جـــامـعـة بـــاجـــي مـخـتـــار - عـنـــابـة
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Chapter 1: Electrostatics I – Coulomb's Law

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In this chapter, we will study the phenomena of electrostatics created by the interaction between electric charges in a vacuum.

We will discuss the basic concepts related to electrostatic forces, electric fields and electric potential.

What is Electrostatics?

It is a combination of two words

Electro

Means: **electric charges**

Statics

Means: **stationary or at rest**

Study of electric charges at rest

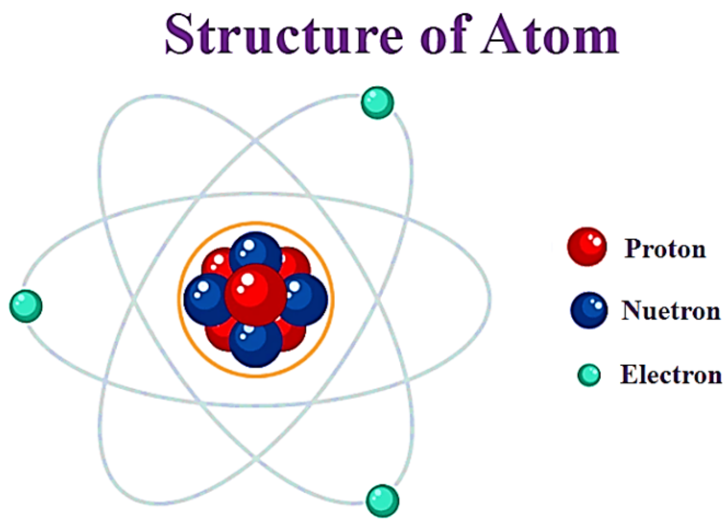
A branch of physics which deals with the study of charges at rest under the action of electric forces. It is also named as : **Static electricity**

The atom, the elementary constituent of all matter, is a neutral particle formed by a nucleus with ***positive charge*** and an electron cloud with ***negative charge*** equal in absolute value to that of the nucleus. The electric charge of a proton is called elementary electric charge, and its value is:

$$e = 1{,}6021892\,.\,10^{-19}\,C$$

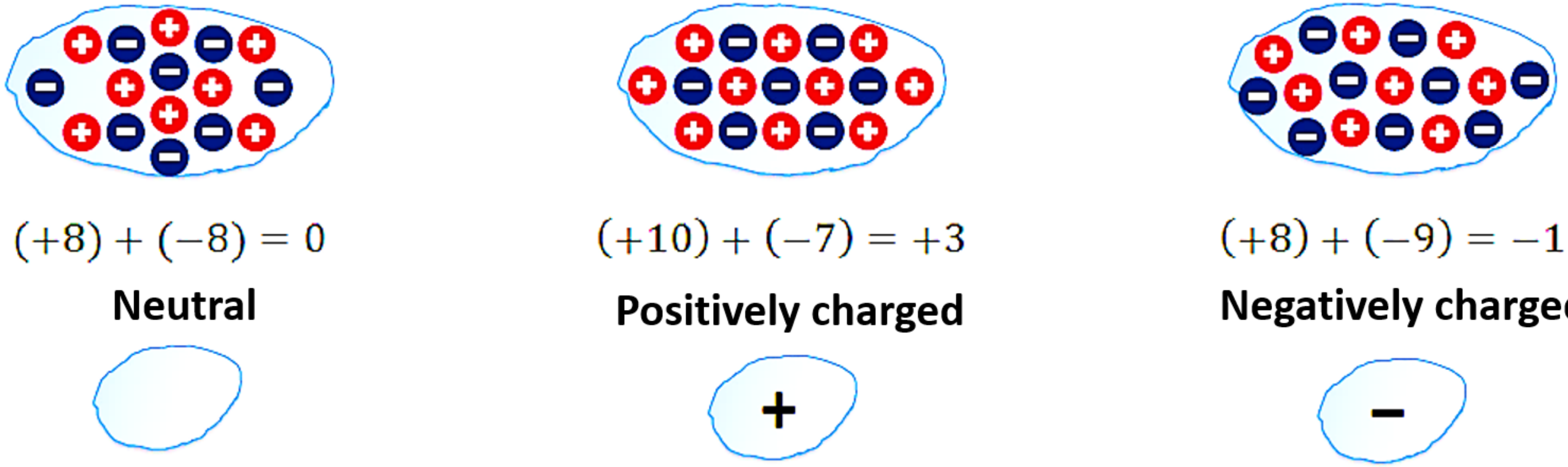
The unit C is the coulomb

The electric charge of an **electron** is $-e$



An atom can lose its neutrality through positive electrification by removing electrons from it, or through negative electrification by gaining additional electrons.

