TP series (Programming tools for mathematics).

TP1- First steps with MATLAB Objective: The objective of this first practical is to become familiar with the MATLAB graphical interface and to learn how to define calculation expressions and how to use certain predefined functions under the MATLAB command line (command window).

Exercise 1: Perform the following functions in Matlab:

1. $f\colon [0, 2\pi] \to \mathbb{R},\ x \mapsto \sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \frac{1}{7}\sin(7x),$
2. $f\colon [-3, 3] \to \mathbb{R},\ x \mapsto \sqrt[3]{x^2|x-2|},$
3. $f\colon [10^{-2}, \pi] \to \mathbb{R},\ x \mapsto \sqrt{x}\sin(1/x).$

Exercise 2: Perform the following function in Matlab:

$$f\colon [-3, 3] \to \mathbb{R}$$
$$x \mapsto (1+x)e^{-x^2+3x\cos(x)} - (1+x^4)^2\sin(x).$$

Exercise 3: Perform the following function in Matlab:

$$g(t) = e^{\cos(t)}\sin(t-1)^2 + 2\sqrt{t^3+7t}.$$

Exercise 4:
Give MATLAB commands to evaluate the following expressions:

1. $logarithme_{10}(2)$
2. $\frac{1}{1+\frac{1}{1+\frac{1}{2}}}$
3. $-x^6 - \frac{5}{7}x^3 + x^2 + 5$, pour $x = 1, x = 4$

4. $\frac{1}{\sqrt{8^3+2}} - \frac{2\sin(45)}{e^2} + \ln(4)$ Les angles sont donnés en degré.

5. $\frac{x^3 \sin(\frac{4\pi}{2})^2}{\cos(2\pi-1)}$, pour $x = e^3$

6. $-2\,ln(5x) + \sqrt{4x^3+1}$, pour $x = -3i$

7. $\frac{4}{3}\pi R^3$ où $R = 3cm$

8. $z \leftarrow \frac{|2n^5-3|}{\sqrt{4n^2+\ln(6n)}}$

9. $x \leftarrow \frac{e^{\sqrt{x}}}{2y-1} + |x| - \frac{1}{y^2+3}$

10. $w \leftarrow \frac{b}{2} \times \sqrt{c^2 - \left(\frac{b}{2,5}\right)^2}$;

11. $y \leftarrow e^{2-\sqrt{b^3-\frac{1}{a}}}$;

Series 2- Vectors in MATLAB

Objective: The objective of this practical work is to learn how to: define, manipulate and operate on vectors in MATLAB.

Exercise 1:

1. Let k be a vector defined from 3 to 5 with a step of 2, N a vector defined from 1 to 2 with a step of 1 and the vector F which is the horizontal concatenation of k and N. Define the vectors K, N and F in MATLAB and display the results obtained;

$$t = (0\ 3\ 6\ 9\ 12\ 15\ 18),\ v = \begin{pmatrix} 1 \\ 3 \\ 7 \\ 9 \\ 5 \end{pmatrix}$$

2. Define the following vectors t (row-vector) and v (column-vector):
Modify the values of the elements in the indices: 5 and 7 by the value -3 of the vector t.

$$t = (0\ 3\ 6\ 9{-}3\ 15{-}3);$$

3. Modify the values of the elements in the indices: 1 to 3 by the values 3, 5 and 9 respectively, then the elements of indices 4 and 5 by 8 and 10 of the vector v;

$$v = \begin{pmatrix} 4 \\ 8 \\ 0 \\ 5 \\ 7 \end{pmatrix}$$

4. Let $v1$ =(1 2 3 4), $v2$ =(7 6 0) be two row vectors. Create a row-vector v3 of size 10 by concatenating vector v1 of size 4 then two vectors of size 3 which correspond to vector v2;

$v3$=(1 2 3 4 7 6 0 7 6 0)

5. Let x and y be column vectors:

$$x = \begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix},\ y = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$$

Define z which is a concatenation of x and y:

$$z = \begin{pmatrix} 3 \\ 5 \\ 6 \\ 9 \\ 6 \end{pmatrix}$$

- Insert the value 2 in the first position of the vector z, the value 20 in the last position of the vector and the value -7 in the 3rd position of z;
- Delete the elements in positions 2 and 4;
- Order the elements of vector z in ascending order;

Exercise 2:

