University Badji Mokhtar-Annaba
Department of Electronics
3rd year License in Automatique L3 (S5)
Academic year: 2025/2026

TP on Linear Systems Control (CSL)

TP5 : Study and synthesis of regulators by pole placement

Introduction: pole placement by state feedback
State feedback control involves modifying the closed-loop behavior of a system given by a state representation in such a way that the closed-loop system's poles are appropriately placed. These poles indeed determine the behavior of the system.
Controllability of a system
The problem of controlling a system consists of controlling a system in such a way that it evolves, from an observed initial state, to a determined final state. In state representation, it will be about determining the control signal u(t) between two given moments, t1 and t1, to bring the system from the state x(t1) to a desired state x(t2).

Principle of state feedback control:
The principle is to determine a control such that the poles of the system's transfer function of the closed-loop system are suitably placed in the complex plane and meet damping, speed, etc. specifications. The poles of the transfer function being the eigenvalues of the state matrix, the goal is therefore to achieve a control that suitably modifies the state matrix of the system.

Figure (2-1) : *Système en boucle ouverte*

Soit un système décrit par l'équation d'état suivant :

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}$$

In the context of this course, we restrict ourselves to linear control constructed by linear feedback of the system state on the input:

$$u(t) = r(t) - Kx(t) \text{ (en boucle ouverte)}$$

Le signal de commande du système (autrement dit l'écart) doit être construit en soustrayant au signal de consigne un signal qui dépend du vecteur d'état. Ce vecteur d'état étant composé de n signaux $x_1(t), x_2(t), \dots, x_n(t)$, on le multiplie par un vecteur ligne (K) appelé vecteur de gain pour pouvoir effectuer cette soustraction. On a alors :

$$K = [k_1 \quad k_2 \quad \dots \quad k_n] \qquad (2\text{-}3)$$

Et :

$$u(t) = r(t) - Kx(t) = r(t) - [k_1 \quad k_2 \quad \dots \quad k_n]\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad (2\text{-}4)$$

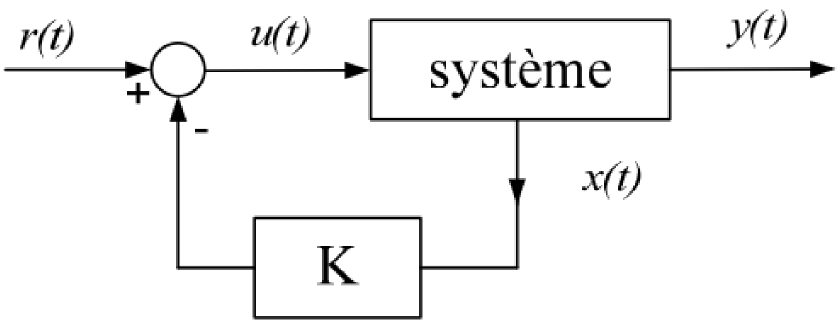

Figure (2-2) : Bouclage du système par un vecteur de gain.

Les équations du système en boucle fermé sont :

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ u(t) = r(t) - Kx(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \qquad (2\text{-}5)$$

L'équation d'état du système en boucle fermé s'écrit :

$$\dot{x}(t) = Ax(t) + B[r(t) - Kx(t)] = [A - BK]x(t) + Br(t) \qquad (2\text{-}6)$$

Par conséquent, la matrice d'état du système en boucle fermé vaut : $(A - BK)$.

La dynamique du système bouclé est donc fixée par les valeurs propres de la matrice $(A - BK)$; ces valeurs propres sont les racines de l'équation caractéristique :

$$det\big(pI - (A - BK)\big) = Q(p)_{A-BK} = 0 \qquad (2\text{-}7)$$

Application:

Consider the following system represented by the following state model:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

Context: - Calculate the feedback matrix ***K*** such that the matrix ***A-BK*** has the following eigenvalues:

$$\begin{cases} s_{1,2} = -2 \mp 4j \\ s_3 = -10 \end{cases}$$

Requested work:

- Study the controllability/observability of the system - What can we conclude about the system?
  - Calculate the state feedback gain theoretically.
  - Calculate the state feedback gain using Matlab.
  - Calculate the poles of the closed-loop system.
  - Plot the step response of the closed-loop system
  - What can we conclude?
  - Interpret the results obtained.