Chapter three

Dynamics of a Material Point

General

- ✓ Kinematics is the study of the movements of material points as a function of time, irrespective of their causes.
- ✓ Dynamics is the science that studies (or determines) the causes of the movements of these material points (forces).

Notion of force

- A material point is in motion as a result of interactions between the particle and its environment. These interactions are called forces. These forces depend on the nature of the particle and the nature of its environment.
- The force is represented by a vector,such as :
- Its origin is the point of contact (force/body)
- Its direction: is the direction of motion supported on the wire, in the case of force: wire tension, for example.
- Its modulus: is the value of the force in Newton (N)

Different forces

- **Weight forces**: gravitational force due to the body's mass $\vec{P} = m\vec{g}$
- **Contact forces**: interaction forces between two bodies in contact. When the two bodies are in relative motion with respect to each other, we have a friction force, which always opposes the motion of the body in question.
- **Tension forces**: these are forces that pull on an element of a body: wire tension, spring tension, etc…
- **Other forces**: electrical forces, magnetic forces, etc.

$\vec{T}$

Example

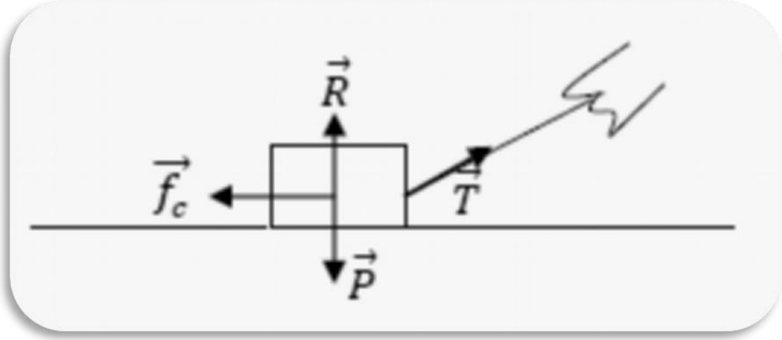
- A body slides on a horizontal surface by a wire (figure).
- The forces exerted on this body are:

✓ Weight force $\vec{P}$,

✓ Wire tension $\vec{T}$,

✓ Reaction force $\vec{R}$

✓ Friction force $\vec{f_e}$.



Mass and center of inertia

- The mass of a system characterizes the quantity of matter it contains. It is invariant in Newtonian mechanics. In relativistic mechanics, it depends on velocity through the expression through the expression :

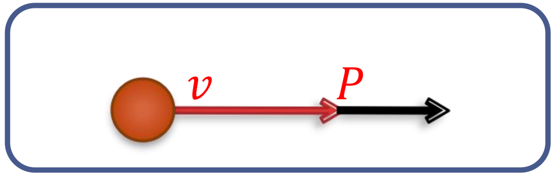
$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- where:
- m0, the mass at rest
- m, mass at speed v
- c, speed of light, c≈3 108 m.s-1

Momentum

- Let B be a material point with mass m moving with respect to a reference frame R .
- The momentum of B with respect to R is a vector denoted by $P_{A/R}$ and defined as:

$$\boxed{\vec{P}_{B/R} = m \, . \, \vec{v}_{B/R}}$$



Momentum

The momentum is a vector quantity that has the same direction as velocity.

The principle of inertia can then be stated as follows:

- A free particle moves with a constant momentum in a Galilean frame of reference.
- Or the total momentum of a system is constant if the principle of inertia is verified.

Forces

- Any cause capable of modifying the momentum vector of a material point in a Galilean reference frame is called a FORCE.

- An average force, such as :

$$\overrightarrow{F_{moy}} = \frac{\Delta\vec{P}}{\Delta t} = \frac{\vec{P}(t') - \vec{P}(t)}{t' - t}$$

- An instantaneous force, such as:

$$\vec{F}(t) = \lim_{t' \to t} \overrightarrow{F_{moy}} = \frac{d\vec{P}(t)}{dt}$$

Galilean Reference Frame

- A Galilean reference frame is a reference frame in which any isolated point is either at rest or moving with a constant velocity.
- If R is a Galilean reference frame, then any reference frame that is uniformly translating relative to R is also Galilean.
- Examples of Galilean reference frames:
- Copernican Reference Frame: *has its origin at the barycenter of the solar system and its axes directed towards three*

distant stars

- Geocentric Reference Frame: *has its origin at the barycenter of the Earth and its axes parallel to those of the Copernican reference frame*
- Earth Reference Frame:

Fundamental Laws of Dynamics

I. The principle of inertia (Newton's first law)
II. The basic principle of motion (Newton's second law)
III. The basic principle of motion (Newton's third law)

Newton's 1st laws: The principle of inertia

- Newton's 1st laws: The principle of inertia

If an isolated object is at rest or in uniform rectilinear motion, then $\Sigma\vec{F} = \vec{0}$

- Newton's 2 nd laws : The resultant of the forces exerted on a body is the derivative of the momentum :

$$\sum \vec{F} = \frac{\overrightarrow{dP}}{dt} = \frac{d}{dt} m\,\vec{v}$$

If mass is constant :

$$\sum \vec{F} = m\frac{d\vec{v}}{dt}$$

$$\sum \vec{F} = m\,\vec{a}$$

Newton's second laws Fundamental Principle of Dynamics (PFD).