1. Construct the following vectors in MATLAB:

$u1$=(3 6 9…27 30)

$u2$=(−π/2 –π/4 0 π/4 π/2 …11π/4 3π)

$u3$=(0 1 49…81 100)

Exercise 3:

1. Propose the MATLAB instructions which allow you to define the following three vectors:

$v1$=(1 2 3 4 5 6 7 8 9 10), $v2$=(−1.5,0,1.5,…,4.5,6), $v3$=(1,1/4,1/9,1/16,1/25,…,1/81,1/100)

2. Create a vector v which contains all the elements of v1, v2, v3 consecutively;

3. Show the elements of v from the 11th position to the 5th position;

Exercise 4:

Let the row vector be v=(-1 12 0 15 23 50 1 8).

1. Calculate and display the sum, average, product of the elements of v;

2. Calculate and display the minimum and maximum in v;

3. Reverse the vector v (display the elements of the vector in reverse order);

4. Find and display values greater than 10 in v.

Series 3 - Manipulation of Matrices in MATLAB

Objective: The objective of this practical work is to learn how to: define, manipulate and operate on matrices (2-dimensional vectors).

Exercise 1 :

1. Give instructions (as simple as possible) to produce the matrix A of type 10 × 10 having the following form:

$$A = \begin{pmatrix} \pi & 0 & 0 & \cdots & -1 \\ 0 & \pi & 0 & \cdots & 0 \\ 0 & 0 & \pi & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 1 & \cdots & 0 & 0 & \pi \end{pmatrix}$$

Note:

The elements represented by dotted lines are all zero except on the main diagonal of A.

2. Calculate the first three elements of the main diagonal of A^{-1} and A^5

Exercise 2: Manipulating a matrix.

Give the result of each of the following MATLAB instructions:

$\gg C=diag(diag(ones(3)))$

$\gg k=[1:1:5]$

$\gg D=(ones(5)+diag(k))+3*zeros(5)$

Exercice 3: matrix operations

1. Define a matrix M = [1 2; 3 4] then try the following operations in the interpreter:

>>2*M+3

>>M^3

>>M.^3

>> M+M

>> M - M

>>M*M and M.*M

>>M/M and M./M

2. What is the difference between the operators * and .*, ^ and .^, / and ./? what is the result and meaning of each command?

3. Easily create a 54 × 42 matrix containing only 7?

4. Using the predefined MATLAB functions, calculate: the transpose matrix of M (M'), the inverse of M (M^{-1}), the determinant of M, the values and the eigenvectors of M.

Exercise 4: special matrices

- Define the matrices A, B, C and D, where: A is a square matrix of order 3 initialized randomly, B is a 3×2 matrix with all zero values, C is a matrix of dimension 2×3 with values all equal to 1 and D is a square matrix of order 3 with the values all zero except the diagonal equal to 1.

$$A = \begin{pmatrix} 0{,}95 & 0.48 & 0{,}45 \\ 0{,}23 & 0{,}89 & 0{,}01 \\ 0{,}60 & 0{,}76 & 0{,}82 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \text{ et } D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

– Give an appropriate nomination for each of these matrices.

– Let x be a column vector with values (3 5 7). Replace the diagonal of matrix D with vector x. As a result you will have:

$$D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{pmatrix}$$

– Create a magic matrix M of dimension 4×4. A magic matrix is a matrix where the sum of the elements of each column and the sum of the elements of each row and those of the two diagonals are identical.

TP4- MATLAB Scripts and Functions

Part A. MATLAB scripts: In order to be able to reuse the calculation lines, it is useful to put them in a script. A script is a text file that MATLAB can read and execute. To access it: (1) Open the MATLAB script editor either by clicking on the blank page in the toolbar, or by going to the "File/New/M-file" menu.

Exercise 1: Conditional Statements

1. Write the MATLAB script that requests two values x and y from the user and displays them, swaps their contents, and displays them again.
2. Write the MATLAB script that asks for a number and then displays its sign (positive, negative, or zero).

Exercise 2: Repeating instructions (loops)

1. Write a program that requires two integers a and b and displays the result of the following sum:

$$\sum_{k=1}^{b} k^{a}$$

2. Write a MATLAB program that takes as input a real $x \in]0.20[$, gives an error message if $x \notin]0.20[$, otherwise calculates and outputs the smallest n such that the sum is greater than x.

$$S_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

3. Write a program that calculates the 10th term of the Fibonacci sequence:

$$u_0 = 0, u_1 = 1, u_{n+2} = u_{n+1} + u_n$$

Part B. MATLAB Functions:
There are many predefined functions in MATLAB, but there will inevitably come a time when you want to use a function that is not defined. Fortunately, it is possible to define your own functions and use them exactly like pre-existing functions.

