Series 1

Exercise 1 :

1. Show that : $\forall x, y \in \mathbb{R},\ x^2 + y^2 = 0 \Longleftrightarrow x = y = 0.$
2. Show that : $\forall x, y \in \mathbb{R}, \quad x \neq 2 \wedge y \neq 2 \Longrightarrow 2x - xy + 2y - 2 \neq 2.$
3. Show that : n is prime $\Longrightarrow n = 2 \vee n$ is odd

Exercise 2 :

1. Show by contradiction that : $\forall x \in \mathbb{R} : x \notin \mathbb{Q} \Longrightarrow 1 + x \notin \mathbb{Q}$.
2. Show by induction that : $\forall n \geq 0, \quad 6^n + 9$ is a multiple of 5.

Exercise 3 :

Consider the following four assertions :
(a) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, \quad x + y > 0.$ (b) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, \quad x + y > 0.$
(c) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \quad x + y > 0.$ (d) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, \quad y^2 > x.$

1. Are the assertions a, b, c and d true or false ?
2. Give their negation.

Exercise 4 :

In $\mathbb{R}$, we define the relation $\mathcal{R}$ by :

$$x\mathcal{R}y \Longleftrightarrow x^2 - y^2 = x - y.$$

1. Show that $\mathcal{R}$ is an equivalence relation in $\mathbb{R}$.
2. Calculate the equivalence class of an element x of $\mathbb{R}$.

Exercise 5 :

Le $f : \mathbb{R} \longmapsto \mathbb{R}$ be the application defined by $f(x) = x^2 + 1$. Consider the sets
$A =]-3, 2], \quad B = [0, 4].$
Compare the sets $f(A \cap B)$ and $f(A) \cap f(B)$.

Exercise 6 :

Let $f : \mathbb{R} \longmapsto \mathbb{R}$ be the application defined by $f(x) = x^2$ and $A = [-1, 4]$. Determine :

a) Direct image of A by f.
b) Reciprocal image of A by f.

Exercise 7 :

Let $g : \mathbb{R} \setminus \{\frac{1}{2}\} \to \mathbb{R}^*$ be the application such that :

$$g(x) = \frac{9}{2x - 1}.$$

Show that g is a bijection. Determine its reciprocal application.

Additional exercises

Exercise 1 : Show that : [n is odd] $\Longleftrightarrow$ [n^2 is odd]

Exercise 2 :

1. Let be the application $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ and $A = [-1; 4]$. Determine :
   (a) The direct image of A by f.
   (b) The reciprocal image of A by f.
2. What is the direct image of sets : $\mathbb{R}$, $[0, 2\pi]$, $[0, \frac{\pi}{2}]$, and the reciprocal image of sets$[0, 1]$, $[3, 4]$, $[1, 2]$ by application $\sin(x)$.