Lecture 3 – Analogue Realisation of Filter Transfer Functions

3.1 LCR filters

The use of all 3 types of linear electric circuit elements – R's, L's and C's – enables poles or zeroes to be placed anywhere in the s-plane, and in particular anywhere to the left of the imaginary axis; hence any Butterworth, Chebyshev, elliptic or Bessel-Thomson filter can in theory be realised with a passive LCR filter. The synthesis of such filters is a subject on its own and we will only cover the basics in this lecture.

Although good band-pass characteristics can be achieved with passive LCR filters, the use of inductors is best avoided, especially at low frequencies. Inductors tend to be lossy (i.e. they have significant series resistance), expensive and bulky over the low-frequency range. The use of active devices allows inductors to be eliminated from the network, as a circuit with R's and C's only but *with gain* can have poles or zeroes anywhere in the s-domain.

3.2 Active filters

These are usually built around operational amplifiers which have a high input impedance (i.e. a JFET input stage). Component accuracy better than 10% is required, and 5% or even 2% may be needed for Chebyshev and higher-order filters. There are a number of circuit designs, and you will already have met some of these in the Electronic Instrumentation course in your second year. Some of what follows should therefore be revision.

3.2.1 Low-pass filters

The general transfer function for a second-order low-pass filter is :

$$G(s) = \frac{K}{1 \; + \; a_1(s/\omega_c) \; + \; a_2(s/\omega_c)^2}$$

where K = d.c. gain of filter and ω_c = cut-off frequency The circuit in Fig. 3.1 will implement the above transfer function. To analyse the circuit, start from the

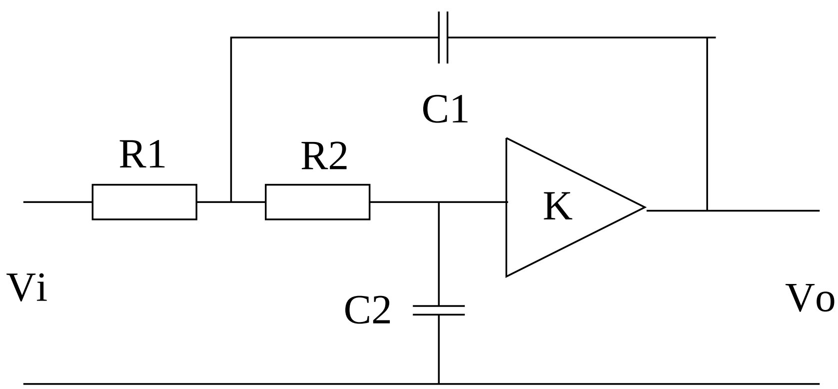

Figure 3.1: *Second-order LPF circuit.*

right and work towards the left; the final result (see Second Year lecture notes) is:

$$\frac{v_o}{v_i}(s) = \frac{K}{1 + s[C_2(R_1 + R_2) \; + \; (1 - K)C_1R_1] + \; s^2C_1C_2R_1R_2}$$

For a Butterworth filter, a_2 = 1 and a_1 = 2ζ

$$\text{i.e.} \quad \omega_c = \frac{1}{\sqrt{C_1C_2R_1R_2}} \qquad\qquad \zeta = \sqrt{\frac{C_2}{C_1}} \cdot \frac{R_1 + R_2}{2\sqrt{R_1R_2}} - \frac{(K-1)}{2}\sqrt{\frac{C_1R_1}{C_2R_2}}$$

To implement the Butterworth second-order section, there are two possibilities:

Sallen-Key filter ($K = 1$)

If $K = 1$, then $\zeta = \frac{1}{2}\sqrt{\frac{C_2}{C_1}} \cdot \frac{R_1+R_2}{\sqrt{R_1R_2}}$ which is greater than $\sqrt{\frac{C_2}{C_1}}$ since $\frac{R_1+R_2}{2} > \sqrt{R_1R_2}$.

So for $\zeta < 1$, we need $C_1 > C_2$. Usually $R_1 = R_2 = R$ and the resistor value is chosen to be as large as possible so that the capacitors will be small (in general, the capacitors will have to be made from a combination of individual components).

