I.1.1. Generalities about Physical Quantities عموميات عن المقادير الفيزيائية

A Physical Quantity is a property that can be measured or calculated, and usually equals a value followed by a unit.

Example : mass (m=5 kg), time (t=12 h)

There are two types of physical quantities: fundamental quantities and derived quantities.

I.1.1.1. Fundamental (Basic) quantities المقادير الأساسية

The seven basic quantities are given in Table 1:

Table1. The seven fundamental quantities

Physical quantity	Symbol
Length	l (x, d)
Mass	m
Time	t
Electric current	i
Temperature	T
Luminous intensity	j (I_v)
Amount of substance	n

1.1.1.2. Derived Quantities المقاديرالمشتقة

These quantities are expressed as a combination of the seven fundamental quantities (multiplication, division, etc.).

Examples:

- Velocity v: v = l/t. (Unit m/s).
- Force F: F = m a = m(v/t) = m(l/t^2), (Unit: Newton N = kg m/s^2).

I.1.2. System of units نظام الوحدات

A physical quantity is characterized by both a numerical value and a unit. The four fundamental units thus chosen define the MKSA system, whose initials mean meter, kilogram, second and ampere respectively**.** The international system of units SI comprises seven basic units and two additional units**:** meter, kilogram, second, ampere, kelvin, mole and candela. (One can add a complementary unit: the angles, one assigns to a plane angle the radian unit). All other units called derived units are obtained by combining these basic units of the international system.

Remark:

Before the adoption of the MKSA system, another system in which the length was measured in centimeters, the mass in grams and the time in seconds already existed, is the CGS system (C=centimeter, g=gram and s=second).

I.1.3. Dimensional equations معادلات الأبعاد

I.1.3.1. Dimension البعد

The nature of a physical quantity is identified by its dimension. The dimension of a physical quantity G is noted by the expression **[G].** For example, if G has the dimension of a length, it is said to be homogeneous to a length, so the relation [G] = L corresponds to the equation to the dimensions (the dimension) of the quantity G.

So if G is the size of a:

The dimension and unity must therefore be coherent with each other. A quantity has a single dimension but can be expressed in several units.

For example, the mass has the dimension M and can be expressed in kg or g.

Table 2. dimension of fundamental quantities and their units in the SI system

Basic quantity	Dimension	Unit Name (SI)	unit symbol (SI)
Length	L	meter	m
Mass	M	kilogram	kg
Time	T	second	s
Electric current	I	ampere	A
Temperature	θ	kelvin	K
Amount of substance	N	mole	mol
Luminous intensity	J	candela	cd
Flat angle	A	radian	rad
Solid angle	Ω	steradian	sr

I.1.3.2. Dimensional equations معادلات الأبعاد

A dimensional equation is a mathematical relationship that expresses the dimension of a physical quantity as a function of the dimensions of the fundamental quantities. Generally, the dimensional equation of a derived physical quantity *G* is written:

$$[G] = M^{\alpha} L^{\beta} T^{\gamma} I^{\sigma}$$

The dimensional equations allows to:

•Determine the unit composed of a quantity according to the fundamental quantities.

• Check if a formula is homogeneous and detect errors in calculations.

• Perform unit conversions.

Example:

velocity: $v = \frac{x}{t}$

The velocity dimension: $[v] = \left[\frac{x}{t}\right] = \frac{[x]}{[t]} = \frac{L}{T} = LT^{-1}$

So, the of velocity dimensional equation: $[v] = LT^{-1}$ and the unit in SI: $m.s^{-1}$

Some of quantities are reported in the table 2.

Table 3. Dimensional equations of derived quantities and their units in SI

Derived quantity	**The expression**	**dimensional equation**	**IS Unit**	**Commonly used unit**
Acceleration	$a=l/t^2$	LT^{-2}	$m.s^{-2}$	
Force	$F=ma$	MLT^{-2}	$Kg.m.s^{-2}$	newton (N)
Pressure	$p=F/S$	$ML^{-1}T^{-2}$	$kg.m^{-1}.s^{-2}$	pascal (Pa)
Energy, Work	$W=Fl$	ML^2T^{-2}	$kg.m^2.s^{-2}$	joule (J)
Power	$P=W/t$	ML^2T^{-3}	$kg.m^2.s^{-3}$	watt (W)
Electric charge	$Q=it$	IT	A.S	coulomb (C)
Electric field	$E=F/q$	$MLT^{-3}I^{-1}$	$kg.m.s^{-3}.A^{-1}$	Volt/meter (V/m)
Potential (voltage)	$U=El$	$ML^2T^{-3}I^{-1}$	$kg.m^2.s^{-3}.A^{-1}$	volt (V)
Electrical capacity	$C=q/U$	$M^{-1}L^{-2}T^4I^2$	$Kg^{-1}.m^{-2}.s^4.A^2$	farad (F)
Resistance	$R=U/i$	$ML^2T^{-3}I^{-2}$	$kg.m^2. s^{-3}.A^{-2.}$	ohm (Ω)

Note:

The functions: sin(x), cos(x), tan(x), ln(x), log(x) and e^x are dimensionless (without dimensions), so $[\sin(x)]= [\cos(x)]= [\tan(x)]=[e^x]=[\ln(x)]=[\log(x)]=1$.

