

Corrige type rathapage

LM. 2024/2025

Exercice 1:

$$q_0(0/1) \supset q_1$$

$$q_1(0/1) \supset q_1$$

$$q_1 \not\subseteq q_2$$

$$q_2 \subseteq q_3$$

$$q_3(0/1) \subseteq q_4$$

$$q_4 \supset q_5$$

$$q_5(0/1) \supset q_5$$

$$q_5 \not\subseteq q_6$$

$$q_6 = 1 \ q_6$$

$$q_6 \supset$$

$$q_4 \subseteq q_4$$

$$q_4 \not\subseteq$$

$$q_2 \supset q_7$$

$$q_7 \supset q_8$$

$$q_7 \subseteq q_4$$

$$q_8(0/1) \supset q_8$$

$$q_8 \not\subseteq q_9$$

$$q_9 \supset q_{10}$$

$$q_{10} \subseteq q_9$$

$$q_9(0/\#) \supset q_6$$

$$q_7 \not\subseteq$$

(4 pts)

Exercice 2:

$F_1:$

$$\forall x_4 \mid \exists y_3$$

(0,5 pt)

$$\rightarrow \begin{cases} \forall x_1 \mid p(x_4, y_1) \\ \exists x_2 \mid \phi(x_2, y_3) \end{cases}$$

$F_2:$

$$\forall x_4$$

$$\exists z_3$$

(0,5 pt)

$$\rightarrow \begin{cases} \forall u_1 \mid p(z_3, u_1) \\ \exists u_2 \mid \phi(u_2, x_4) \end{cases}$$

$F_3:$

$$\forall z_4$$

$$\exists u_3$$

(0,5 pt)

$$\rightarrow \begin{cases} \forall u_1 \mid p(z_4, u_1) \\ \exists z_2 \mid \phi(z_2, u_3) \end{cases}$$

$$F_1: \forall x_4 \exists y_3 (\forall x_1 p(x_4, y_1) \rightarrow \exists x_2 \phi(x_2, y_3))$$

$$F_1: \forall y \exists z (\forall x p(x, y) \rightarrow \exists z \phi(z, y))$$

$$F_2: \forall x_4 \exists z_3 (\forall u_1 p(z_3, u_1) \rightarrow \exists u_2 \phi(u_2, x_4))$$

$$F_2: \forall u \exists z (\forall x p(z, x) \rightarrow \exists z \phi(z, u))$$

(1)

$F_3: \forall x_4 \exists x_3 (\forall x_1 \phi(x_4, x_1)) \rightarrow \exists x_2 \phi(x_2, x_3))$ (0,5 pt)
 $F_3': \forall x_4 \exists x_3 (\forall x_1 \phi(x_4, x_1)) \rightarrow \exists x_2 \phi(x_2, x_3))$
 $F_1 \equiv F_3'$ donc les deux formules sont équivalentes. (0,5 pt)
 si tout est juste.

Exercice 3:

m) D e D n e x x $\rightarrow$ D F_1

T	T	T	F	T
T	F	F	T	F
F	T	F	F	F
F	F	F	T	T

(2,5 pts)

Le nombre de modèles générés = 2.

(0,5 pt)

n))

la table locale de $P(x)$:

x	$f_1(x)$	$f_2(x)$	$f_3(x)$	$f_4(x)$
1	T	T	F	F
2	T	F	T	F

(1 pt)

la table globale de F_2 :

	$P(x)$	Q	F_2
I_1	f_1	T	T
I_2	f_1	T	T
I_3	f_2	T	T
I_4	f_2	F	T
I_1'	f_3	T	T
I_2'	f_3	F	T
I_3'	f_4	T	F
I_4'	f_4	F	T

(0,5 pt)

$I_1: T \rightarrow \exists x \bigwedge_{x \in \{x_1, x_2\}} f_1(x) \rightarrow$ Vrai
 Donc T (0,25)