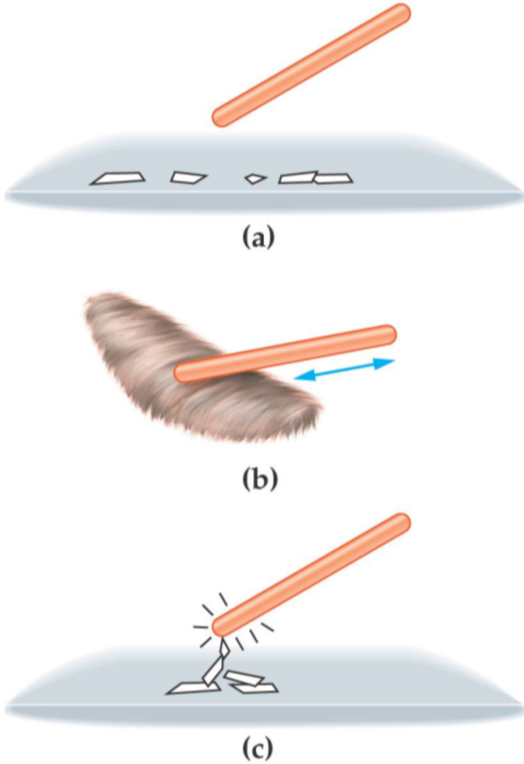
The matter thus becomes positively or negatively electrified.



Electrification process: Static electricity is obtained by **friction**, **contact** or **influence**.

Friction electrification

The electrons are removed from the rubbed neutral body then transferred to the second body which becomes positively electrified (Glass, Silk, Wood, Rabbit Hair, Cat Hair, Mica, Sulfur, Amber, Resin, ...)



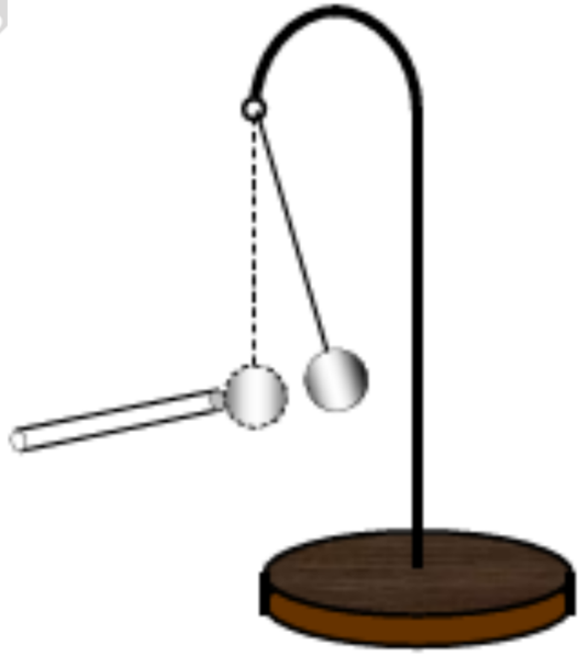
Charging an Amber Rod

(a) Uncharged amber exert no force on papers.
(b) Rod amber is rubbed against a piece of fur.
(c) Amber become charged and then attracts the papers.

Electrification by contact

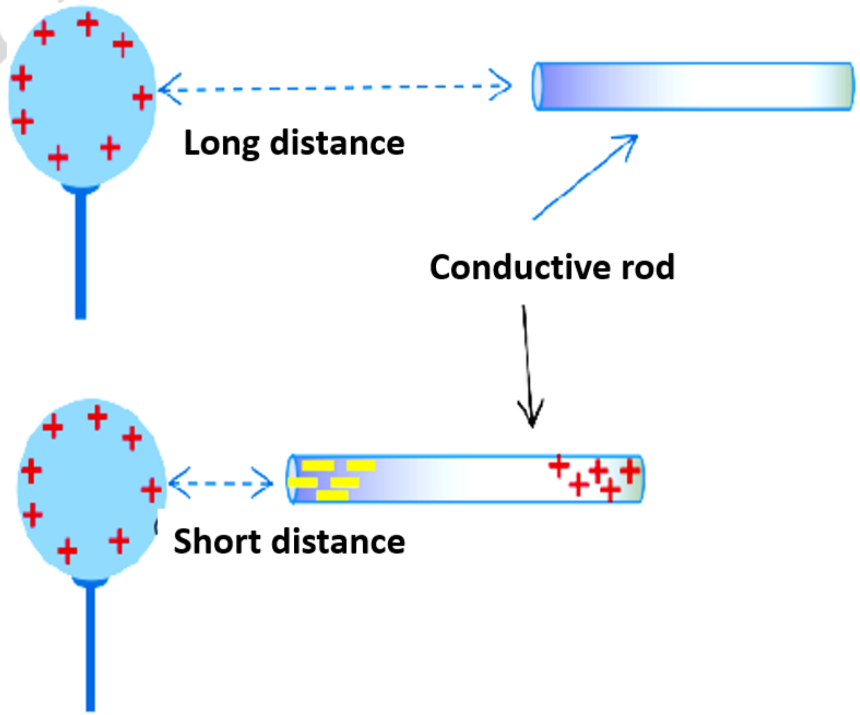
A body electrified by friction (glass) is brought closer to another neutral body (polystyrene ball surrounded by a conductive material) until contact.

By interaction, the two bodies (the glass and the ball) will be charged with an electricity of the **same sign** and repel each other. This results in **a repulsion**.



