L1 Informatics | Badji Mokhtar – Annaba University | 2025/2026
Algebra 1 | Faculty of Technology
Department of Informatics

Series of Exercises 02
Sets, Functions and Binary Relation

Exercice 1 *Let the following sets :* $A =]-\infty, 3]\,, B = [-2, 8]\,, C =]-5, +\infty[$, $D = \{x \in \mathbb{R}, |x-3| \leq 5\}$

1. *What are the equality or inclusion relationships that exist between these sets ?*
2. *Find the complement in the following cases :* $C_{\mathbb{R}}A, C_{\mathbb{R}}B, C_{\mathbb{R}}C, C_{C}B$.
3. *Find* $A \cap B, A \cup B, A \cap C, A \cup C, A \setminus C, (\mathbb{R} \setminus A) \cap (\mathbb{R} \setminus B)$, *and* $A \triangle B$.

Exercice 2 *Let the set* E *and* A, B, D *are three parts of* E

a. Show that :

1. $A \subset B \Rightarrow C_E B \subset C_E A$
2. $(A \setminus B) \setminus D = A \setminus (B \cup D)$
3. $C_E A \triangle C_E B = A \triangle B$
4. $(A \times D) \cup (B \times D) = (A \cup B) \times D$

b. Simplify

1. $C_E(A \cup B) \cap C_E\left(D \cup C_E A\right)$
2. $C_E(A \cap B) \cup C_E\left(D \cap C_E A\right)$. ***(homework)***

Exercice 3 *Let the functions* $f : [0, 1] \rightarrow [0, 2]$ *with* $f(x) = 2 - x$ *and* $g : [-1, 1] \rightarrow [0, 2]$ *with* $g(x) = x^2 + 1$

1. *Find* $f(\{\frac{1}{2}\}), f^{-1}(\{0\}), g([-1, 1]), g^{-1}[0, 2])$
2. *Study the injectivity and surjectivity of* f, *is the function* f *bijective ?*
3. *Study the injectivity and surjectivity of* g, *is the function* g *bijective ?*
4. *Can we calculate* $g \circ f$ *and* $f \circ g$ *Justify.*

Exercice 4 *Let* f *the application defined by :*

$$\begin{aligned} f : E &\longrightarrow \mathbb{R} \\ x &\longmapsto f(x) = \tfrac{1}{\sqrt{x^2-1}} \end{aligned}$$

1. *Find* E *so that* f *is an application.*
2. *We take :* $E =]-\infty, -1[\cup]1, +\infty[$
   a) *Determine* $f\left(\left\{-\sqrt{2}, \frac{5}{3}, \sqrt{2}\right\}\right)$ *and* $f^{-1}\left(\{0\}\right)$.
   b) *Is the application* f *injective ? Is it surjective ? Justify your answer.*
3. *Show that the restriction* $g :]1, +\infty[\longrightarrow]0, +\infty[, \quad g(x) = f(x)$ *is bijective.*
4. *Determine the inverse application* g^{-1}.

Exercice 5 *Let $f : E \to F$ be a function. Let A and A' be two subsets of E, and let B and B' be two subsets of F. Show that :*

1- $A \subset f^{-1}(f(A))$	$2 - f\left(f^{-1}(B)\right) \subset B($ *homework* $)$
3- $f\left(A \cup A'\right) = f(A) \cup f\left(A'\right)$	$4 - f$ *injective* $\Rightarrow f\left(A \cap A'\right) = f(A) \cap f\left(A'\right)$ (*homework*)
$5 - f^{-1}\left(B \cap B'\right) = f^{-1}(B) \cap f^{-1}\left(B'\right)$	$6 - f^{-1}\left(B \cup B'\right) = f^{-1}(B) \cup f^{-1}\left(B'\right)$ *(homework)*

Exercice 6 *We define the relation $\Re$ on $\mathbb{R}$:*

$$\forall x, y \in \mathbb{R}, \quad x \Re y \Leftrightarrow x^4 - y^4 = x^2 - y^2$$

1. *Show that $\Re$ is an equivalence relation.*
2. *Find the equivalence class of 0, and deduce that of 1.*
3. *Determine the equivalence class of x for any real x.*

Exercice 7 *We define the binary relation $\mathcal{R}$ in $\mathbb{N}^*$ by :*

$$\forall x, y \in \mathbb{N}^*, \quad x \mathcal{R} y \Leftrightarrow \exists n \in \mathbb{N} \ \text{ such that : } y^n = x$$

1. *Show that $\mathcal{R}$ is a order relation.*
2. *Is this order total ? Justify your answer.*

Exercice 8 *We define the relation $\mathcal{R}$ in $\mathbb{R}^2$:*

$$\forall (x, y); (x', y') \in \mathbb{R}^2, (x, y) \mathcal{R} (x', y') \Leftrightarrow |x - x'| \leq y' - y$$

1. *Verify that : $(1,2)\mathcal{R}(4,7)$ and $(2,3)\mathcal{R}(5,3)$.*
2. *Show that $\mathcal{R}$ is a order relation.*
3. *Is the order total or partial ?*