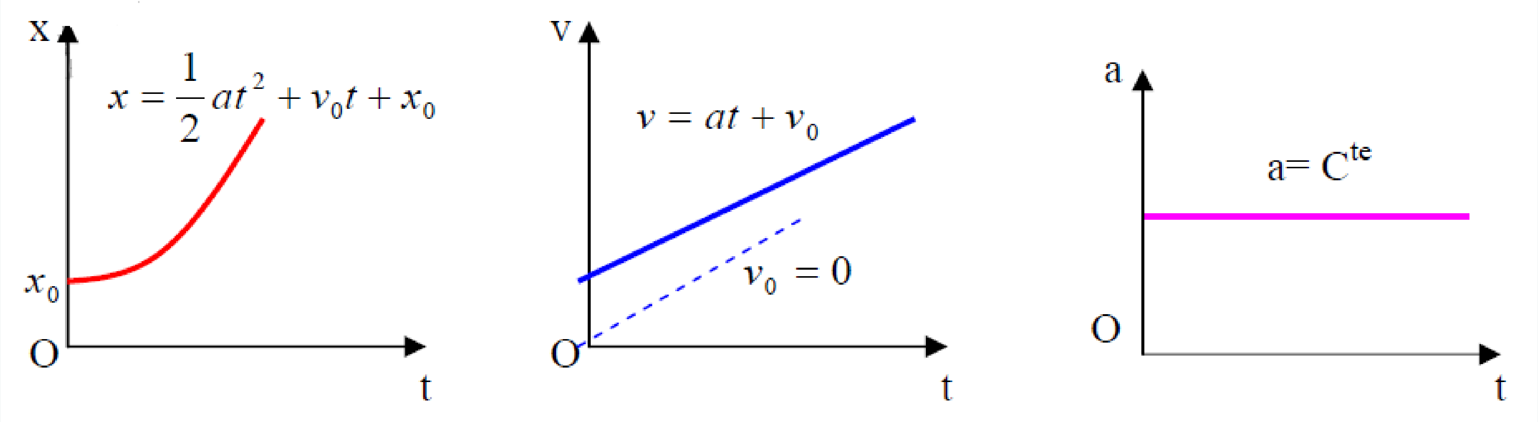
- By compensation in ① We find:

$$a = \frac{dv}{\frac{dx}{v}} = \frac{dv}{dx}.v \quad \Rightarrow adx = v.dv$$

$$\int_{x_0}^{x} adx = \int_{v_0}^{v} vdv \quad \Rightarrow a(x - x_0) = \frac{1}{2}(v^2 - v_0^2)$$

$$2a(x - x_0) = v^2 - v_0^2$$

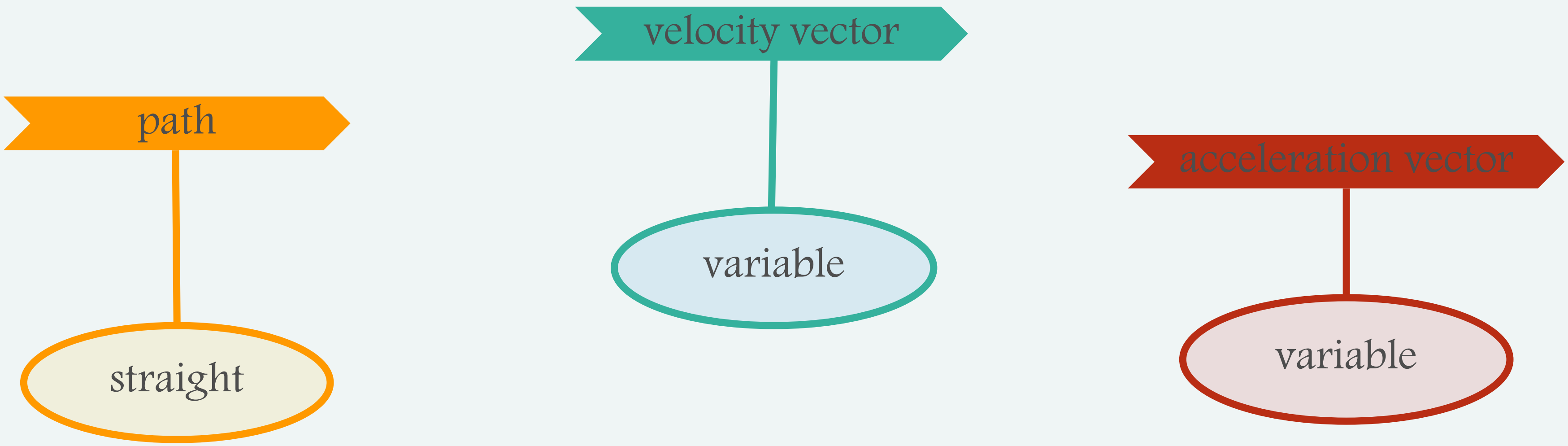
Movement diagrams



- So the three fundamental kinematics formulas are :
- $v = at + v_0$
- $x = \frac{1}{2}at^2 + v_0 t + x_0$
- $2a(x - x_0) = v^2 - v_0^2$

Rectilinear motion with variable acceleration:

The motion of a straight material point is uniformly variable



Example:

A point body moves along a straight line with acceleration $a = 4 - t^2$

♦ Find the expressions for velocity and translation as a function of time and By taking the conditions below:

$$t = 3s \quad , \qquad v = 2ms^{-1}, \qquad x = 9$$

- **Answer:**
- To obtain the literal expression of the velocity we must integrate the acceleration equation:

$$v = \int_0^t a\,dt + v_0 \Rightarrow v = v_0 + \int_0^t (4 - t^2)\,dt$$

$$v = v_0 + 4t - \frac{1}{3}t^3 \quad \Rightarrow \quad v = 4t - \frac{1}{3}t^3 + v_0$$

- We integrate again to get the phrase: literal transference

$$x = \int_0^t v\,dt + x_0 \Rightarrow x = \int_0^t \left(4t - \frac{1}{3}t^3 + v_0\right) + x_0$$

$$x = 2t^2 - \frac{1}{12}t^4 + v_0 t + x_0$$

$$x = -\frac{1}{12}t^4 + 2t^2 + v_0 t + x_0$$

- It now remains for us to determine both the initial interval and the speed $x_0 v_0$ For the body
- According to our data

$$t = 3s \quad , \qquad v = 2m/s, \qquad n = 9m$$

- We make up In the first phrase we find

$$2 = 4(3) - \frac{1}{3}(3)^3 + v_0 \Rightarrow 2 = 12 - 9 - v_0$$

$$v_0 = -1ms^{-1}$$

$$9 = -\frac{1}{12}(3)^4 + 2(3)^2 + (-1)(3) + x_0$$

$$9 = -\frac{1}{12}(81) + 2 \times 9 - 3 + x_0 \quad \Rightarrow \quad 9 + \frac{27}{4} - 18 + 3 = x_0$$

$$-6 + \frac{27}{4} = x_0 \quad \Rightarrow \quad \frac{-24 + 27}{4} = x_0 \quad \Rightarrow x_0 = \frac{3}{4}m$$

- Finally, we write the expressions for speed and translation:

$$x = 2t^3 - \frac{1}{12}t^4 - t + \frac{3}{4}$$

$$v = 4t - \frac{1}{3}t^3 - 1$$