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3rd Year License in Automatique (S5)

Linear Systems Control (LSC)
TD 4

Exercise 1 : Consider the following system represented by the following state model:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad \text{avec } A = \begin{bmatrix} -1 & -2 \\ 0 & -4 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ C = [1 \ \ 2]$$

1- Calculate the transition matrix e^{At} .
2- Deduce the state of the system in free regime for $x(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$
3- Calculate the system output $y(\mathrm{t})$ in free regime.
4- Calculate the state of the system in forced regime, knowing that the input is a unit step.
5- Calculate the system output $y(\mathrm{t})$ in forced regime.
6- Express the complete solution $x(\mathrm{t})$
7- Deduce the complete output $y(\mathrm{t})$ of the system.

Exercise 2 : Let the matrix A be given by:

$$A = \begin{pmatrix} 2 & 3 \\ -1 & 6 \end{pmatrix}$$

- Calculate the transition matrix using the Cayley-Hamilton method.

Exercise 3 :

Study the stability of the following system:

$$\dot{x} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

Exercise 4 :
Let the state representation of the system be:

$$\dot{x} = \begin{bmatrix} 3 & 1 \\ 6 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = [1 \quad -1\,] x$$

- Study the controllability and observability of this system.

Exercise 5 :

- Study the controllability and observability for the following system.

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [-2 \quad 1 \quad 0] x$$

Additional exercises:

Exercise 1 :

Calculate the transition matrix using the Cayley-Hamilton method for the system given by the following model:

$$\dot{x} = \begin{bmatrix} 2 & 2 \\ -1 & 5 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

Calculate the state and response of the system given by:

Exercise 2 :

- Determine the transfer function of the system defined by the following state model:

$$\begin{cases} \dot{x} = \begin{bmatrix} -2 & -4 \\ -2 & -9 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y = [1 \quad 0]\, x(t) \end{cases}$$

- Calculate the poles of the transfer function and deduce the expression of its output in response to a unit step.

<u>Exercise 3</u> :

Study the stability of the following system:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}3 & 1\\ 6 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}u$$

<u>Exercise 4</u> :

Study the controllability and observability of the following system:

$$\dot{x} = \begin{bmatrix}0 & 1 & -1\\ 1 & 0 & 1\\ 2 & 2 & -1\end{bmatrix}x + \begin{bmatrix}-1\\ 0\\ 1\end{bmatrix}u$$

$$y = [-1\ \ 0\ \ 2]x$$