جــامـعـة بــاجـي مخـتــار - عـنـابـة
BADJI MOKHTAR - ANNABA UNIVERSITY

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Introduction: Reminder of some mathematical tools

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In this chapter, we will give some mathematical definitions (such as the scalar product, the vector product as well as the total derivative and the partial derivative) which are necessary to carry out calculations later in the different chapters of the General Electricity course.

The different concepts in electricity are either:

Scalar

Positive or **negative** number used to represent quantities defined by their magnitudes without being associated with a specific direction:

- potential V,
- work W,
- charge q, ….

Vector

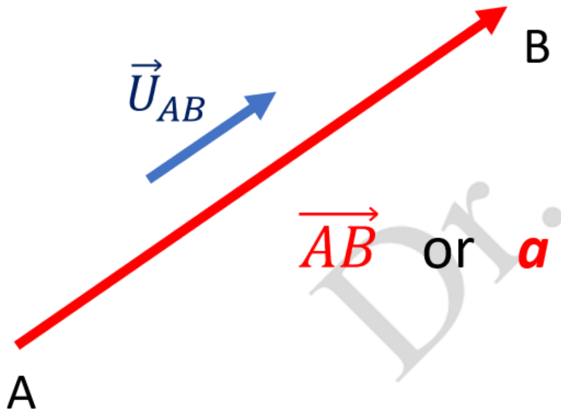
$\vec{A}$ is defined by a direction, a holder, a magnitude (also known as the modulus), and the point of application, such as :

- force $\vec{F}$,
- electric field $\vec{E}$,
- magnetic field $\vec{B}$, ….

It is important when writing vectors to distinguish them from scalars.

Various notations are used. We emphasise that we are dealing with the vector from A to B by using an arrow and writing $\overrightarrow{AB}$.

Often, in textbooks, vectors are indicated by using a bold typeface such as ***a***.



$\vec{U}_{AB}$ *is the **unit vector** of the vector* $\overrightarrow{AB}$

Example

Consider the vector : $\vec{V} = \vec{\imath} - 2\,\vec{\jmath} + 3\,\vec{k}$.

Calculate the modulus V then deduce the unit vector $\vec{U}$.

Solution

1- The modulus V:

$$V = \sqrt{1^2 + (-2)^2 + 3^2}$$

$$V = \sqrt{14}$$

2- The unit vector $\vec{U}$:

$$\vec{U} = \frac{\vec{V}}{V}$$

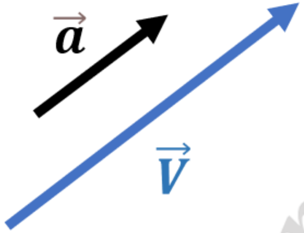
$$\vec{U} = \frac{1}{\sqrt{14}}\,\vec{\imath} - \frac{2}{\sqrt{14}}\,\vec{\jmath} + \frac{3}{\sqrt{14}}\,\vec{k}$$

1. Operations on Vectors

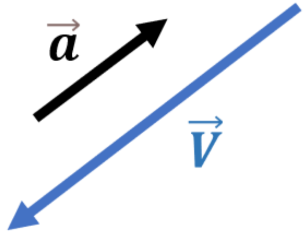
Multiplication of a Vector by a Scalar

When we multiply the vector $\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$ by a number α, we obtain a new vector $\vec{V} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$

- The vector $\vec{V} = \alpha.\vec{a}$.
- The magnitude $V = |\alpha|.\, a$.
- The components : $\begin{cases} V_x = \alpha.\, a_x \\ V_y = \alpha.\, a_y \\ V_z = \alpha.\, a_z \end{cases}$



if $\alpha > 0$



if $\alpha < 0$

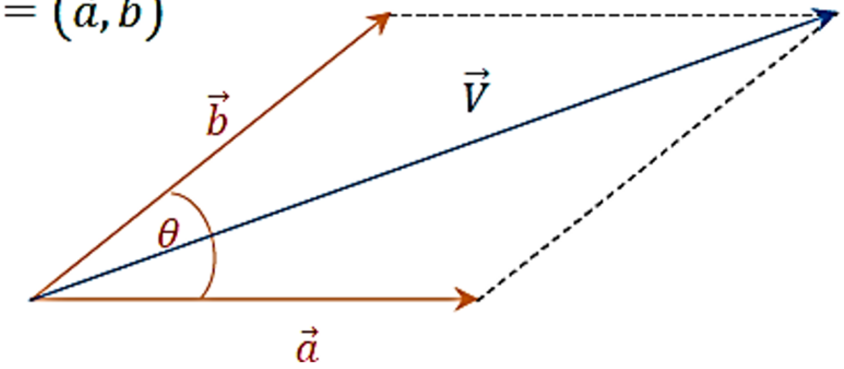
Vector Addition

The sum of the vectors $\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}$ is the resultant vector $\vec{V} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$ such that :

- The vector : $\vec{V} = \vec{a} + \vec{b}$
- The magnitude : $V = \sqrt{\vec{V}^2} = \sqrt{\left(\vec{a} + \vec{b}\right)^2} = \sqrt{\vec{a}^2 + \vec{b}^2 + 2\left(\vec{a}.\vec{b}\right)}$

$V = \sqrt{a^2 + b^2 + 2ab\cos\theta}$ where $\theta = \left(\vec{a}, \vec{b}\right)$

- The components : $\begin{cases} V_x = a_x + b_x \\ V_y = a_y + b_y \\ V_z = a_z + b_z \end{cases}$



Scalar product (Dot product .)