VCVS filter ($K > 1$)

An ideal voltage–controlled voltage source (VCVS) is a voltage amplifier with $Z_{in} = \infty$, $Z_{out} = 0$ and a constant voltage gain. The high input impedance, low output impedance and stable gain of an op-amp configured as a non-inverting amplifier make it a good approximation to an ideal VCVS. With the VCVS circuit,

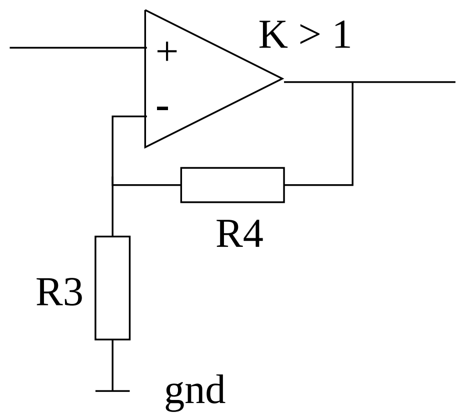

Figure 3.2: $K > 1$ *gain stage for VCVS filter.*

we can now set $C_1 = C_2$ and $R_1 = R_2$. The disadvantage is that the dc gain is constrained to be some odd value, difficult to obtain precisely unless close–tolerance resistors are used for R_3 and R_4.

3.3 High–pass filters

$$G(s) = \frac{K(s/\omega_c)^2}{1 + 2\zeta(s/\omega_c) + (s/\omega_c)^2}$$

where K = high–frequency gain[4] and ω_c = cut–off frequency

[4] in practice, the response of the high–pass filter must fall off at high frequencies because of the open–loop bandwidth limitations of the op–amp.

Such a filter can be realised simply by interchanging the resistors and capacitors in the circuit of Fig. 3.1.

3.4 Band-pass filters

$$G(s) = \frac{K \,.\, 2\zeta(s/\omega_n)}{1 + 2\zeta(s/\omega_n) + (s/\omega_n)^2}$$

where K = mid-band gain and ω_n = centre frequency.

The above is normally re–written as :

$$G(s) = \frac{(K/Q) \,.\, (s/\omega_n)}{1 + \frac{1}{Q}(\frac{s}{\omega_n}) + (\frac{s}{\omega_n})^2}$$

since Q = quality factor = $1/2\zeta$ = $\omega_n/\Delta\omega$

$\Delta\omega$ is the 3 dB bandwidth of the bandpass filter, ie. $\omega_u - \omega_l$ where ω_u is the upper limit of the passband and ω_l the lower limit.

The band-pass version of the VCVS filter circuit (see Fig. 3.3) can be analysed in the same way as the low-pass filter circuit, although one ends up with very cumbersome expressions for K, ω_n and Q (see G.B. Clayton, *Linear Integrated Circuit Applications*, page 63). The multiple-feedback design that follows is a much more popular alternative for band-pass filters.

