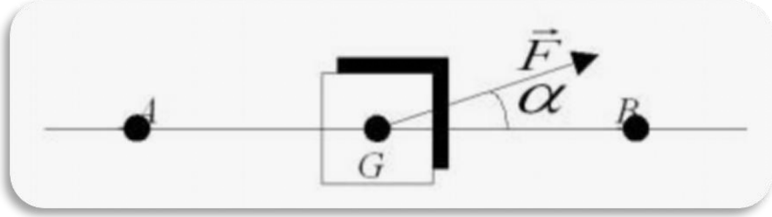
Chapter four
Work and Energy

Work of a force

Constant force on a rectilinear displacement:

- Let a material point M move along the line segment $[A\ B]$ under the effect of a force F .
- By definition, the work of force F on rectilinear displacement AB is given by:

$$W(\vec{F}) = \int_A^B \vec{F}\,\overrightarrow{dx} = F\ AB\ \cos\alpha$$



- α is the angle which F makes with $\overrightarrow{AB}$
- **Note**

➢ Work is positive ("driving work") if $\cos\alpha > 0$. $0 \le \alpha < \pi/2$
➢ It is negative (resistive work) if $\cos\alpha < 0$, $\pi/2 < \alpha \le \pi$
➢ It is zero when $\cos\alpha = 0$, $\alpha = \pi/2$.

Kinetic Energy and its theorem

Kinetic Energy:

- The kinetic energy of a material point of mass m and velocity v is given by :

$$E_c = \frac{1}{2}mv^2$$

Kinetic energy theorem:

- The change in kinetic energy of a point of matter between two positions A and B is equal to the sum of the work of all the forces acting on the point of matter during its movement between these two positions.

$$\sum W_{AB}(\vec{F})_{AB} = \Delta E_C = E_C(B) - E_C(A)$$

Kinetic Energy and its theorem

- The kinetic energy theorem is used to determine the velocity of a material point.
- It is based on the determination of the work of all external forces applied to this point.
- It is possible to define a second state function called the potential energy of the system.
- To do this, we need to distinguish between two types of external forces:

1. Conservative forces are those whose work does not depend on the path taken, but only on the starting and end points, such as weight, spring tension and the work of a constant force.
2. Non-conservative forces whose work depends on the path followed, such as friction forces.

Potential Energy

- Elastic potential energy :

$$E_p = \frac{1}{2}k(l - l_0)^2$$

- Gravitational potential energy :

$$E_p = mgh$$

- Gravitational potential energy of mass m in the field created by mass M :

$$E_p = -G\frac{mM}{r}$$

- All these quantities are defined within a constant.

Potential energy theorem

- If a force derives from potential energy (conservative force), its work between two positions A and B is equal and opposite to the variation in potential energy between these two positions.

$$W_{AB}=-\Delta E_p=E_p(A)-E_p(B)$$

Mechanical energy (total)

- The mechanical energy of a material point is the sum of kinetic and potential energies :

$$E_M = E_C + E_P$$

Principle of conservation of mechanical energy

- The (total) mechanical energy of a material point subjected to a force deriving from a potential energy, is conserved if
- $E_c(A)+E_P(A)=E_c(B)$
  $E_P(B)=c\ \ o\ \ s\ \ t\ \ e$
- so the change in mechanical energy iszero(ΔEm = 0).
- If the same system contains at least one force that does not derive from potential energy (a non-conservative force), then the mechanical energy is not conserved. In this case :
- ΔEm =W (non-conservative forces)
- $\Delta E\ \ m=\Sigma W(F\ \ \ n\ \ c\ \ )$ such that $F\ \ \ n\ \ c$ are the non-conservative forces.

Thank you for your attention