جــامعة بــاجــي مخــتـــار - عـنــابــة
BADJI MOKHTAR - ANNABA UNIVERSITY

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Chapter 1: Electrostatics II – Gauss's Theorem

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Calculating the value of the electrostatic field for a somewhat complex charge distribution using Coulomb's law alone can quickly become complicated.

In this chapter, we will investigate a theorem that simplifies calculations in cases where the system creating the field possesses particular symmetries and invariances.

Gauss's theorem is another form of Coulomb's law that can be used to determine the force between two point charges or the electric field created by a point charge.

It is used to calculate the electric field created by a continuous charge distribution with a high level of symmetry when it would be difficult to solve the problem by calculating the field created by one element of charge before integrating it over the entire distribution.

1. Gauss's theorem

*"The flux of the electrostatic field through a **closed surface** is proportional to the charge contained within that surface"*

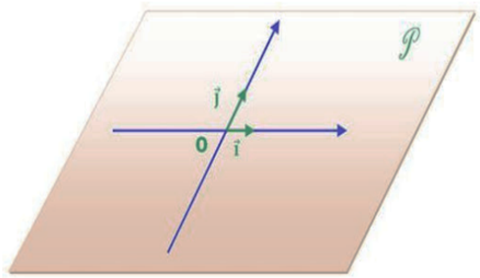
What is an electrostatic flux ???

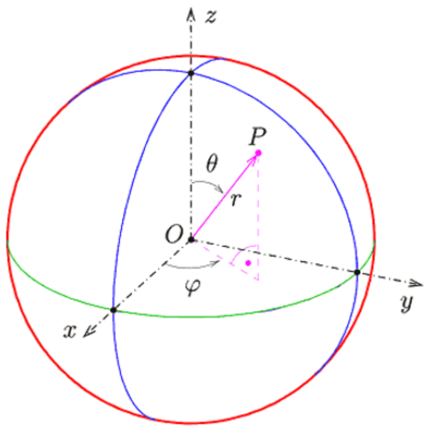
It's a mathematical tool that simply explains the multiplication of a field by an **imaginary surface**.

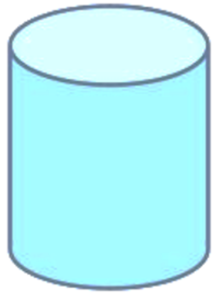
What is an oriented surface ???

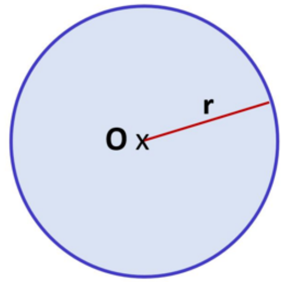
A surface is a set of points in space constrained to two dimensions.

Plane	**Sphere (*but not a ball*)**	**Hollow cylinder**	**Disk**



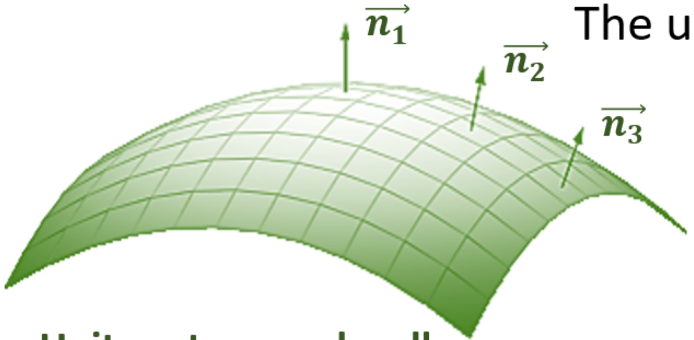






To define a surface mathematically, we can specify **all the points** that belong to it, or we can give the set of **<u>normal vectors to this surface</u>**.

In the case where the surface is flat, a single vector is sufficient.
The unit vector is **normal** to the plane locally tangent to the surface.

