# Series 3 : Real functions of real variable

**Exercise 1** : Calculate the following limits :

1) $\lim\limits_{x\to 0} \dfrac{\ln(1+x^2)}{\sin^2 x}$ 2) $\lim\limits_{x\to 0} \dfrac{x\sin x}{1-\cos x}$

3) $\lim\limits_{x\to 0} x\exp\left(\dfrac{1}{x}-1\right)$ 4) $\lim\limits_{x\to +\infty} \left(\dfrac{x}{x-2}\right)^x$.

**Exercise 2** : Consider the function $f:\mathbb{R}\to\mathbb{R}$ defined by :

$$f(x)=\begin{cases}\cos^2(\pi x) & \text{if } x\in]-\infty,1]\\ 1+\dfrac{\ln x}{x} & \text{if } x\in]1,+\infty[\end{cases}$$

1. Study the continuity and differentiability of $f$ on $\mathbb{R}$.
2. Is the function $f$ of class $C^1$ on $\mathbb{R}$ ? Justify your answer.

**Exercise 3** :Let $f$ be the function defined by :

$$f(x)=x^2\cos\left(\frac{1}{x}\right),\quad x\in\mathbb{R}^*$$

1. Is the function $f$ extendable by continuity at 0 ?
2. Show that the equation $f(x)-1=0$ has at least one solution in the interval $\left[\dfrac{3}{\pi},\dfrac{4}{\pi}\right]$. Is this solution unique ?

**Exercise 4** : Let the function defined by :

$$f(x)=\arccos(2x-1)-\arcsin(3x^2)$$

1. Determine the domain of definition of $f(x)$.
2. Calculate the derivative of $f(x)$.

**Exercise 5** :

1. Solve the equations : $\arcsin x=\arcsin\dfrac{2}{5}+\arcsin\dfrac{3}{5}$.
2. Verify that : $\forall x\in]0,+\infty[$; $\arctan(x)+\arctan(\dfrac{1}{x})=\dfrac{\pi}{2}$.

**Exercise 6** : Simplify the following expressions :

1. $\sinh(\arg\cosh(x))$.
2. $\dfrac{2\cosh^2 x-\sinh 2x}{x-\ln(\cosh x)-\ln 2}$.

3 $\cos(\arctan(x))$.