University of Badji-Mokhtar Annaba | Mathematics 1

Sciences and Technology Depatement | 1 ST year : 2024-2025

Series 5 : LINEAR ALGEBRA

Exercise 1

$$\forall x, y \in IR^{+} : xTy = \ln\left(e^{x} + e^{y} - 1\right)$$

Show that T is an internal composition law of IR^{+} *and show that* $\left(IR^{+}, T\right)$ *is a commutative group.*

Exercise 2

The following families are free or linearly dependent in IR^{3}

1) $U_1 = (1,2,3), V_2 = (-1,4,6)$
2) $U_2 = (,12,-1), V_2 = (1,0,1), W_2 = (0,0;1)$
3) $U_3 = (,12,-1), V_3 = (1,0,1), W_3 = (-1,2;-3)$
4) $U_4 = (,12,-1), V_4 = (1,0,1), W_4 = (-1,2;-3), Z_4 = (-1,2,-3)$

Exercise 3

Show that the vectors $U_1 = (0,1,1), U_2 = (1,0,1), U_3 = (1,1,0)$ *form a base of* IR^{3}*, and find in this base the coordinates of the vector V=(1,1,1).*

Exercise 4 *Of the following sets, which are or are not vector subspaces?*

If yes, give a generating family, a base and the dimension

1) $E_1 = \{(x, y, z) \in IR^{3}, x + y + 3z = 0\}$
2) $E_2 = \{(x, y, z) \in IR^{3}, x + y + 3z = 2\}$
3) $E_3 = \{(x, y) \in IR^{2}, xy = 0\}$