1st year LMD
Computer Science

2025/2026

Solution of
* Series 1 *

Ex 1:

We have : $\vec{V_1} = \vec{AB}$; $A(1,3,8)$ and $B(6,7,8)$

$\vec{V_2} = -8\vec{i} + \alpha\vec{j} + 7\vec{k}$

* First we determine the vector $\vec{V_1}$:

$$\vec{V_1} = (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k}$$

$$= (6-1)\vec{i} + (7-3)\vec{j} + (8-8)\vec{k}$$

$$\Rightarrow \boxed{\vec{V_1} = 5\vec{i} + 4\vec{j}}$$

* The two vectors are perpendicular $\perp$ $\Rightarrow$
The scalar product $\boxed{\vec{V_1} \cdot \vec{V_2} = 0}$

We have : $\vec{V_1} \cdot \vec{V_2} = \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix}\begin{pmatrix} -8 \\ \alpha \\ 7 \end{pmatrix} = (5\times(-8)) + (4\times\alpha) + (0\times 7)$

$$= -40 + 4\alpha = 0$$

$$\Rightarrow \boxed{\alpha = 10}$$

So: $\boxed{\vec{V_2} = -8\vec{i} + 10\vec{j} + 7\vec{k}}$

Ex 2:

We have: $\vec{A} = 2\vec{i} + 3\vec{j} + 5\vec{k}$; $\vec{B} = \vec{i} - 2\vec{j} + 3\vec{k}$

a- The vector product:

$$\vec{C} = \vec{A} \wedge \vec{B} = \begin{vmatrix} +\vec{i} & -\vec{j} & +\vec{k} \\ 2 & 3 & 5 \\ 1 & -2 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 5 \\ -2 & 3 \end{vmatrix}\vec{i} - \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix}\vec{j} + \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix}\vec{k}$$

$$= \left[(3\times 3) - ((-2)\times 5)\right]\vec{i} - \left[(2\times 3) - (1\times 5)\right]\vec{j} + \left[(2\times(-2)) - (1\times 3)\right]\vec{k}$$

$\Rightarrow \boxed{\vec{C} = 19\vec{i} - \vec{j} - 7\vec{k}}$ The result is a vector.

b- The unit vector $\vec{u} \perp (\vec{A}, \vec{B})$

$\vec{u}$ unit vector $\Rightarrow \boxed{\|\vec{u}\| = 1}$

As the vector product of $\vec{A}$ and $\vec{B}$ is a vector $\vec{C}$ perpendicular to the plane $(\vec{A}, \vec{B}) \Rightarrow \vec{u}$ is the unit vector of $\vec{C}$.

$\vec{C} = \|\vec{C}\| \vec{u} \Rightarrow \boxed{\vec{u} = \dfrac{\vec{C}}{\|\vec{C}\|}}$; $\boxed{\|\vec{C}\| = C = \sqrt{C_x^2 + C_y^2 + C_z^2}}$

modulus of $\vec{C}$

D.A: (Digital application)

$C = \sqrt{(19)^2 + (-1)^2 + (-7)^2} = \sqrt{361 + 1 + 49} = \sqrt{411}$

So: $\boxed{\vec{u} = \dfrac{19}{\sqrt{411}}\vec{i} - \dfrac{1}{\sqrt{411}}\vec{j} - \dfrac{7}{\sqrt{411}}\vec{k}}$

C- The angle between $\vec{A}$ and $\vec{B}$:

We know that: $\boxed{\vec{C} = \vec{A} \wedge \vec{B} = [A\, B \sin\theta]\, \vec{u}}$

$\Rightarrow \|\vec{C}\| = \|\vec{A} \wedge \vec{B}\| = A\, B \sin\theta \Rightarrow \boxed{\sin\theta = \dfrac{\|\vec{A} \wedge \vec{B}\|}{A\, B}}$