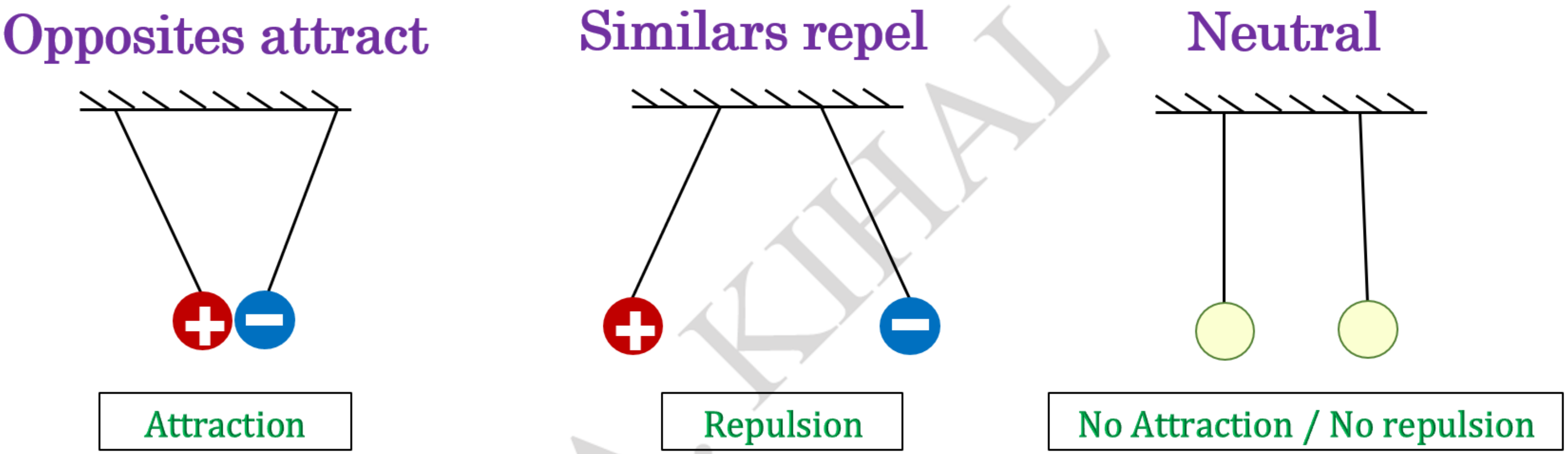
Electrification by influence

The positively charged ball influences the free electrons in the conductive rod. It attracts them towards it and "repels" the positive charges to the other side.



IMPORTANT

By influence, charges do not move from one body to another.



Conservation of Electric Charge

The total electric charge of the universe is constant:

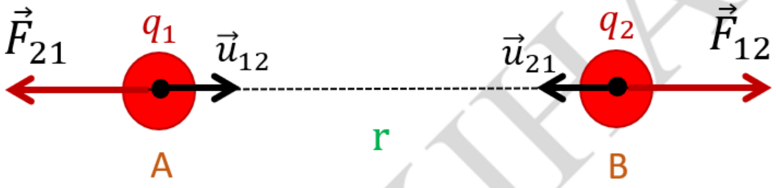
No physical process can increase or decrease the total amount of electric charge.

Point charges: assumed to be dimensionless, which is analogous to the material point hypothesis in mechanics.

1. COULOMB'S LAW

Let two charges be stationary and placed at points A and B in a vacuum.

$q_1, q_2 > 0$

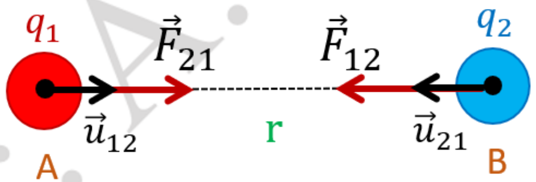


Force exerted by q_1 on q_2 :

$$\vec{F}_{12} = K\frac{q_1\, q_2}{r^2}\vec{u}_{12}\ , \quad \vec{u}_{12} = \frac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|}\ , \quad r = \|\overrightarrow{AB}\|$$

$$\boxed{\vec{F}_{12} = K\frac{q_1\, q_2}{\|\overrightarrow{AB}\|^3}\overrightarrow{AB}}$$

$q_1 > 0\ , q_2 < 0$



Force exerted by q_2 on q_1 :

$$\vec{F}_{21} = K\frac{q_1\, q_2}{r^2}\vec{u}_{21}\ , \quad \vec{u}_{21} = \frac{\overrightarrow{BA}}{\|\overrightarrow{BA}\|}\ , \quad r = \|\overrightarrow{BA}\| = \|\overrightarrow{AB}\|$$

$$\boxed{\vec{F}_{21} = K\frac{q_1\, q_2}{\|\overrightarrow{AB}\|^3}\overrightarrow{BA}}$$

$$K = \frac{1}{4\,\pi\,\varepsilon_0} = 9\ 10^9\ \text{(S. I.)}\ ,\ [F\,] = \textit{Newton (N)}$$

$$\Rightarrow \quad \|\vec{F}_{12}\| = \|\vec{F}_{21}\|$$

Example

Calculate the electric force exerted by the charge $q_1 = -10^{-6}\ C$, located at point M_1 (1, 2, 3) on the charge $q_2 = 2\ 10^{-6}\ C$, located at point M_2 (−1, 3, 4).