$I_2: F \rightarrow \exists x \bigwedge_{x \in \{x_1, x_2\}} f_1(x) \rightarrow$ Faux
 Donc T (0,25)

$I_3: T \rightarrow \exists x \bigwedge_{x \in \{x_1, x_2\}} f_2(x) \rightarrow$ Vrai
 Donc T (0,25)

$I_4: F \rightarrow \exists x \bigwedge_{x \in \{x_1, x_2\}} f_2(x) \rightarrow$ Faux
 Donc T (0,25)

$I_1': T \rightarrow \exists x \bigwedge_{x \in \{x_1, x_2\}} f_3(x) \rightarrow$ Vrai
 Donc T (0,25)

$I_2': F \rightarrow \exists x \bigwedge_{x \in \{x_1, x_2\}} f_3(x) \rightarrow$ Faux
 Donc T (0,25)

(2)

$$I_7: \mathcal{T} \rightarrow \exists n \begin{cases} \bigvee_{i=1}^{n-1} f_i(n) \text{ F} \\ \bigwedge_{i=2}^n f_i(n) \text{ F} \end{cases} \text{ Bonne F 0,2 ✓}$$

$$I_8: \mathcal{F} \rightarrow \exists n \begin{cases} \bigvee_{i=1}^{n-1} f_i(n) \text{ F} \\ \bigwedge_{i=2}^n f_i(n) \text{ F} \end{cases} \text{ Bonne F 0,2 ✓}$$

le nombre de modèles génériques est 7. 0,2 ✓ si la table est juste.

R3) la forme FNC de F_1

$$\begin{aligned} & (\Delta \wedge C) \leftrightarrow (\neg C \rightarrow \Delta) \\ & \equiv ((\Delta \wedge C) \rightarrow (\neg C \rightarrow \Delta)) \wedge ((\neg C \rightarrow \Delta) \rightarrow (\Delta \wedge C)) \\ & \equiv ((\neg \Delta \vee \neg C) \vee (C \vee \Delta)) \wedge ((C \vee \Delta) \rightarrow (\Delta \wedge C)) \\ & \equiv (C \vee \Delta) \rightarrow (\Delta \wedge C) \\ & \equiv (\neg C \wedge \neg \Delta) \vee (\Delta \wedge C) \\ & \equiv (\neg C \vee (\Delta \wedge C)) \wedge (\neg \Delta \vee (\Delta \wedge C)) \\ & \equiv (\neg C \vee \Delta) \wedge (\neg C \vee C) \wedge (\neg \Delta \vee \Delta) \wedge (\neg \Delta \vee C) \\ & \equiv (\neg C \vee \Delta) \wedge (\neg \Delta \vee C). \text{ FNC.} \end{aligned} \quad (3 \text{ pts})$$

il faut faire toutes les étapes.

Exercice 4:

$$P, \neg P \rightarrow (B \rightarrow C) \vdash \neg P \rightarrow C$$

$$1 \vdash P \text{ hyp 1}$$

$$2 \vdash \neg P \rightarrow (B \rightarrow C) \text{ hyp 2}$$

$$3 \vdash (\neg P \rightarrow (B \rightarrow C)) \rightarrow ((\neg P \rightarrow B) \rightarrow (\neg P \rightarrow C)) \text{ sih 1C on a remplacé } B \text{ par } \neg P.$$

$$4 \vdash (\neg P \rightarrow B) \rightarrow (\neg P \rightarrow C) \text{ mp 2, 3}$$

(3 pts)

5 $\vdash P \rightarrow (\neg B \rightarrow P)$ sch 1a
on a remplacé
le A par P
le B par $\neg B$.

6 $\vdash \neg B \rightarrow P$ mp 1,5

7 $\vdash (\neg B \rightarrow P) \rightarrow (\neg P \rightarrow B)$ sch 8.

8 $\vdash \neg P \rightarrow B$ mp 6,7

9 $\vdash \neg P \rightarrow C$ mp 8,4.