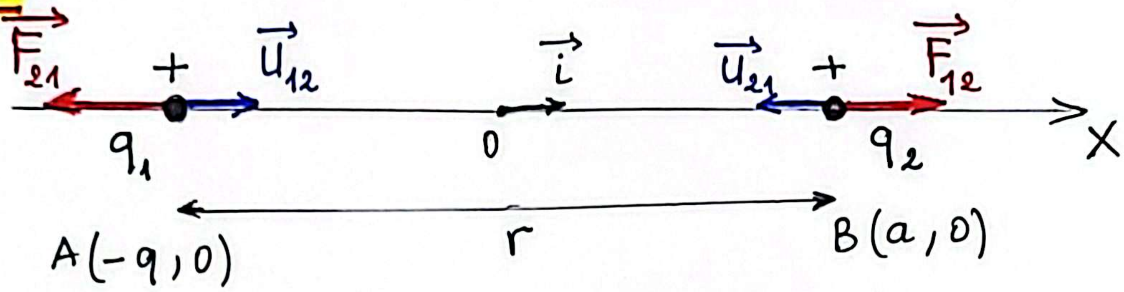
1st year LMD 2025 / 2026

Computer Science

# Solution of * Series 2 - Part 1 *

## Ex1:

**1**




$q_1, q_2 > 0 \Rightarrow$ we have a repulsive force.

$\vec{F}_{12}$ : Force exerted by $q_1$ on $q_2$.

$\vec{F}_{21}$ : " " by $q_2$ on $q_1$.

**2** According to Coulomb's law :

* $\boxed{\vec{F}_{12} = K \dfrac{q_1 q_2}{r_{12}} \cdot \vec{u}_{12}}$ ; $K = \dfrac{1}{4\pi\varepsilon_0} = 9.10^9$ (SI)

The unit vector $\vec{u}_{12} = \dfrac{\vec{AB}}{\|\vec{AB}\|}$ ; $r_{12} = \|\vec{AB}\|$

$\Rightarrow \boxed{\vec{F}_{12} = K \dfrac{q_1 q_2}{\|\vec{AB}\|^3} \cdot \vec{AB}}$ ; $\vec{AB}\begin{pmatrix} 2a \\ 0 \end{pmatrix} \Rightarrow \boxed{r_{12} = \|\vec{AB}\| = 2a}$

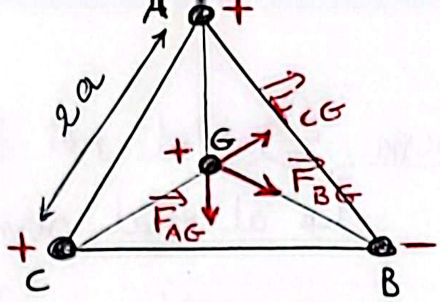
$\vec{u}_{12} = \vec{i} \Rightarrow \boxed{\vec{F}_{12} = K \dfrac{q_1 q_2}{4a^2} \cdot \vec{i}}$

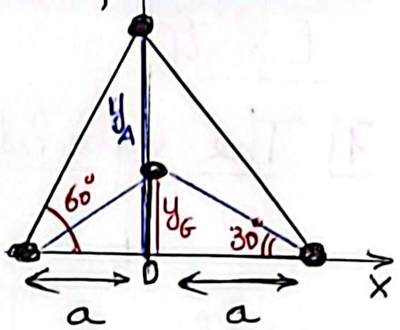
* $\boxed{\vec{F}_{21} = K \dfrac{q_2 q_1}{r_{21}^2} \cdot \vec{u}_{21}}$ ; $\vec{u}_{21} = -\vec{i}$

$\Rightarrow \boxed{\vec{F}_{21} = -K \dfrac{q_1 q_2}{4a^2} \cdot \vec{i}}$

We conclude that : $\boxed{\vec{F}_{12} = -\vec{F}_{21}}$ and $\boxed{\|\vec{F}_{12}\| = \|\vec{F}_{21}\|}$