In a Galilean frame of reference, the sum of the external forces applied to a system is equal to the derivative of the momentum vector of the system's center of inertia.

$$\sum \overrightarrow{F_{ext}} = \frac{d\overrightarrow{P}}{dt} = \frac{d}{dt}\left(m\overrightarrow{v}\right) = m\frac{d\overrightarrow{v}}{dt} = m\overrightarrow{\gamma}$$

Newton's third laws
Principle of action and reaction:

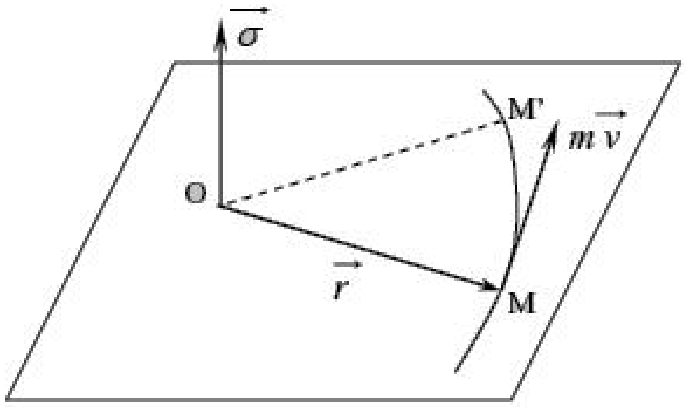
- Let two material points (1) and (2) interact with each other; the action exerted by (1) on (2) F12is equal and opposite to that exerted by (2) on (1) F21
- F12 is equal and opposite to that exerted by (2) on (1) F21 :
-

$$\overrightarrow{F_{12}} = -\overrightarrow{F_{21}} \qquad \left(\left|\overrightarrow{F_{12}}\right| = \left|\overrightarrow{F_{21}}\right|\right)$$

Angular momentum

- The angular momentum of point M moving at velocity v and having mass m relative to O is defined by:

$$\vec{\sigma} = \vec{r} \wedge m\vec{v}$$



Angular momentum

- Its derivative with respect to time is given by:

$$\frac{d\vec{\sigma}}{dt} = \frac{d\vec{r}}{dt} \wedge m\vec{v} + \vec{r} \wedge m\frac{d\vec{v}}{dt}$$

$$= \underbrace{\vec{v} \wedge m\vec{v}}_{0} + \vec{r} \wedge \frac{d\vec{P}}{dt}$$

$$\Rightarrow \quad \frac{d\vec{\sigma}}{dt} = \vec{r} \wedge \vec{F} = \overrightarrow{M^{t}_{0\vec{F}}}$$

Example of forces

Interaction forces at a distance

a. Newtonian gravitational forces
b. Coulombic interaction
c. Electromagnetic interaction

Contact forces

a. Reaction of the support
b. Friction forces
c. Viscous friction
d. Solid friction
e. Tension forces

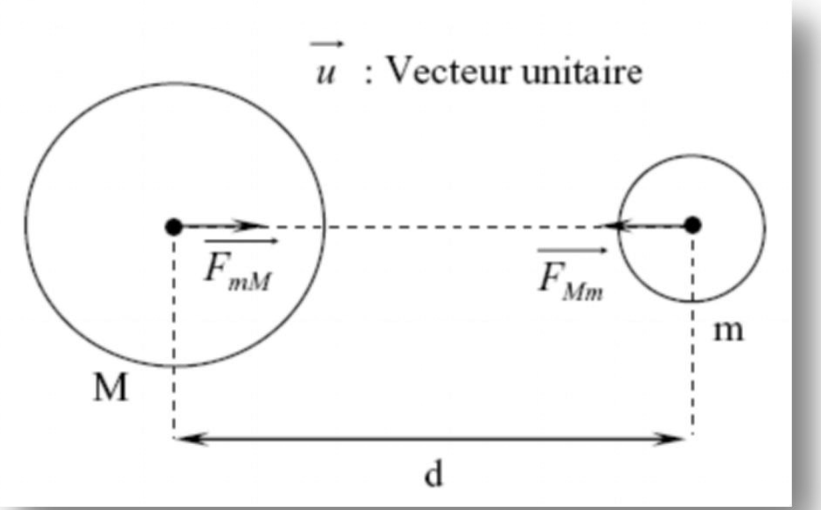
Newtonian gravitational forces

- The force exerted by a mass M on another mass m is called the gravitational force or gravitational interaction force.