Also, a constant is dimensionless ($[\pi]=1$).

I.1.3.3. Homogeneity of dimensions تجانس الأبعاد

Dimensional equations are used to verify the homogeneity of formulas, that is, both its members have the same dimension.

I.1.3.4. Conversion from SI to CGS

Table 4 summarizes some conversions from SI to CGS

Table 4. Conversion from SI to CGS

Quantity	**IS Unit**	**Symbol**	**CGS Unit**	**Symbol**	**Equivalence**
length	m	m	cm	cm	$1m=10^2cm$
mass	kg	kg	g	g	$1kg=10^3g$
time	s	s	s	s	
acceleration	$m.s^{-2}$	$m.s^{-2}$	$cm.s^{2}$	Gal	$1m.s^{-2}=10^2cm.s^{-2}(1m.s^{-2}=10^2$ Gal)
force	$Kg.m.s^{-2}$	N (newton)	$g.cm.s^{2}$	Dyn (dyne)	$Kg.m.s^{-2} =10^5g.cm.\ s^{2}(1N=10^5dyn)$
energy	$Kg.m^2s^{-2}$	J (joule)	$g.cm^2.s^{-2}$	erg	$kg.m^2.s^{2}=10^7g.cm^2.s^{-2}(1J=10^7erg)$
pressure	$Kg.m^{-1}.s^{-2}$	Pa (pascal)	$g.cm^{-1}.s^{2}$	Ba (barye)	$Kg.m^{-1}.s^{-2}=10g.cm^{-1}s^{-2}(1Pa=10Ba)$

Exercises

Exercise 1

Write the dimensional equations of the following quantities and deduce their units in the international system (IS):

1. The pressure $P = \frac{F}{S}$

2. The quantity of movement $\vec{P}$: $\left(\vec{F} = \frac{d\vec{P}}{dt}\right)$

3. The momentum of $\vec{F}$: $\vec{\mathcal{M}}_{/O}\left(\vec{F}\right) = \vec{r} \wedge \vec{F}$

4. The angular momentum $\vec{L} = \vec{r} \wedge \vec{P}$

5. The electric field $E = F/q$

6. The electric potential $V = E.l$

Exercise 2

The T-period of a circular Earth satellite may depend on the mass of the Earth m, the radius of the circle described R and the constant of the universal gravitation G.

We will write: $T = k.m^a.R^b.G^c$, where k is a dimensionless constant.

- Determine by a dimensional analysis the values of a, b and c. Deduce the expression of the formula of the period T.

Exercise 3

Experience has shown that the force experienced by a sphere immersed in a moving fluid depends on:

- The viscosity coefficient η of the fluid.
- The radius of the sphere R.
- Their relative speed v.

Find the expression for this force by assuming the form: $F = k\eta^a R^b v^c$

(k is a dimensionless numerical coefficient).We recall that $[\eta]=L^{-1}MT^{-1}$.

Solution

Exercise 1

1. Pressure (P):

$$\boldsymbol{P} = \frac{\boldsymbol{F}}{\boldsymbol{S}}$$

$$[P] = \left[\frac{F}{S}\right] = \frac{[F]}{[S]}$$

F : force : F=ma and $a = \frac{v}{t} = \frac{\frac{x}{t}}{t} = \frac{x}{t^2}$

$F = m\frac{x}{t^2}$ So $[F] = [m]\frac{[x]}{[t^2]} = M.\frac{L}{T^2} = M.L.T^{-2}$

S : area : $S=l^2$ So $[S]=L^2$

$[P] = \frac{[F]}{[S]} = \frac{M.L.T^{-2}}{L^2}$ $\boxed{[P] = M.L^{-1}.T^{-2}}$

Unit of pressure in IS : $kg\cdot m^{-1}\cdot s^{-2}$= pascals (Pa) So 1 Pa = 1 N/m^2

2. Quantity of movement $\vec{P}$:

$$\vec{\boldsymbol{F}} = \frac{\boldsymbol{d\vec{P}}}{\boldsymbol{dt}}$$

$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow d\vec{p} = \vec{F}.dt = m.\vec{a}.dt.$$

$$\Rightarrow p = \int m.a.dt = m\int \frac{dv}{dt}dt = m\int dv = mv$$

So $p = mv \Rightarrow [p] = [mv] = [m][v] = [m]\left[\frac{x}{t}\right] = M.\frac{L}{T}$

$$\boxed{[p] = M.L.T^{-1}}$$

Unit of quantity of movement in IS: kg·m/s.