The scalar product of vectors $\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}$ is defined as the scalar A that can be calculated in two ways :

- Using the magnitudes of the vectors and the angle θ between them : $A = \vec{a}.\vec{b} = ab\cos\theta$
- Using the components of the vectors : $A = \vec{a}.\vec{b} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}.\begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = a_x b_x + a_y b_y + a_z b_z$

Example

Calculate the scalar product A of the vectors : $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$ and $\vec{b} = 3\vec{i} + \vec{j} - 2\vec{k}$.

Solution

$A = \vec{a} \, . \, \vec{b} = -5$

Some applications of the scalar product

- **Using the scalar product to test whether two vectors are perpendicular**

*If both the vectors **a** and **b** are non-zero and we find that **a** · **b** = 0 then we can deduce cos θ must be zero, so that **θ = 90°** , i.e. a and b are **perpendicular**.*

- **Using the scalar product to find the angle between two vectors**

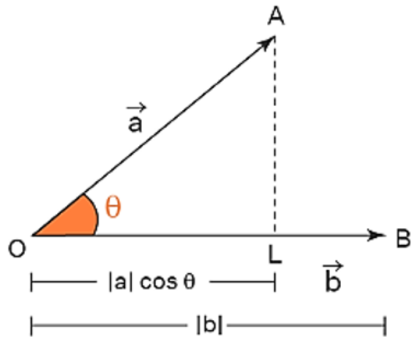
From the definition of the scalar product $\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}|\,|\mathbf{b}|\cos\theta \Rightarrow \cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|}$

*If we are given **a** and **b** in cartesian form, we can calculate their modulus*

$$|a| = \sqrt{a_1^2 + a_2^2 + a_3^2} \quad \text{and} \quad |b| = \sqrt{b_1^2 + b_2^2 + b_3^2}$$

- **Finding the component of a vector in the direction of another vector**

*The vector **projection** of one vector over another vector is the length of the shadow of the given vector over another vector. The resultant of a vector projection formula is a scalar value.*



a . b = |a|.|b| cos θ

Some properties of scalar product

- **Commutative property:**

With the usual definition, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

- **Distributive property:**

- $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
- $(\vec{a} + \vec{b}) \cdot \vec{c} = \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c}$
- $\vec{a} \cdot (\vec{b} - \vec{c}) = \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}$
- $(\vec{a} - \vec{b}) \cdot \vec{c} = \vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{c}$

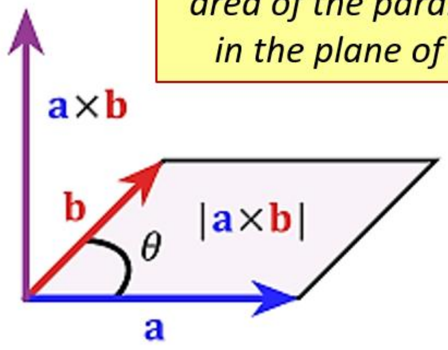
- **Natural property**

- We know that $0 \leq \theta \leq \pi$.
- If $\theta = 0$ then $\vec{a} \cdot \vec{b} = ab$ [Two vectors are parallel in the same direction $\Rightarrow \theta = 0$].
- If $\theta = \pi$, $\vec{a} \cdot \vec{b} = -ab$ [Two vectors are parallel in the opposite direction $\Rightarrow \theta = \pi$.].
- If $\theta = \pi/2$, then $\vec{a} \cdot \vec{b} = 0$ [Two vectors are perpendicular $\Rightarrow \theta = \pi/2$]
- If $0 < \theta < \pi/2$, then $\cos\theta$ is positive and hence $\vec{a} \cdot \vec{b}$ is positive.
- If $\pi/2 < \theta < \pi$ then $\cos\theta$ is negative and hence $\vec{a} \cdot \vec{b}$ is negative.

Vector product (Cross product x)

The vector product of the vectors $\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}$ is the vector $\vec{V}$ that is directly perpendicular to the plane $(\vec{a}\,, \vec{b}\,)$.

