Series 1 : Properties of the real set $\mathbb{R}$

Exercise 1 :

Determine (if they exist) : the upper bounds, the lower bounds, the supremum, the infimum, the maximum, and the minimum of the following sets :

1. $[-1,3]\,;\quad]0,1[\,;\quad [-1,3]\cap\mathbb{Z}\,;\quad \mathbb{N}$
2. $A=\left\{\dfrac{1}{n}+\dfrac{1}{n^2};n\in\mathbb{N}^*\right\}$
3. $B=\left\{\dfrac{1}{2x+1};x\in[0,1]\right\}$

Exercise 2 :

1. Let f and g be two functions such that :

$$f(x)=3+|x-1|,\quad g(x)=|x-2|+|2x+3|$$

   Write the above expressions without absolute value.
2. Solve the following equations and inequalities in $\mathbb{R}$:
   a) $|x-2|=3$
   b) $|2x-6|=|x+1|$
   c) $2\leq|x^2-1|$

Exercise 3 :

Prove that for all $x,y\in\mathbb{R}$:

$$\max(x,y)=\frac{x+y+|x-y|}{2}\quad\text{and}\quad\min(x,y)=\frac{x+y-|x-y|}{2}$$

Exercise 4 :

Solve the following equations :

1. $E\left(-\dfrac{x-1}{2}\right)=-2$
2. $E(2x)=x-1$
3. $E(x)+|x-1|=x$

Exercise 5 : **Additional Exercise**

Let x,y be two real numbers such that : $x\geq\dfrac{1}{2}$, $y\leq 1$, $x-y=3$

1. Calculate $E=\sqrt{(2x-1)^2}+\sqrt{(2y-2)^2}$
2. Show that : $\dfrac{1}{2}\leq x\leq 4$, and $-\dfrac{5}{2}\leq y\leq 1$
3. Calculate $F=|x+y-5|+|x+y+2|$