Exercise 3:

1. Write a function *pair* that can tell whether an integer x is even or not;
2. Write the *fonction somme* which calculates the sum of two matrices A and B.
3. Write the *fonction produit* which calculates the product of two matrices A and B.

Exercise 4:

Let the following calculation function have two nested loops:

```
function M=calcul(M)
[n,m]=size(M);
for i=1:n
v=M(i,:);
for j=1:m
M(i,j)=v(m-j+1)
end
end
```

1. Give the value of B after executing the following instructions:

```
>> A=[1 2 3 4;5 6 7 8 ;9 10 11 12];
>> B=calcul(A)
```

2. Derive what this function does.
3. Rewrite the calculation function to obtain the same result using a single loop.

Part C. Using functions and scripts together:
In case of complex and difficult to solve programs use functions with the MATLAB program.

Exercise 5 :

1. Write a MATLAB product function that takes two square matrices of the same dimensions A and B as argument and calculates the product $A \times B$, then displays the result to matrix C.

2. Write the MATLAB script which allows you to enter two square matrices of the same dimensions A and B, checks if their size is compatible. If they are not, gives an error message and no output, if they are, calculates $AB+BA$. The script calls the product function.

NB: We refrain from using operations of the type $A \times B$ or $A + B$, we must instead use an assignment element by element.

Exercise 6:

1. Write a MATLAB "fact" function that takes a positive integer $\boldsymbol{n}$ as an argument and returns $\boldsymbol{n}!$ as an answer. Knowing that $n!=\mathbf{1}\times\mathbf{2}\times\mathbf{3}\times\ldots\times\boldsymbol{n}$.
2. Write a MATLAB script which allows you to read two numbers $\boldsymbol{n}$ and $\boldsymbol{p}$ and which calculates and displays

$$\boldsymbol{n}!/(\boldsymbol{p}! * (\boldsymbol{n} - \boldsymbol{p})!)$$

TP5- Graphical representation in MATLAB

Objective: The objective of this practical work is to learn how to represent and analyze data graphically using graphic windows. We also learn how to build a graphical user interface using the MATLAB GUIDE.

Exercise 1: Represent the graphs of the following functions:

1. $f\colon [0, 2\pi] \to \mathbb{R},\ x \mapsto \sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \frac{1}{7}\sin(7x),$
2. $f\colon [-3, 3] \to \mathbb{R},\ x \mapsto \sqrt[3]{x^2|x-2|},$
3. $f\colon [10^{-2}, \pi] \to \mathbb{R},\ x \mapsto \sqrt{x}\sin(1/x).$

Exercise 2: Represent the graph of the function:

$$f\colon [1, 10] \to \mathbb{R}$$

$$x \mapsto \begin{cases} (\ln(x)+2)^2 & \text{si } \ln(x) - x + 2 \geq 0 \\ x^2 - 4x & \text{si } \ln(x) - x + 2 < 0. \end{cases}$$

Exercise 3: Represent the graph of the function:

$$f\colon [-3, 3] \to \mathbb{R}$$

$$x \mapsto (1+x)e^{-x^2+3x\cos(x)} - (1+x^4)^2\sin(x).$$

Exercise 4:

1.) Create, using the linspace function, a vector V of 120 points, with values between -13 and 13, then draw the graph of the function $2V^2 + 5$ as a function of V.

2.) Draw the curve corresponding to the function ( ):

$$y = 4\exp\left(-\frac{(x-5)^2}{2}\right) \text{ pour } 0 \leq x \leq 10$$

(We will start by creating a value table for x with a step of 0.01).

a) Annotate the axes by indicating the abscissa x in (cm) and the ordinate y in (u.a).

b) Write its legend on this curve.

c) The curve must be red and marked “star” size 3.

Exercise 5 :

Write a Matlab script which represents on the same graph, the functions sin(x), cos(x), $\sin^2(x)$, $\sin(x^2)$ in different colors.

Exercise 6:

Draw on the interval [−5,5] the function x2cosx in blue solid line and the function xcosx in red dotted line.