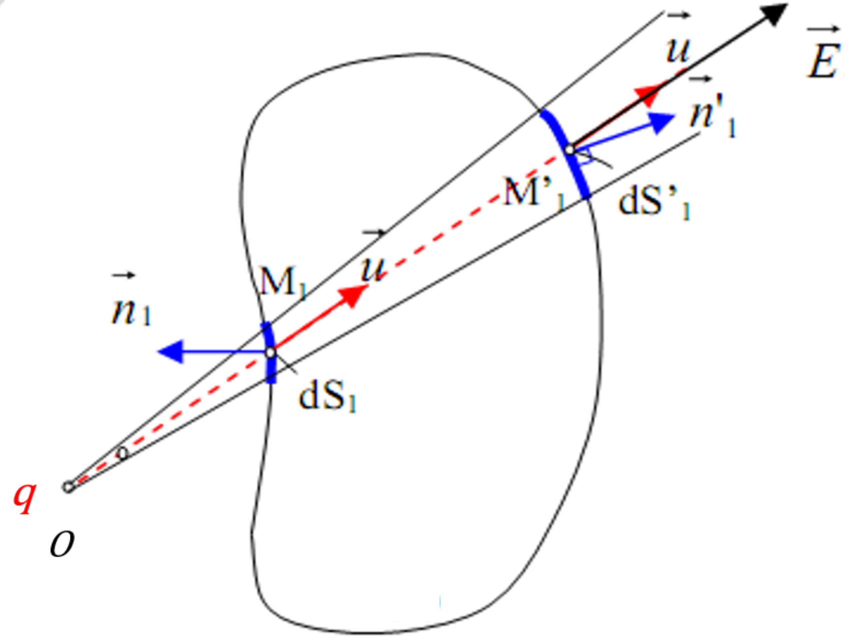


Unit vectors are locally Normal ⊥ to the surface

By convention, to define the same surface, we take the ***normal vector that goes towards the outside*** *of the surface*

If we take an *elementary surface dS* sufficiently small, then the surface will be locally planar.

The surface element vector is called the vector $\overrightarrow{dS} = dS\,\vec{n}$



Electrostatic flux

The electric field flux across the surface S is called the quantity ϕ such that:

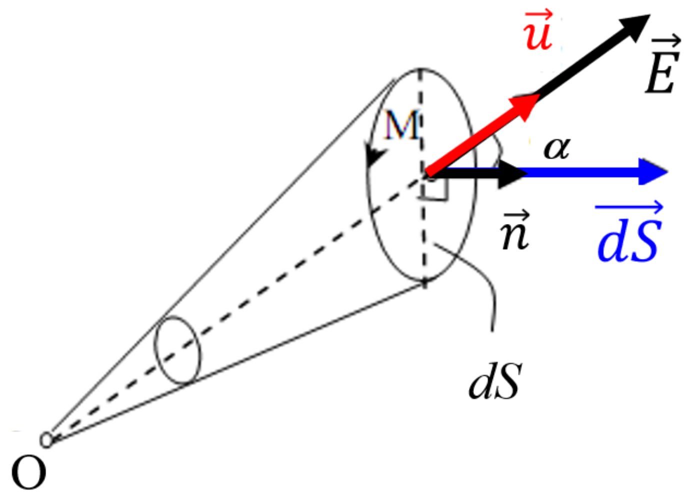
$$\phi = \oiint \vec{E}\,.\overrightarrow{dS}$$

$$= \oiint E\,\vec{u}\,.\,dS\,\vec{n}$$

$$= E \oiint (\vec{u}\,.\vec{n})\;dS$$

$$= E\cos\alpha \oiint dS$$

$$\phi = E\,S\cos\alpha$$



*When calculating a flux through a **closed surface**, we indicate that the integral is performed on a closed surface by a **small circle on the integral sign**.*

$$\int_{close\ S} \longrightarrow \oint \quad or \quad \oiint$$

Global form of Gauss's theorem

The flux of the electrostatic field through any closed surface is equal to the charge contained in the volume enclosed by the closed surface, divided by the *permitivity of the vacuum* ε_0.

$$\phi = \oiint \vec{E} \cdot \overrightarrow{dS} = \frac{\sum Q_{int}}{\varepsilon_0}$$

$\vec{E}$ is the electrostatic field created by the entire charge distribution (*included in the volume delimited by the surface S*) at points *M* of the surface *S*.

dS is a surface element taken around *M*.

The closed surface through which the electric field flux is calculated is called a Gaussian surface.

Conditions for Applying Gauss's Theorem

The theorem is used to calculate the field in cases of very high symmetry.