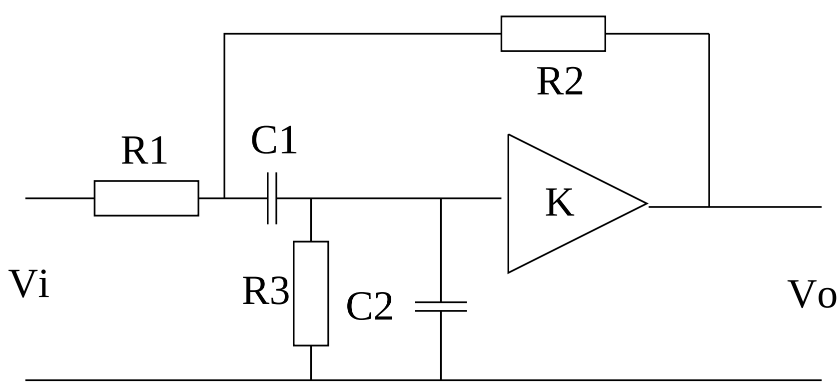

Figure 3.3: *VCVS band-pass filter circuit*

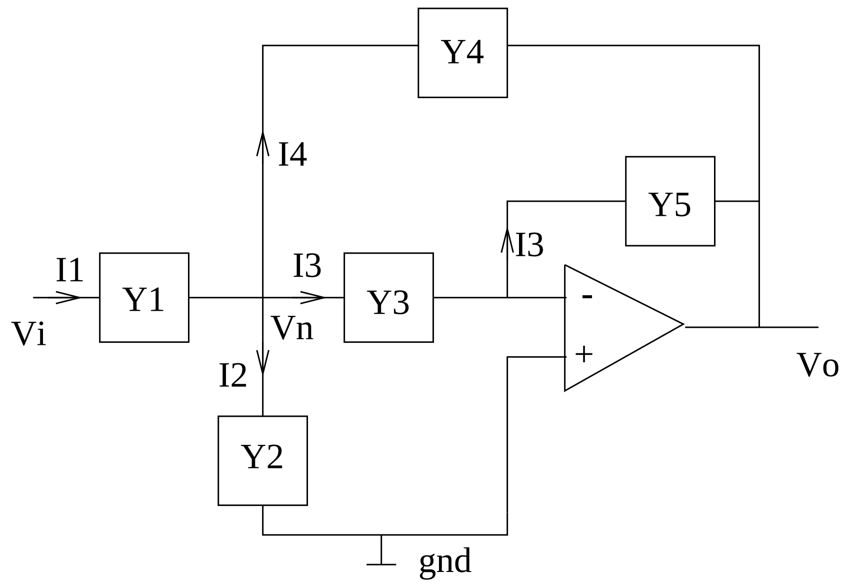

Figure 3.4: *Generalised multiple feedback circuit.*

3.5 Multiple feedback circuits

Summing the currents at node N gives:

$$I_1 = Y_1(v_i - v_N) = I_2 + I_3 + I_4 = Y_2 v_N \ + \ Y_3 v_N \ + \ Y_4(v_N - v_o)$$

Re-arranging the above leads to:

$$Y_1 v_i = v_N(Y_1 + Y_2 + Y_3 + Y_4) \ - \ Y_4 v_o$$

We also have:

$$Y_3 v_N \ = -Y_5 v_o \quad \rightarrow \quad v_N = -\frac{Y_5}{Y_3} v_o$$

$$Y_1 Y_3 v_i = v_o[-Y_5(Y_1 + Y_2 + Y_3 + Y_4) \ - \ Y_3 Y_4]$$

and so we end up with:

$$\frac{v_o}{v_i} = \frac{-Y_1Y_3}{Y_5(Y_1 + Y_2 + Y_3 + Y_4)\ +\ Y_3Y_4}$$

For a low–pass filter, $Y_1 = 1/R_1; Y_3 = 1/R_2; Y_4 = 1/R_3$ and $Y_2 = sC_1; Y_5 = sC_2$.

For a high–pass filter, swap the R's and C's, as before.

For a band–pass filter, $Y_1 = 1/R_1; Y_2 = 1/R_2; Y_5 = 1/R_3$ and $Y_3 = sC_1; Y_4 = sC_2$.

Example of a band-pass filter using multiple feedback design

For the case when $C_1 = C_2$, we have the following circuit,

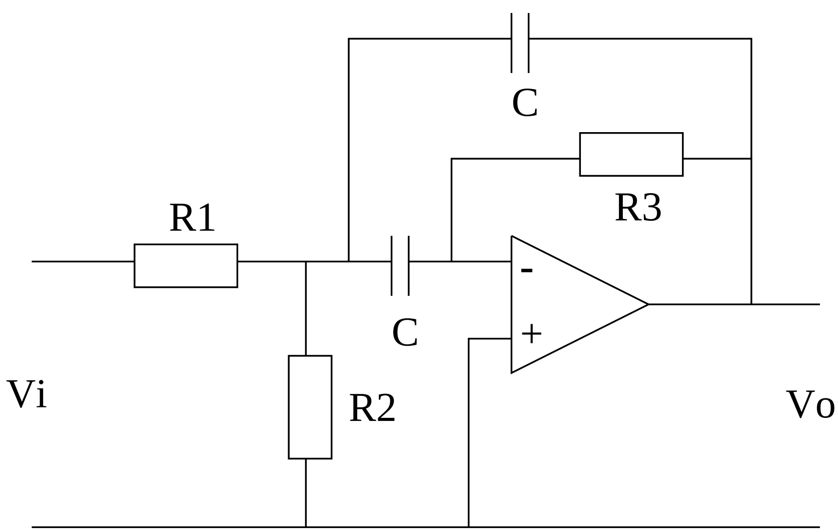

Figure 3.5: *Multiple feedback 2nd-order band-pass filter circuit.*

$K = -\frac{1}{2}\frac{R_3}{R_1}$ (gain at resonance)

