Series 4 : Limited development

Exercise 1 : Using Taylor's formula , find the limited development of order 3 at 0 of the following functions :

1. $f(x) = \dfrac{1}{1-x}$

2. $g(x) = e^{x^2}$

Exercise 2 : I.Calculate the limited development to order 3 of the following functions :

1. $f(x) = \dfrac{\ln(1+x)}{\sin x}$ in the neighborhood of 0.

2. $f(x) = \dfrac{e^x}{x+e^x}$. in the neighborhood of 0.

3. $f(x) = \cos x$ in the neighborhood of $\dfrac{\pi}{3}$.

4. $f(x) = \ln(x + \sqrt{1+x^2}) - \ln x$. in the neighborhood of $+\infty$.

II. Calculate the limited development to order 10 of the function : $f(x) = \arccos(x^2)$

Exercise 3 : Calculate the following limits :

1. $\lim\limits_{x \to 0} \dfrac{1 + \ln(1+x) - e^x}{1 - \cos x}$

2. $\lim\limits_{x \to 0} \dfrac{\cos x - \sqrt{1-x^2}}{x^4}$

3. $\lim\limits_{x \to 0} \dfrac{2\,\text{th}\,x - \text{sh}\,2x}{x(1 - \text{ch}\,3x)}$

4. $\lim\limits_{x \to 0} \dfrac{e^x - e^{\sin x}}{x - \sin x}$

Additional exercises :

Exercise 4 : Calculate the $LD_3(0)$ of the following functions :

1. $f(x) = \ln(2 + x)$

2. $g(x) = \ln(\dfrac{\sin x}{x})$.

Exercise 5 : Calculate the following limits :

1. $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x^2}$

2. $\lim\limits_{x \to 0} \dfrac{2x}{\ln(\frac{1+x}{1-x})}$.