3. The momentum of force $\vec{F}$:

$\overrightarrow{\mathcal{M}}_{/O}(\vec{F}) = \vec{r}\wedge\vec{F} \Rightarrow M_{/O}(\vec{F}) = r.F \Rightarrow [M] = [r].[F]= L.M.L.T^{-2}$

$$\boxed{M.L^2.T^{-2} = [M]}$$

Unit of momentum of a force in IS: $kg\cdot m^2/s^2$ = joule (J)

4. The angular momentum $\overrightarrow{\mathcal{L}}$:

$\overrightarrow{\boldsymbol{\mathcal{L}}} = \vec{\boldsymbol{r}} \wedge \vec{\boldsymbol{P}}$ **(p is a quantity of movement)**

$\vec{\mathcal{L}} = \vec{r} \wedge \vec{p} = \vec{r} \wedge m\vec{v} \Rightarrow \mathcal{L} = r.m.v \Rightarrow [p] = [r][m][v] = [r][m]\left[\frac{x}{t}\right]$

$$\boxed{[\mathcal{L}] = M.L^2.T^{-1}}$$

Unit of angular momentum in IS: kg·m²/s

5. Electric Field (E):

$\boldsymbol{E} = \frac{\boldsymbol{F}}{\boldsymbol{q}} \quad \Rightarrow [E] = \frac{[F]}{[q]}$

$[F] = M.L.T^{-2}$

q: charge elctrique: $q = it \Rightarrow [q] = [i].[t] = IT$

$$[E] = \frac{[F]}{[q]} = \frac{M.L.T^{-2}}{IT}$$

$$\boxed{[E] = M.L.T^{-3}I^{-1}}$$

Unit of electric field in IS: kg·m·s⁻³·A⁻¹= (V/m).

6. Electric p otential (V):

V = E·l $\Rightarrow [V] = [E].[l] = M.L.T^{-3}I^{-1}.L$

$$\boxed{[V] = ML^2T^{-3}I^{-1}}$$

Unit of electric potential in IS: kg·m²·s⁻³·A⁻¹.= Volt (V)

Exercise 2

The equation of dimensions for period *T* is given by the following formula:

$[T]=[k.m^a.R^b.G^c] = [k].\ [m^a].\ [R^b].\ [G^c] = [m]^a.\ [R]^b.\ [G]^c$ (k is a dimensionless constant)

The dimension of each term must be determined:

[T] = T

[m] = M

[R] = L

[G] = ?

So according to the law of universal gravitation :

$$F = G\frac{mm'}{r^2}$$

Or *F* is the gravitational force between two masses *m* and m' separated by distance *r*

$\Rightarrow G = \frac{Fr^2}{mm'}$, so the dimension of G is : $[G] = \frac{[F][r]^2}{[m][m']}$

F is a force: $F=ma \Rightarrow [F] = [m][a] = MLT^{-2}$

$[r] = L \quad and \quad [m] = [m'] = M$

$$[G] = \frac{MLT^{-2}L^2}{M^2} = M^{-1}L^3T^{-2}$$

Replace [G] in the [T] expression

$T = M^a L^b (M^{-1}L^3T^{-2})^c = M^{a-c}L^{b+3c}T^{-2c}$

$$\begin{cases} a - c = 0 \\ b + 3c = 0 \\ -2c = 1 \end{cases} \Rightarrow \begin{cases} a = c \\ b = -3c \\ c = -1/2 \end{cases} \Rightarrow \begin{cases} a = -1/2 \\ b = 3/2 \\ c = -1/2 \end{cases}$$

So $T = k.m^{-1/2}.R^{3/2}.G^{-1/2}$

$$\boxed{T = kR\sqrt{\frac{R}{mG}}}$$

Exercise 3

$F = k\eta^a R^b v^c \Rightarrow [F] = [k][\eta]^a[R]^b[v]^c$

The dimensions of the quantities are:

Viscosity coefficient η: $[\eta] = M L^{-1}T^{-1}$

Radius of the sphere R: $[R] = L$

Relative speed v: $[v] = LT^{-1}$

Force F: $[F] = M.L.T^{-2}$

k is a dimensionless numerical coefficient, so $[k] = 1$

$\Rightarrow [F] = [k][\eta]^a[R]^b[v]^c$

$\Rightarrow M.L.T^{-2} = (ML^{-1}T^{-1})^a(L)^b(LT^{-1})^c$

$M.L.T^{-2} = (M)^a L^{-a+b+c} T^{-a-c}$

$$\begin{cases} \boxed{1 = a} \\ 1 = -a + b + c \Rightarrow b = 1 + a - c = 1 + 1 - 1 = \boxed{1 = b} \\ -2 = -a - c \Rightarrow c = -a + 2 = -1 + 2 = \boxed{1 = c} \end{cases}$$

$\Rightarrow F = k\eta^1 R^1 v^1$

So, the expression for this force $\boxed{F = k\eta Rv}$