Mathematical Logic
Series of tutorials N°02 : Formal systems

Exercise 1

1. Define a formal system such that we can produce the theorems kst, kstst, kststst,........
from an axiom k.

2.Define a formal system such that can produce the theorems ca, caba, cababa, cabababa,
cababababa,..., etc. The axiom is c.

3. Define a formal system such that we can produce theorems b, ba, baa, baaa, baaaa,...,
etc. The axiom is b.

Exercise 2 Consider the MIU system which includes :
— The alphabet S = $\{M, I, U\}$
— The axiom A = $\{MI\}$
— the rules :
    — R_1 : if a string ends with an I we can add a U at the end,
    — R_2 : If we have a string Mx, we can form Mxx (where x is any string),
    — R_3 :we can replace III with a U in a string,
    — R_4 : we can delete any UU pair.

1. Prove that MUIUI is a theorem.

2. Is UM a theorem ?

3. Is MU a theorem ?

Exercise 3 Let the formal system p-q
S= $\{p, /, q\}$ A= $\{pq\}$ R =
    — a- $x \rightarrow /x/$
    — b- $xpy \rightarrow xp/y/$ (x and y are system words)
Can we derive the following strings : $//p/q///$; $/p//q/$; $//////p///q/////////$?

Exercise 4 Consider a formal system composed of :
An alphabet $\{A, B, C, D\}$,
Axioms : D, DD,
Deductive rules :
    — a- add C to the end of any string.
    — b- add an A at the beginning and end of any string.
    — c- replace a C with a B in a string.

Which of the following strings are theorems ? Give the proofs
DC, DCCC, DCCA, AAADAAA, AAADAAAA, AADCCCABBA.

Exercise 5 Let the formal system S (Σ, A, W, R) such that :

— Σ : it is the alphabet set such that $\Sigma = \{a, b, c\}$,
— A :it is the set of axioms which have the following form A = $\{a^{2i+1}bc^{2i-1} | i \geq 1\}$,

— W : represents the set of wffs generated from the axioms and wffs already generated,
— R : it is the set of rules such as R $= \{r_1 : (a^k bc^m, a^p bc^n) \longrightarrow a^{k+n} bc^{m+p}\}$

Q1 : Are the following formulas theorems $a^4 bc^4, a^5 bc^5, a^6 bc^6$?

Q2 : Give the different possible forms of theorems.