Scalar value of *a* x *b*, is the area of the parallelogram in the plane of *a* and *b*



We can calculate the vector $\vec{V}$ in two ways :

- ***Using the magnitudes and the angle between the vectors :***

$$\vec{V} = \vec{a} \wedge \vec{b} = \vec{a} \times \vec{b}$$

$\vec{V} = V\,\vec{U}_V = \|\,\vec{a} \wedge \vec{b}\,\|\,\vec{U}_V = a\,b\,sin\theta\,\vec{U}_V$ where $\theta = (\vec{a}\,, \vec{b}\,)$ and $\vec{U}_V \perp (\vec{a}\,, \vec{b}\,)$

➢ ***Using the components of the vectors :***

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}=\vec{i}\cdot\begin{vmatrix}a_y & a_z\\ b_y & b_z\end{vmatrix}-\vec{j}\cdot\begin{vmatrix}a_x & a_z\\ b_x & b_z\end{vmatrix}+\vec{k}\cdot\begin{vmatrix}a_x & a_y\\ b_x & b_y\end{vmatrix}$$

$$\vec{a}\times\vec{b}=(a_yb_z-a_zb_y)\cdot\vec{i}-(a_xb_z-a_zb_x)\cdot\vec{j}+(a_xb_y-a_yb_x)\cdot\vec{k}.$$

Special Mention

While calculating the vector product, the following set of rules should be kept in mind.

$\vec{\imath}\wedge\vec{\jmath}=\vec{k}$ $\qquad$ $\vec{\jmath}\wedge\vec{k}=\vec{\imath}$ $\qquad$ $\vec{k}\wedge\vec{\imath}=\vec{\jmath}$

Where : $\vec{\imath}$, $\vec{\jmath}$, $\vec{k}$ are the unit vectors in x, y, and z directions.

Some properties of vector product

- **Non-Commutative property:**

Cross-products are non-commutative.

$$\vec{a} \wedge \vec{b} \neq \vec{b} \wedge \vec{a}$$

- **Distributive property:**

Just like scalar products, cross products are also distributive in nature.

$$\vec{a} \wedge (\vec{b} + \vec{c}) = (\vec{a} \wedge \vec{b}) + (\vec{a} \wedge \vec{c})$$

- **If** $\vec{a} \wedge \vec{b} = \vec{0}$ and $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0} \Rightarrow$ the two vectors are **parallel //** to each other.

Some applications of vector poduct

- *Determining the **Area** of a Parallelogram*
- *Computing Torque*
- *Magnetic Field*
- *Calculating the Normal Vector, etc ....*

Mixed product

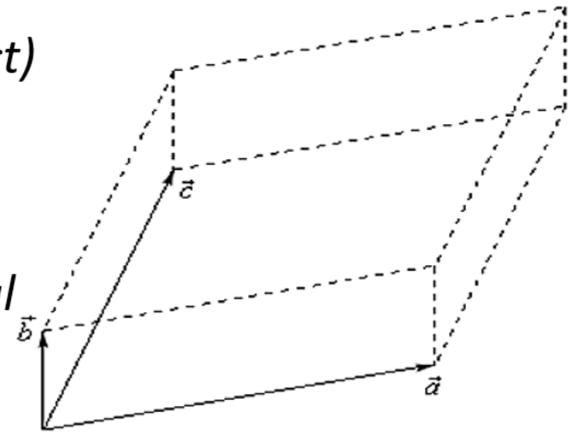
The mixed product of the vectors $\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$, $\vec{b} = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}$ and $\vec{c} = \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix}$

is the ***volume*** V of the parallelepiped formed by the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ such that :

- $V = \vec{a}.\left(\vec{b} \wedge \vec{c}\right) = \left(\vec{a} \wedge \vec{b}\right).\vec{c} = \left(\vec{a}, \vec{b}, \vec{c}\right)$
- $V = \left(\vec{a}, \vec{b}, \vec{c}\right) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$

$$V = \left(\vec{a}, \vec{b}, \vec{c}\right) = a_x\left(b_y c_z - b_z c_y\right) - a_y\left(b_x c_z - b_z c_x\right) + a_z\left(b_x c_y - b_y c_x\right)$$

Suppose we have three vectors $\vec{a}, \vec{b}, \vec{c}$ and we form the three-dimensional figure shown below.



- *The area of the parallelogram (two-dimensional front of this object) is given by:* Area $= \| \vec{a} \wedge \vec{b} \|$

- *and the volume of the parallelepiped (the whole three-dimensional object) is given by:* Volume $= \left| \vec{a} \,.\, (\vec{b} \wedge \vec{c}) \right|$

Note that the absolute value bars are required since the quantity could be negative and volume isn't negative.

We can use this volume fact to determine if three vectors lie in the same plane or not.
If three vectors lie in the same plane then the volume of the parallelepiped will be zero.