$\omega_n = \frac{1}{C\sqrt{R_{12}.R_3}}$

$$Q = 0.5\sqrt{R_3/R_{12}}$$

where $R_{12} = \frac{R_1 R_2}{R_1 + R_2}$.

Since $Q^2 = \frac{1}{4}\frac{R_3}{R_{12}}$ and $\left|\frac{K}{2}\right| = \frac{1}{4}\frac{R_3}{R_1}$, we must have $Q^2 > \left|\frac{K}{2}\right|$.

Note that, since Q depends on the square root of component ratios, a high-Q filter (≈ 100) requires very high component ratios. In practice, R_2 needs to be adjustable in order to get the correct resonant frequency, as the latter is affected by the non-ideal behaviour of the op-amp.

3.6 State-variable filters

The performance of second-order sections can be improved by the introduction of additional op-amps. The disadvantage of increased power consumption is more than offset by reduced sensitivity to component variations and the use of a standard topology to realize the basic frequency responses.

One such circuit is the state-variable filter which can provide second-order low-pass, band-pass and high-pass outputs simultaneously. The circuit consists of two integrators and an inverting gain stage. In the version shown below, negative feedback around all 3 stages is provided by R_5, whilst R_6 and R_7 form a positive feedback loop around the first two stages. It is a simple matter to show that v_A, v_B and v_C are the high-pass, band-pass and low-pass outputs, respectively.

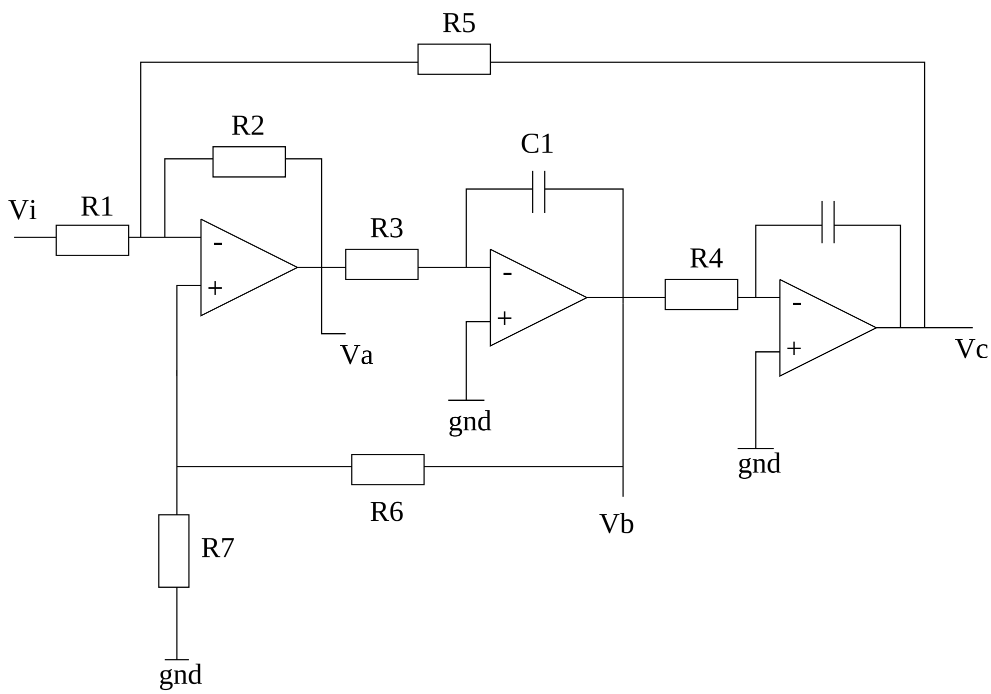

Figure 3.6: *Universal or state-variable filter.*