$A = \sqrt{x_A^2 + y_A^2 + z_A^2}$; $B = \sqrt{x_B^2 + y_B^2 + z_B^2}$

D.A:

$A = \sqrt{2^2 + 3^2 + 5^2} \Rightarrow \boxed{A = \sqrt{38}}$

$B = \sqrt{1^2 + (-2)^2 + 3^2} \Rightarrow \boxed{B = \sqrt{14}}$

$\Rightarrow \sin\theta = \dfrac{\sqrt{411}}{\sqrt{38} \cdot \sqrt{14}} \Rightarrow \boxed{\theta \simeq 61{,}5°}$

Ex 3:

We have three vectors:

$\vec{A} = 5\vec{i} - \vec{j} + 2\vec{k}$; $\vec{B} = 3\vec{j} - \vec{k}$; $\vec{C} = 4\vec{i} + \vec{k}$

a- The volume V of the parallelepiped P is the absolute value of the mixed product:

$$\boxed{V = |(\vec{A}, \vec{B}, \vec{C})| = |\vec{A} \cdot (\vec{B} \wedge \vec{C})|}$$

$$\vec{A} \cdot (\vec{B} \wedge \vec{C}) = \begin{vmatrix} +5 & -(-1) & +2 \\ 0 & 3 & -1 \\ 4 & 0 & 1 \end{vmatrix} = (3-0)5 - (0-(-4))(-1) + (0-12)2$$

$$\boxed{\vec{A} \cdot (\vec{B} \wedge \vec{C}) = -5} \Rightarrow \boxed{V = 5} \text{ (unit of volume)}$$

b- The three vectors are not coplanar.

Justification: The three vectors are coplaner if and only if their mixed product is equal to zero.

Since $\vec{A} \cdot (\vec{B} \wedge \vec{C}) = -5 \neq 0 \Rightarrow$ vectors are not contained in the same plane.

Ex 4:

* $\overrightarrow{grad} f(x, y, z) = ?$ $\longrightarrow$ Gradient operator

$$\boxed{\overrightarrow{grad} f(x,y,z) = \vec{\nabla} f} \quad ; \quad \vec{\nabla} \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix} \rightarrow \text{Differential opperators}$$

$$\Rightarrow \boxed{\overrightarrow{grad} f = \frac{\partial f}{\partial x} \vec{i} + \frac{\partial f}{\partial y} \vec{j} + \frac{\partial f}{\partial z} \vec{k}}$$

The function $f(x, y, z) = x^2 y + 2y^2 z^3$

$\partial/\partial x$, $\partial/\partial y$, $\partial/\partial z$ are the partial derivatives.

$$\frac{\partial}{\partial x} f(x,y,z) = 2xy$$

$$\frac{\partial}{\partial y} f(x,y,z) = x^2 + 4yz^3$$

$$\frac{\partial}{\partial z} f(x,y,z) = 6y^2z^2$$

$$\Rightarrow \boxed{\overrightarrow{grad}\, f = 2xy\,\vec{i} + (x^2 + 4yz^3)\,\vec{j} + 6y^2z^2\,\vec{k}}$$

At the point $(1,0,0) \Rightarrow \boxed{\overrightarrow{grad}\, f = \vec{j}}$ → the gradient is a vector

* $\operatorname{div} \vec{A} = ?$ → Divergence operator

$$\boxed{\operatorname{div} \vec{A} = \vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

The vector $\vec{A} = 2xyz\,\vec{i} + (2x^2 - y)\,\vec{j} + yz^2\,\vec{k}$

$$\left.\begin{aligned} \frac{\partial A_x}{\partial x} &= 2yz \\ \frac{\partial A_y}{\partial y} &= -1 \\ \frac{\partial A_z}{\partial z} &= -2yz \end{aligned}\right\} \Rightarrow \boxed{\operatorname{div} \vec{A} = -1}$$

→ the divergence is a scalar

At the point $(1,0,0)$