For this, the choice of surface (*S*) must satisfy the following conditions:

- *The form of (S), simple and easy to calculate.*
  - *The orientation of the electrostatic field $\vec{E}$ on (S) must be known.*
    - *The modulus of the field $\vec{E}$ on (S) must be constant.*

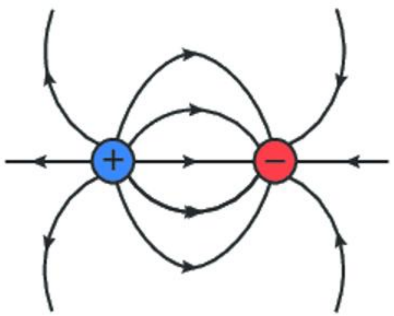
To calculate the electrostatic field $\vec{E}$

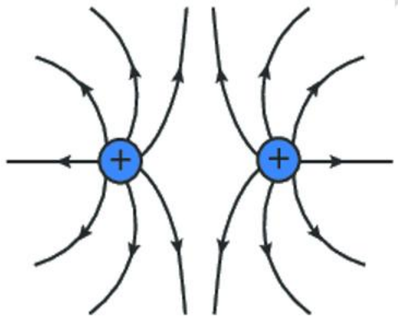
Coulomb's Law

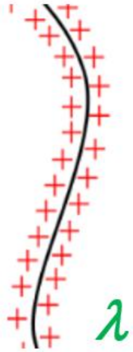
Gauss's Theorem

Point charges

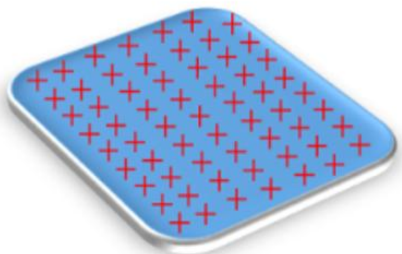
Continuous charge distribution



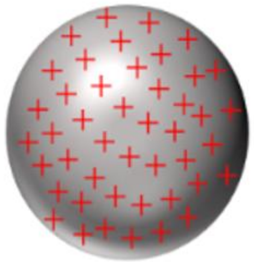




λ



σ



ρ

2. Applications of Gauss's theorem

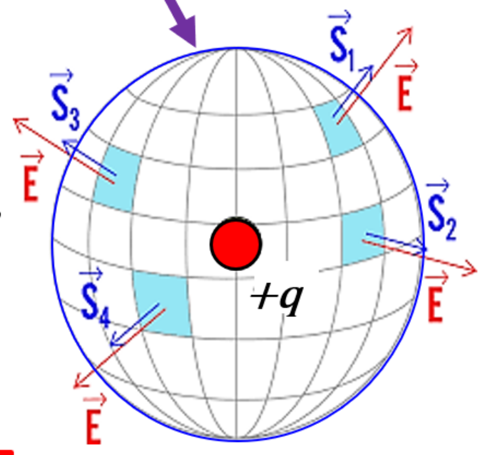
Field created by a point charge

We consider the charge $q > 0$ as the center of a sphere of radius r.

Therefore the electric field $\vec{E}$ is **radial** and **directed outwards**

*By reason of symmetry, $\vec{E}$ has the **same modulus** at every point on the sphere*

Gaussian Surface (Sphere)



$$\phi = \oiint \vec{E}\,.\,\overrightarrow{dS} = \frac{\sum Q_{int}}{\varepsilon_0}$$

$$= \oiint E\,\vec{u}\,.\,dS\,\vec{n}$$

$$= E \oiint (\vec{u}\,.\,\vec{n})\, dS$$

$$\phi = E\,S\cos\alpha = E\,S = \frac{q}{\varepsilon_0}$$

$S = 4\,\pi\,r^2$ (Surface of a sphere)

$$\boxed{E = \frac{q}{\varepsilon_0\,S} = \frac{1}{4\,\pi\varepsilon_0}\cdot\frac{q}{r^2} = \frac{K\,q}{r^2}}$$

