

SERIES 1

(Mathematical tools)

Exercise 1 :

In an oriented space Oxyz, two vectors are given by :

$$\vec{v}_1 = \overrightarrow{AB} \quad \text{with } A(1, 3, 8) \text{ and } B(6, 7, 8)$$

$$\vec{v}_2 = -8\vec{i} + \alpha\vec{j} + 7\vec{k}$$

- Find the value of α so that both vectors are perpendicular.

Exercise 2 :

Let the vectors : $\vec{A} = 2\vec{i} + 3\vec{j} + 5\vec{k}$ and $\vec{B} = \vec{i} - 2\vec{j} + 3\vec{k}$

- a-* Calculate the vector product $\vec{C} = \vec{A} \wedge \vec{B}$
- b-* Find the unit vector perpendicular to the plane $(\vec{A}, \vec{B})$.
- c-* Deduce the angle between these two vectors.

Exercise 3 :

In an orthonormal coordinate system $(O, \vec{i}, \vec{j}, \vec{k})$, consider the parallelepiped P defined by the three vectors:

$$\vec{A} = 5\vec{i} - \vec{j} + 2\vec{k}, \quad \vec{B} = 3\vec{j} - \vec{k} \quad \text{and} \quad \vec{C} = 4\vec{i} + \vec{k}$$

- a-* Calculate the volume of the parallelepiped P .
- b-* Are the three vectors coplanar? Justify your answer.

Exercise 4 :

Let the vector $\vec{A} = 2xyz\vec{i} + (2x^2 - y)\vec{j} + yz^2\vec{k}$ and the function

$$f(x, y, z) = x^2y + 2y^2z^3$$

- Calculate : $\overrightarrow{\text{grad}} f(x, y, z)$ and $\text{div } \vec{A}$ at the point $(1, 0, 0)$.