Solution

We know that : $$\vec{F}_{12} = K \frac{q_1\ q_2}{{r_{12}}^3} \vec{r}_{12}$$

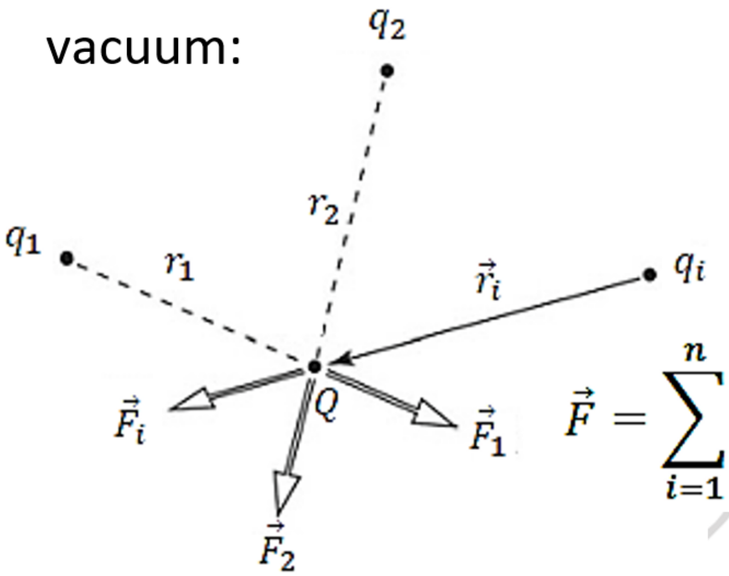
$$\vec{r}_{12} = \overrightarrow{M_1M_2} = \begin{pmatrix} -1-1 \\ 3-2 \\ 4-3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \quad ; \quad r_{12} = \sqrt{4+1+1} = \sqrt{6}$$

$$\vec{F}_{12} = 9\ 10^9 \frac{(-10^{-6})\ 2\ 10^{-6}}{6\sqrt{6}} \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \quad ; \quad \boxed{\vec{F}_{12} = \frac{3\ 10^{-3}}{\sqrt{6}} (2\vec{i} - \vec{j} - \vec{k})}$$

Superposition Principle

Discontinuous Distribution of Charges

In the case where there are more than two-point charges, the total Coulomb force $\vec{F}$ exerted on a charge Q is the linear sum of the electric forces $\vec{F_i}$ individually exerted on it by all the other point charges: $q_1, q_2, \dots, q_n$ distributed in the vacuum:



$$\vec{F_i} = K \frac{Q\, q_i}{{r_i}^3} \vec{r_i}$$

where $i = 1, 2, \dots, n$

$$\vec{F} = \sum_{i=1}^{n} \vec{F}_i = \sum_{i=1}^{n} K \frac{Q \cdot q_i}{r_i^3} \vec{r}_i = K \frac{Q \cdot q_1}{r_1^3} \vec{r}_1 + K \frac{Q \cdot q_2}{r_2^3} \vec{r}_2 + \dots + K \frac{Q \cdot q_n}{r_n^3} \vec{r}_n$$

2. ELECTRIC FIELD

DEFINITION

When a point charge q is placed at point O, an electric field vector $\vec{E}$ is assigned to every point M. In this case, similar to the gravitational field, any charge Q located at point M will experience an electric force depending on the field $\vec{E}$, given by:

$$\vec{F} = Q\vec{E}$$

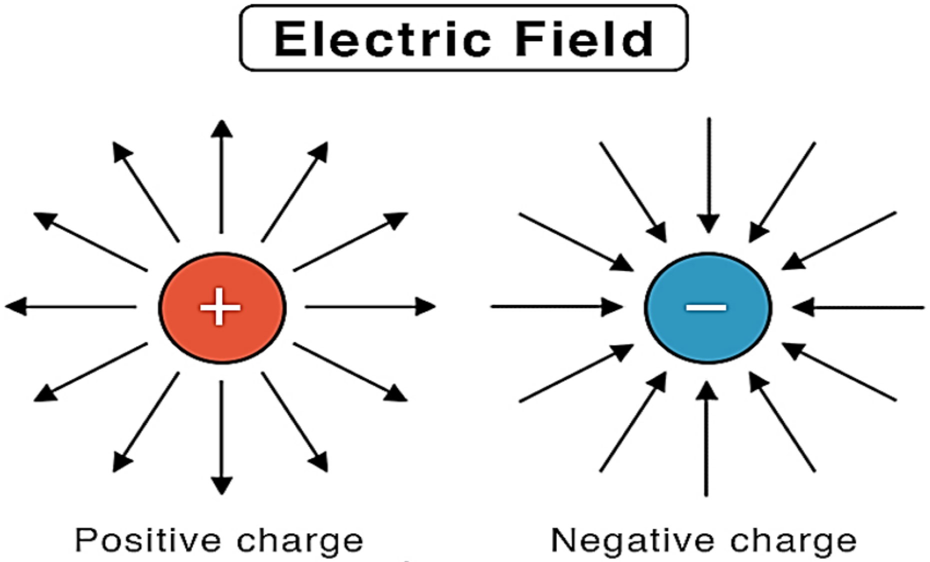
$\vec{E}$ is called the electric field generated by the charge q and has the same direction **or** the opposite direction as $\vec{F}$, depending on the sign of q.

