

Series 4 : Limited development.

Exercise 1 :

Determine the limited development in 0 to order 3 of the following functions :

1. $f_1(x) = \sqrt{1+x}$
2. $f_2(x) = \frac{e^x}{x+e^x}$
3. $f_3(x) = \ln(x+e^x)$.

Exercise 2 :

1. Calculate the limited development in 0 to order 3 of the function

$$f(x) = \sqrt{1 + \ln(1+x)}$$

2. Deduce

$$\lim_{x \rightarrow 0} \frac{1}{\sin^2 x} [f(x) - e^{\frac{x}{2}} + 1 - \cos x]$$

Exercise 3 :

Calculate the following limites :

1. $\lim_{x \rightarrow 0} \frac{1 + \ln(1+x) - e^x}{1 - \cos x}$
2. $\lim_{x \rightarrow 0} \frac{\cos x - \sqrt{1-x^2}}{x^4}$
3. $\lim_{x \rightarrow 0} \frac{2 \tanh x - \sinh 2x}{x(1 - \cosh 3x)}$
4. $\lim_{x \rightarrow 0} \frac{e^{\sin x} - e^{\tan x}}{\sin x - \tan x}$

Additional exercises

Exercise 4 :

Calculate the $DL_3(0)$ of the functions

1. $f_1(x) = (\cos x - 1)(\sinh x - x)$
2. $f_2(x) = \ln(2+x)$
3. $f_3(x) = \ln\left(\frac{\sin x}{x}\right)$.

Exercise 5 :

Calculate the following limites :

1. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$
2. $\lim_{x \rightarrow 0} \frac{2x}{\ln\left(\frac{1+x}{1-x}\right)}$.