$$\overrightarrow{F_{M-m}} = -G\frac{mM}{d^2}\vec{u}$$

G : une constante

$G = 6{,}67\ 10^{11}\ m^3 Kg^{-1} s^{-2}$

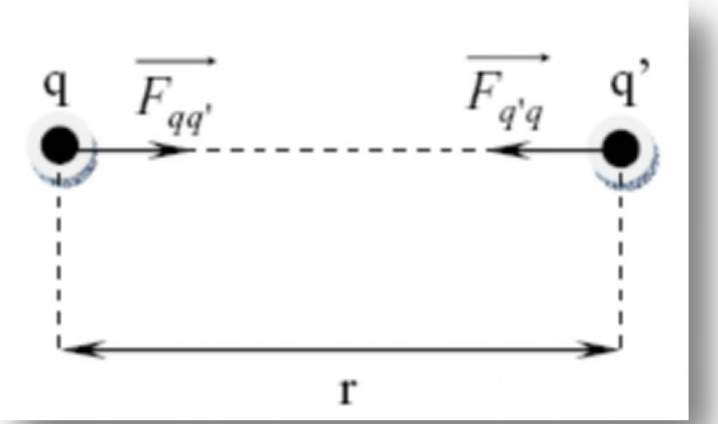


Coulombic interaction

- The coulombic interaction is the analogue of the gravitational interaction for electric charges.

$$\left|\overrightarrow{F_{qq'}}\right| = -\frac{1}{4\pi\varepsilon_0}\frac{q\,q'}{r^2} = K\frac{q\,q'}{r^2}$$

$$K = \frac{1}{4\pi\varepsilon_0} = 9\;10^9\;\; mc^2 Kg\,s^{-2}$$



Electromagnetic interaction

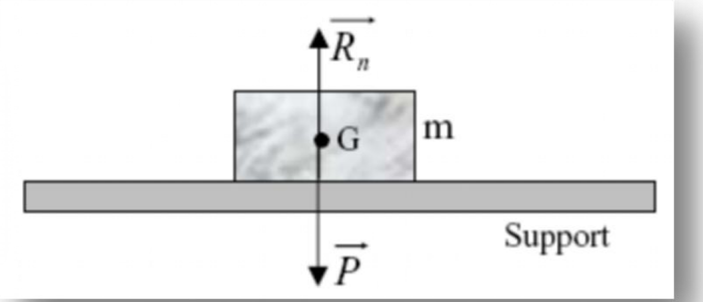
- The force experienced by an electric charge placed in fields E (electric) and B (magnetic) is called the electromagnetic or Lorentz force:

$$\vec{F} = q\left(\vec{E} + \vec{v} \wedge \vec{B}\right)$$

Reaction of the support

- The force exerted by the support on an object resting on a horizontal support is called support reaction.
- The reaction of the support on object m is distributed over the entire support-object contact surface Rn , represents the resultant of all the actions actions exerted on the contact surface.
- The object is in equilibrium:

$$\vec{P} + \vec{R_n} = \vec{0} \quad \Rightarrow \quad \vec{P} = -\vec{R_n}$$



Friction forces

- Friction forces are forces that arise either when an object moves. or the object is subjected to a force that tends to move it.
- There are two types of friction :

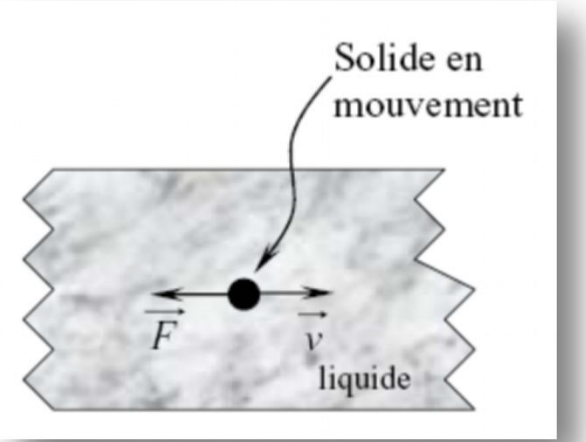
1. Viscous friction (solid-fluid contact)
2. Solid friction (solid-solid contact)

Viscous friction

- In this type of friction, the force is proportional to speed:

$$\overrightarrow{F} = -k\,\overrightarrow{v}$$

k: constante positive.



Tension forces

- Tension force or restoring force. The simplest example is the return force of a spring:

$$\vec{F} = -k(l - l_0)\vec{u}$$

k : coefficient d'allongement