The unit of $\vec{E}$ is: $[N/C]$ *(Newton/Coulomb)* and it is also: $[V/m]$ *(Volt/Meter)*

$\vec{E}$ is defined as a function of the charge q that produced it:

$$\vec{E} = K\frac{q}{r^3}\vec{r} \quad \text{and} \quad E = K\frac{|q|}{r^2}$$

$\vec{E}$ is pointing **away from** the charge q

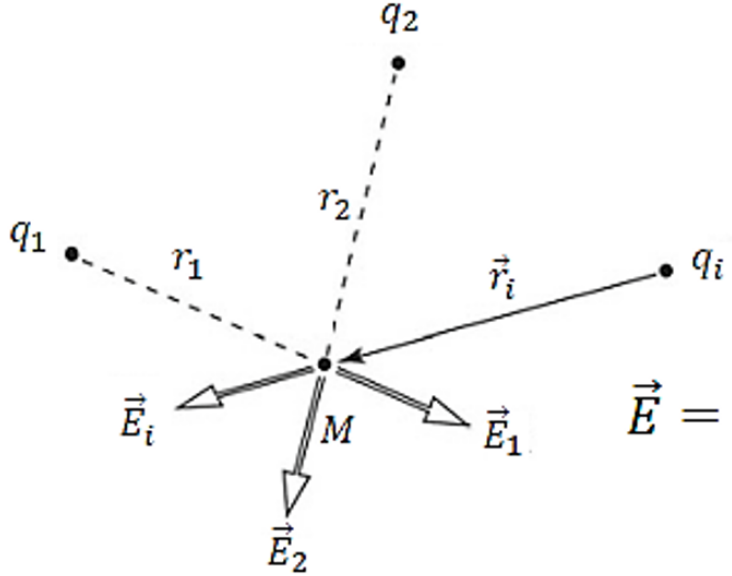


$\vec{E}$ is pointing **towards** the charge q

Superposition Principle

Discrete Distribution of Charges

The total electrostatic field generated at a point M by a discrete distribution of point charges: $q_1, q_2, \dots, q_n$ is the linear summation of the electric fields $\overrightarrow{E_i}$ individually produced by each of these charges:



$$\overrightarrow{E_i} = K \frac{q_i}{{r_i}^3} \overrightarrow{r_i}$$

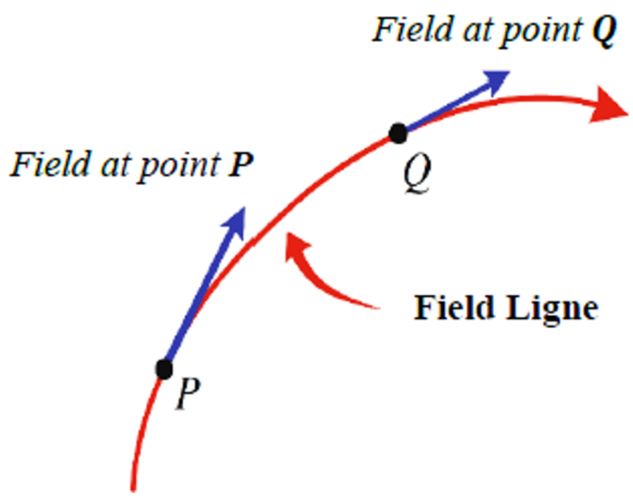
Where i = 1, 2, 3, …… , n

$$\vec{E} = \sum_{i=1}^{n} \vec{E}_i = \sum_{i=1}^{n} K \frac{q_i}{r_i^3} \vec{r}_i = K \frac{q_1}{r_1^3} \vec{r}_1 + K \frac{q_2}{r_2^3} \vec{r}_2 + \dots + K \frac{q_n}{r_n^3} \vec{r}_n$$

LINES OF FORCE (FIELD LINES)

When a point charge is placed in an electric field with intensity E, the charge experiences the Coulomb force which is parallel to $\vec{E}$. If this charge is small enough, its motion caused by the force will not significantly alter the electric field.

The direction of an electric field line is defined as the direction of the force exerted on a positive charge. Therefore, we can define an electric field line as an electric line of force such that the direction of the electric field at an arbitrary point on that line is tangent to that line at that point.

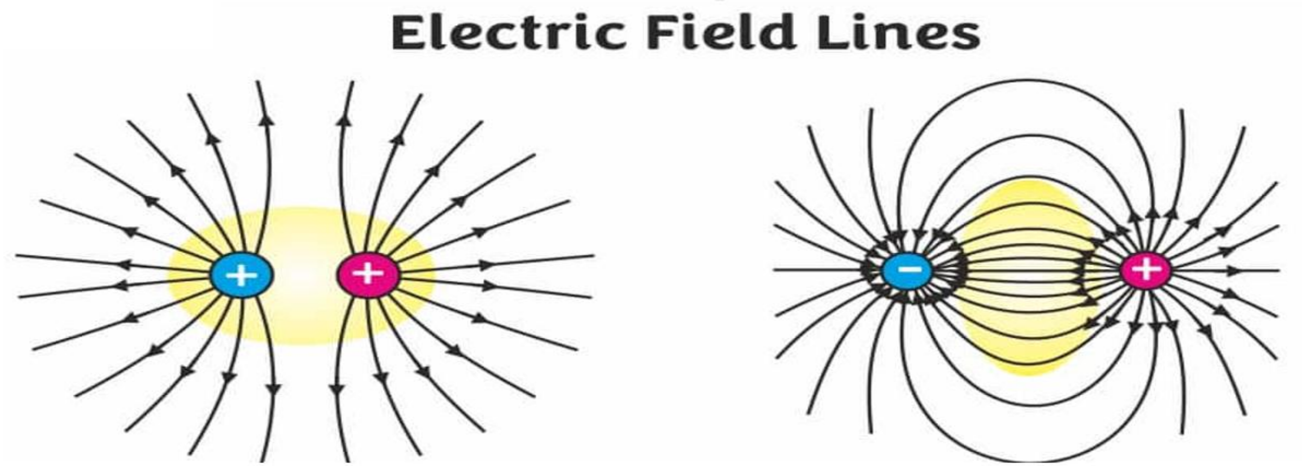


A field line always begins with a positive charge and ends with a negative charge.
It can be seen that a field line never begins or ends at a point without a charge.