$\vec{E}\ //\ \vec{S}$

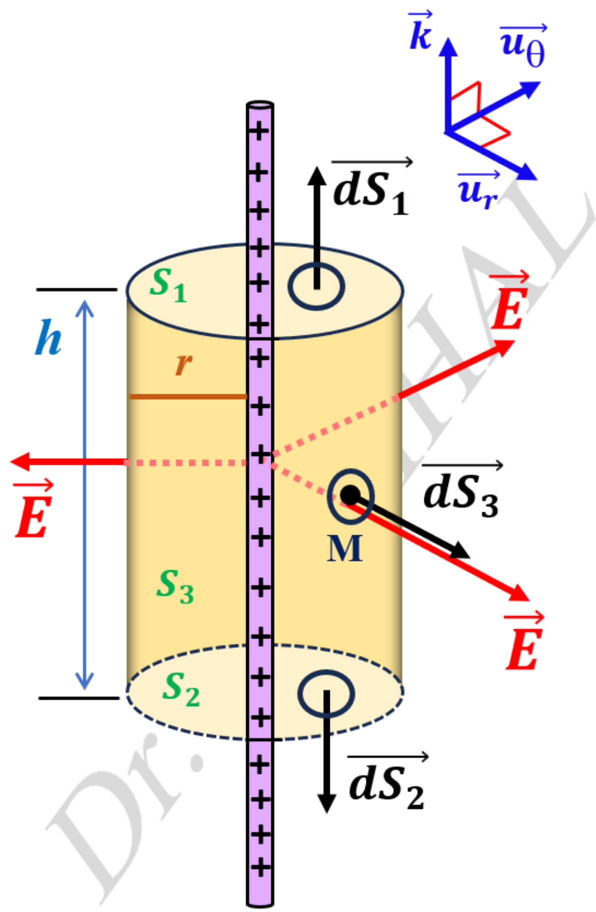
Same direction

$\cos\alpha = 1$

Field created by an infinite wire

Calculation of the electrostatic field $\vec{E}$ created by an infinite length wire, uniformly charged by $\lambda > 0$, at any point in space.

☞ *For the Gauss surface, we generally choose either a* ***sphere*** *or a* ***cylinder***



- The ***Gaussian Surface*** that is suitable for this case is a ***cylinder*** of length h, and whose axis coincides with the wire.
- In this case, the surface of Gauss is made up of three parts. **Two bases** which are two parallel **discs** (S_1 and S_2) and a **curved side** surface (S_3).

The flux through all the surfaces that make up the Gaussian cylinder is the sum of the fluxes through each surface:

$$\phi = \Sigma\phi_i = \phi_1 + \phi_2 + \phi_3$$

$$\phi = \oiint \vec{E}\,.\,\overrightarrow{dS} = \oiint \vec{E}\,.\,\overrightarrow{dS_1} + \oiint \vec{E}\,.\,\overrightarrow{dS_2} + \oiint \vec{E}\,.\,\overrightarrow{dS_3} = \frac{\sum Q_{int}}{\varepsilon_0}$$

The general expression of the electric field $\vec{E}$ at any point M in space is:

$$\vec{E}_M\,(r, \theta, z) = E_r(r, \theta, z)\,\vec{u}_r + E_\theta(r, \theta, z)\,\vec{u}_\theta + E_z(r, \theta, z)\,\vec{k}$$

By ***symmetry*** and ***invariance***, the electric field at point M is written as:

$$\boxed{\vec{E}_M = E_r(r)\,\vec{u}_r = E\,\vec{u}_r}$$

*The charge distribution **does not depend** on the variable z and θ*

On the surfaces of the bases ($\boldsymbol{S_1}$ and $\boldsymbol{S_2}$) the field $\vec{E}$ is **perpendicular** $\perp$ to the vector $\overrightarrow{dS}$,

so there is **no flux crossing these two surfaces** ($\cos \pi/2 = 0$).

On the other hand, on the lateral surface ($\boldsymbol{S_3}$), the vectors $\overrightarrow{dS}$ are all **radial** like $\vec{E}$, ($\cos 0 = 1$).

$$\phi = \oint E\, dS_1 \underbrace{\cos\frac{\pi}{2}}_{0} + \oint E\, dS_2 \underbrace{\cos\frac{\pi}{2}}_{0} + \oint E\, dS_3 \cos 0 = E\, S_3$$

Knowing that: $\sum Q_{int} = \int_0^h \lambda\, dz = \lambda\, h$ *and* $S_3 = 2\pi\, r\, h$

It results: $E\, S_3 = \dfrac{\lambda\, h}{\varepsilon_0} \Longrightarrow \boxed{E = \dfrac{\lambda}{2\pi\, \varepsilon_0\, r}}$ *and* $\boxed{\vec{E}_M = \dfrac{\lambda}{2\pi\, \varepsilon_0\, r}\vec{u}_r}$

Potential produced by a uniformly charged wire of infinite length

The relationship between electrostatic field and electrostatic potential is given as follows:

$$\vec{E} = -\overrightarrow{grad}\, V$$

and as: $\vec{E}_M = E_r(r)\,\vec{u}_r$ so $E = -\frac{dV}{dr}$

$$\Rightarrow \quad dV = -E\,dr \qquad \Rightarrow \quad V = -\int E\,dr = -\int \frac{\lambda}{2\pi\,\varepsilon_0\,r}\,dr$$

Therefore, we find: $\boxed{V = -\frac{\lambda}{2\pi\,\varepsilon_0}\ln r + Const}$

In summary, to calculate the electrostatic field using ***Gauss's law***, we must:

*calculate the **first term** of Gauss's law,*

*then calculate the **second term** and **equate them** to find **E**.*

Thank you for your attention