2- DIFFERENTIAL

Derivative Formulas

Power Formula

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Sum Formula

$$\frac{d}{dx}[f(x)+g(x)] = f'(x) + g'(x)$$

Difference Formula

$$\frac{d}{dx}[f(x)-g(x)] = f'(x) - g'(x)$$

Chain Formula

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

Product Formula

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

Constant Multiple Formula

$$\frac{d}{dx}[cf(x)] = cf'(x)$$

Quotient Formula

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \left[\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}\right]$$

Summary of the basic rules of derivation.

Table of common derivatives required in our course

Function	Derivative
x^n	nx^{n-1}
$\left(f(x)\right)^n$	$nf(x)^{n-1}f'(x)$
$\frac{1}{f(x)^n} = f(x)^{-n}$	$-nf(x)^{-n-1}f'(x)$
$\sqrt[m]{x} = x^{\frac{1}{m}}$	$\frac{1}{m}.x^{\frac{1}{m}-1}$
$\sqrt[m]{f(x)} = f(x)^{\frac{1}{m}}$	$\frac{1}{m}.f(x)^{\frac{1}{m}-1}f'(x)$
e^x	e^x
$e^{f(x)}$	$e^{f(x)}.f'(x)$

Function	Derivative
$\sin x$	$\cos x$
$\sin f(x)$	$\cos f(x)\ .f'(x)$
$\cos x$	$-\sin x$
$\cos f(x)$	$-\sin f(x)\,.f'(x)$
$\mathrm{tg}\,x$	$\frac{1}{\cos^2 x}$
$\mathrm{tg}\,f(x)$	$\frac{f'(x)}{\cos^2 f(x)}$
$\log x$	$\frac{1}{x}$
$\log f(x)$	$\frac{f'(x)}{f(x)}$

3- INTEGRALS

Basic rules in integral calculus
$\int \big(f(x) + g(x)\big)dx = \int f(x)dx + \int g(x)dx$
If $c = const \Rightarrow \quad \int c\, f(x)dx = c \int f(x)dx$
$\int f(x)dg(x) = \int f(x).\, g(x)dx - \int g(x)df(x)$
If $u = g(x) \quad \Rightarrow \quad \int f(u)du = \int f\big(g(x)\big)g'(x)dx$

Summary of the main integration rules

Table of common integrals required in our course

Function	Integral
$\int x^n dx$	$\frac{x^{n+1}}{n+1} + c$ (où $n \neq -1$)
$\int \frac{xdx}{\sqrt{a \pm x^2}}$	$\sqrt{a \pm x^2} + c$
$\int e^{ax} dx$	$\frac{1}{a} e^{ax} + c$
$\int \frac{dx}{x+a}$	$\log\lvert x+a \rvert + c$
$\int \frac{xdx}{x^2+a}$	$\frac{1}{2}\log\lvert x^2+a \rvert + c$

Function	Integral
$\int \sin(ax)\ dx$	$-\frac{1}{a}\cos(ax) + c$
$\int \cos(ax)\ dx$	$\frac{1}{a}\sin(ax) + c$
$\int \frac{dx}{\cos^2(ax)}$	$\frac{1}{a}\mathrm{tg}(ax) + c$
$\int \frac{dx}{\sin^2(ax)}$	$-\frac{1}{a}\mathrm{ctg}(ax) + c$

Applications

Differential operators	$$\vec{\nabla}=\begin{pmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{pmatrix}=\frac{\partial}{\partial x}\vec{i}+\frac{\partial}{\partial y}\vec{j}+\frac{\partial}{\partial z}\vec{k}$$
Gradient operator	Let $f(x,y,z)$ be a scalar function $$\overrightarrow{\text{grad}}\,\text{f} = \vec{\nabla}.f = \frac{\partial f}{\partial x}\vec{i}+\frac{\partial f}{\partial y}\vec{j}+\frac{\partial}{\partial z}\vec{k}$$
Divergence operator	Let $\vec{V} = V_x\vec{i} + V_y\,\vec{j} + V_z\,\vec{k}$ be a vector $$div\,\vec{v} = \vec{\nabla}.\vec{v} = \left(\frac{\partial v_x}{\partial x}\right)_{y,z=cst} + \left(\frac{\partial v_y}{\partial y}\right)_{x,z=cst} + \left(\frac{\partial v_z}{\partial z}\right)_{x,y=cst} = \text{scalaire}$$
Rotational Operator	$$\overrightarrow{Rot}\,\vec{v} = \vec{\nabla}\wedge\vec{v} = \vec{i}\left(\frac{\partial}{\partial y}v_z - \frac{\partial}{\partial z}v_y\right) - \vec{j}\left(\frac{\partial}{\partial x}v_z - \frac{\partial}{\partial z}v_x\right) + \vec{k}\left(\frac{\partial}{\partial x}v_y - \frac{\partial}{\partial y}v_x\right)$$

Thank you for your attention