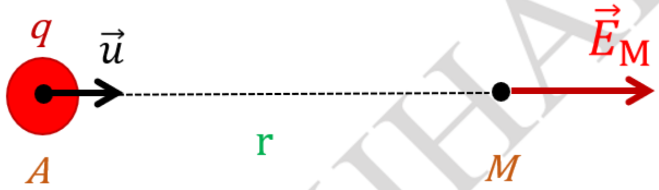
The electric field lines never intersect.

Electric field lines provide information about the direction of electric field intensity at a point.

The figure below shows the electric field lines for a pair of equal and opposite charges, also called an ***electric dipole***

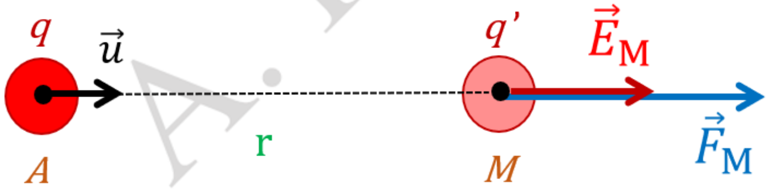


Relation between $\vec{E}$ and $\vec{F}$



$$\vec{E}_M = K\frac{q}{r^2}\vec{u}$$

If we place another charge *q'* at point *M*, it is subjected to a force:



q and q' are positive charges

$$\vec{F}_M = K\frac{q\,q'}{r^2}\vec{u}$$

Where

$$\boxed{\vec{F}_M = q'\vec{E}_M}$$

Example

Let $q_1 = +\,2\ 10^{-9}$ C be a point charge located at point $M_1(1, 0)$.

1- Calculate the electric field vector $\vec{E}$ created by q_1 at point $M_2\,(4, 3)$.

2- If we place a charge $q_2 = -\,q_1$ at point M_2, what is this pair of charges called?

Solution

1- The electric field created by q_1 at point M_2, is given by : $\vec{E} = K\,\dfrac{q_1}{{r_{12}}^3}\,\vec{r}_{12}$

$$\vec{r}_{12} = \overrightarrow{M_1M_2} = \begin{pmatrix} 4-1 \\ 3-0 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad ; \quad r_{12} = 3\sqrt{2}$$

$$\vec{E} = \frac{(9\ 10^{9})\ (2\ 10^{-9})}{(3\sqrt{2})^3}\begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad ; \quad \boxed{\vec{E} = \frac{1}{\sqrt{2}}\,(\,\vec{\imath} + \vec{\jmath}\,)}$$

2- A system composed of two electric charges of the same values and opposite signs $+\,q$ and $-\,q$ separated by a very small distance is called an **electric dipole**.

3. ELECTROSTATIC POTENTIAL

The electrostatic or electric field $\vec{E}$ derives from the potential V, according to the following relation:

$$\vec{E} = -\overrightarrow{grad}\, V \Rightarrow \vec{E} = -\frac{\partial V}{\partial r}\vec{u}$$

$$\text{So, } dV = -\vec{E}\,.\,\overrightarrow{dr} \quad \text{and} \quad \vec{E} = \frac{Kq}{r^2}\vec{u}$$

$$dV = -\frac{Kq}{r^2}\,dr$$

$$\Rightarrow V = -Kq\int\frac{dr}{r^2} = \frac{Kq}{r} + C$$

We assume that $V = 0$ when $r \rightarrow \propto \Rightarrow C = 0$

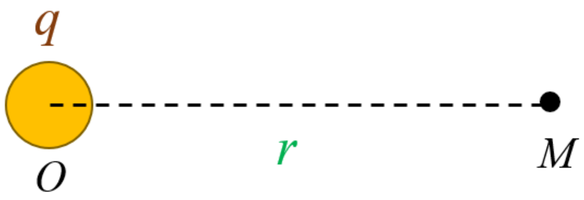
$$V(r) = K\frac{q}{r}$$

The electrostatic potential is a scalar field.

The electrostatic potential of a point charge is spherically symmetric, it only depends on r.

Single charge case

At a point M located at a distance r from a point O, there exists a potential V such that:

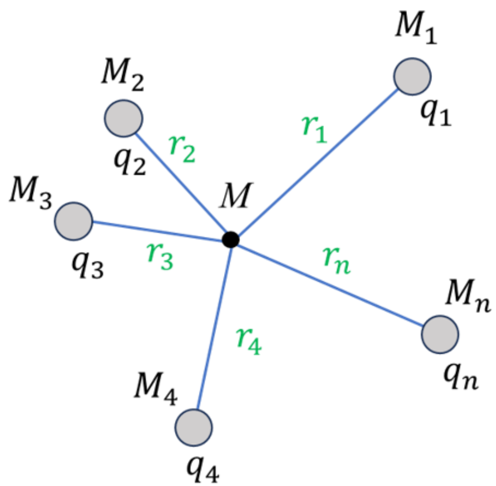


$$V_M = K\frac{q}{r} + Cste$$

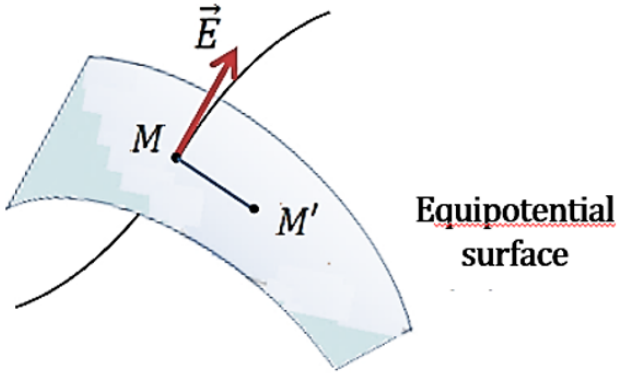
$$V_M(\infty) = 0 \quad ; \quad [V_M] = volt\ (V)$$

Case of n charges

$$V_M = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} K\frac{q_i}{r_i} + Cste$$



Equipotential surfaces



An equipotential surface is an area of points M such that:

$$V_M = \text{cste}$$

The equipotential surfaces are always ⊥ to the field lines:

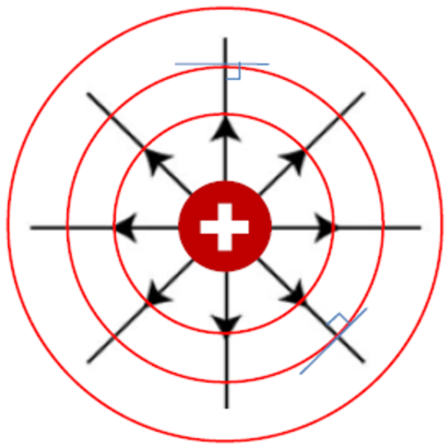
$$\overrightarrow{MM'} = \overrightarrow{dl} \qquad dV = -\vec{E}.\overrightarrow{dl}$$

$$\int_{V_M}^{V_{M'}} dV = -\int_{\overrightarrow{MM'}} \vec{E}.\overrightarrow{dl}$$

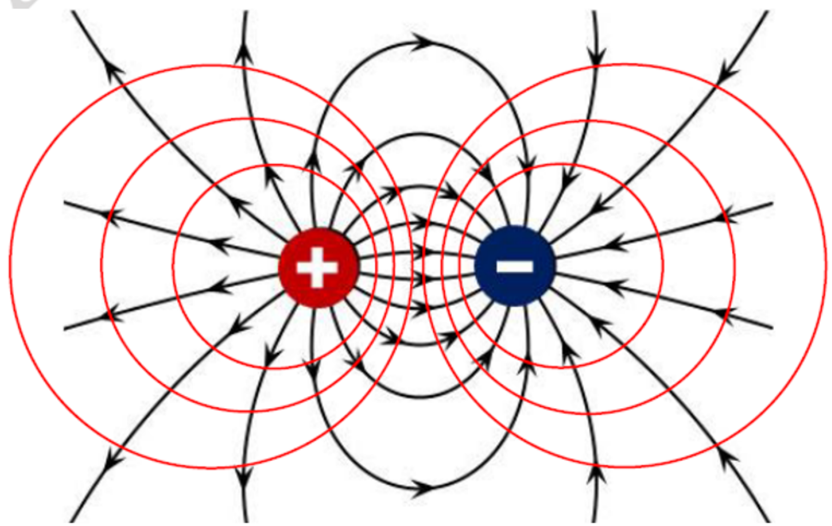
$$V_{M'} - V_M = \vec{E}\,.\overrightarrow{MM'}$$

$$V_{M'} - V_M = 0 \qquad \Rightarrow \qquad \vec{E} \perp \overrightarrow{MM'}$$

Case of a point Charge:



Case of two point charges:



Example

Let $q_A = 2\ \mu C$ and $q_B = -1\ \mu C$ be two point charges placed at points A and B, respectively. The distances between A and M and between B and M are given by $r_A = 50\ cm$ and $r_B = 20\ cm$.
Calculate the electric potential V created by these charges at a point M.

Solution

The electric potential V created by q_A and q_B at a point M is: $V_M = V_A + V_B$

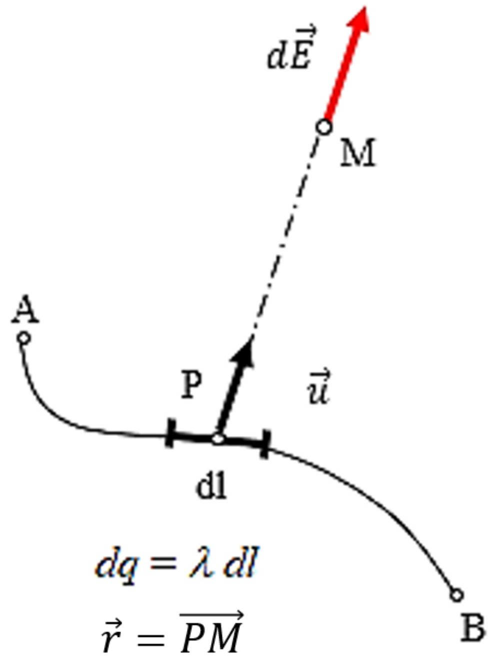
$$V_M = K\frac{q_A}{r_A} + K\frac{q_B}{r_B} = K(\frac{q_A}{r_A} + \frac{q_B}{r_B})$$

$$V_M = 9\ 10^9\ (\frac{2\ 10^{-6}}{50\ 10^{-2}} + \frac{(-1\ 10^{-6})}{20\ 10^{-2}}) \quad ; \quad \boxed{V_M = -\ 9000\ \mathrm{V}}$$

4. Electric field created by continuous distribution of charges

- When the charge q is distributed over a wire with a linear density λ, each element dl carries a charge $dq = \lambda\, dl$, and creates an elementary field:

$$d\vec{E} = \frac{K.\,dq}{r^3}\,\vec{r} = \frac{K.\,\lambda\, dl}{r^3}\,\vec{r}$$



Total field created by a linear distribution with charge density λ :

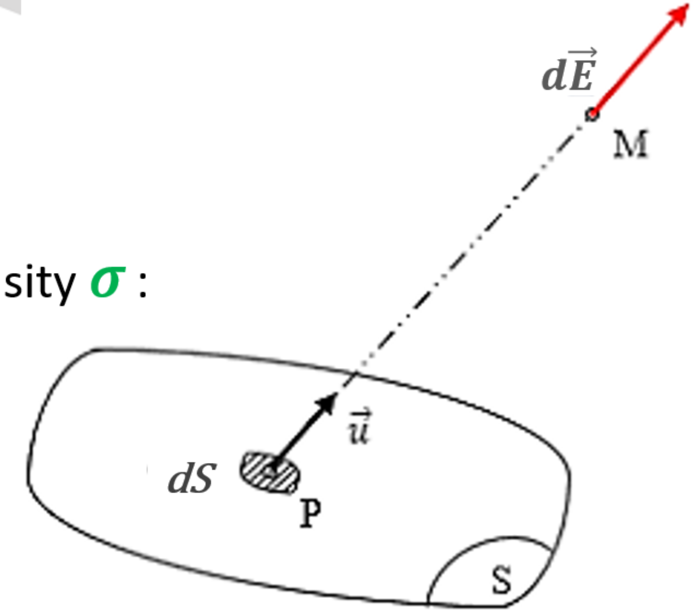
$$\vec{E} = \int d\vec{E} = K\int \frac{dq}{r^3}\,\vec{r} = K\int \frac{\lambda\, dl}{r^3}\,\vec{r}$$

Where $\lambda = \frac{dq}{dl}$ is the *linear charge density*

- In the case of a surface ***S*** charged with a surface density ***σ*** such that $dq = \sigma\ dS$,

we find in the same way:

$$d\vec{E} = \frac{K.\,dq}{r^3}\,\vec{r} = \frac{K.\,\sigma\,dS}{r^3}\,\vec{r}$$



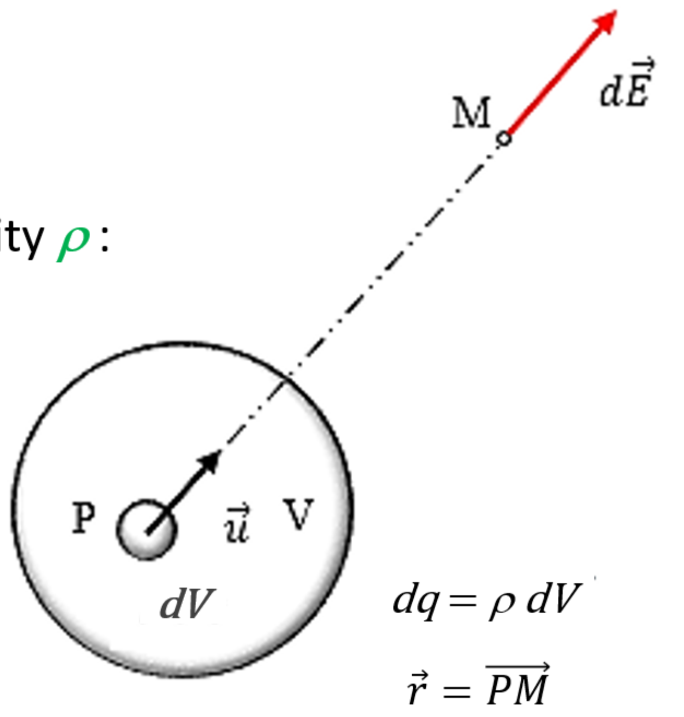
Total field created by a surface distribution with charge density ***σ*** :

$$\vec{E} = \int d\vec{E} = K\int \frac{dq}{r^3}\,\vec{r} = K\iint \frac{\sigma\,dS}{r^3}\,\vec{r}$$

Where $\sigma = \frac{dq}{dS}$ is the *surface charge density*

➢ Similarly, in the case of a volume V charged with a volume density ρ such that $dq = \rho\, dV$ we obtain:

$$d\vec{E} = \frac{K.\,dq}{r^3}\,\vec{r} = \frac{K.\,\rho\, dV}{r^3}\,\vec{r}$$



Total field created by a volume distribution with charge density ρ :

$$\vec{E} = \int d\vec{E} = K \int \frac{dq}{r^3}\,\vec{r} = K \iiint \frac{\rho\, dV}{r^3}\,\vec{r}$$

Where $\rho = \dfrac{dq}{dV}$ is the *volume charge density*

Thank you for your attention