## EX2:




The force exerted by the charges on the center of the triangle is :

$$\boxed{\vec{F}_G = \vec{F}_{AG} + \vec{F}_{BG} + \vec{F}_{CG}}$$

$$\vec{F}_{AG} = K\frac{q^2}{\|\vec{AG}\|^3}\cdot\vec{AG} \quad ; \quad \vec{F}_{BG} = -K\frac{q^2}{\|\vec{BG}\|^3}\cdot\vec{BG}$$

$$\vec{F}_{CG} = K\frac{q^2}{\|\vec{CG}\|^3}\cdot\vec{CG}$$

we know that : $\|\vec{AG}\| = \|\vec{BG}\| = \|\vec{CG}\|$ in equilateral triangle

$$\Rightarrow \boxed{\vec{F}_G = \frac{Kq^2}{\|\vec{AG}\|^3}\cdot\left(\vec{AG} - \vec{BG} + \vec{CG}\right)} \text{ --- } (*)$$

The coordinates of the charges are :

$A(0, y_A)$ ; $B(a, 0)$ ; $C(-a, 0)$ ; $G(0, y_G)$

we have: $\sin 60° = \frac{y_A}{2a} \Rightarrow \frac{\sqrt{3}}{2} = \frac{y_A}{2a} \Rightarrow \boxed{y_A = a\sqrt{3}}$

$$tg\,30° = \frac{y_G}{a} \Rightarrow \frac{1}{\sqrt{3}} = \frac{y_G}{a} \Rightarrow \boxed{y_G = \frac{a}{\sqrt{3}}}$$

$$\vec{AG} = (y_G - y_A)\vec{j} = \left(\frac{a}{\sqrt{3}} - a\sqrt{3}\right)\vec{j} \Rightarrow \boxed{\vec{AG} = -\frac{2a}{\sqrt{3}}\vec{j}}$$

$$\boxed{\vec{BG} = -a\vec{i} + \frac{a}{\sqrt{3}}\vec{j}} \quad ; \quad \boxed{\vec{CG} = a\vec{i} + \frac{a}{\sqrt{3}}\vec{j}}$$

$$\Rightarrow \boxed{\vec{AG} - \vec{BG} + \vec{CG} = \frac{2a}{\sqrt{3}}\left(\sqrt{3}\,\vec{i} - \vec{j}\right)} \quad ; \|\vec{AG}\| = \frac{-8a^3}{3\sqrt{3}}$$

We replace this result in $(*)$ $\Rightarrow$

$$\vec{F}_G = -\frac{Kq^2}{8a^3}\cdot 3\sqrt{3}\left[\frac{2a}{\sqrt{3}}\left(\sqrt{3}\,\vec{i} - \vec{j}\right)\right] = \frac{3Kq^2}{4a^2}\left(-\sqrt{3}\,\vec{i} + \vec{j}\right)$$

$$\Rightarrow \boxed{F_G = \|\vec{F}_G\| = \frac{3Kq^2}{2a^2}} \quad ; \quad \underline{\text{D.A:}}\; q = 1\,\mu m \; ; 2a = 1\,cm$$

$$\Rightarrow \boxed{F_G = 135\ N}$$

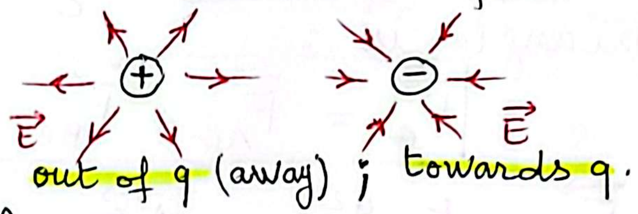
Il faut convertir les unités en (SI)

# EX 3:

**1** The equilibrium $\Rightarrow$ Not net force (Resultant force is zero)

$\hookrightarrow$ at that point $\boxed{E = 0}$ Total electric field

we have: $\boxed{\vec{E} = \dfrac{K q}{r^2} \cdot \vec{u}}$

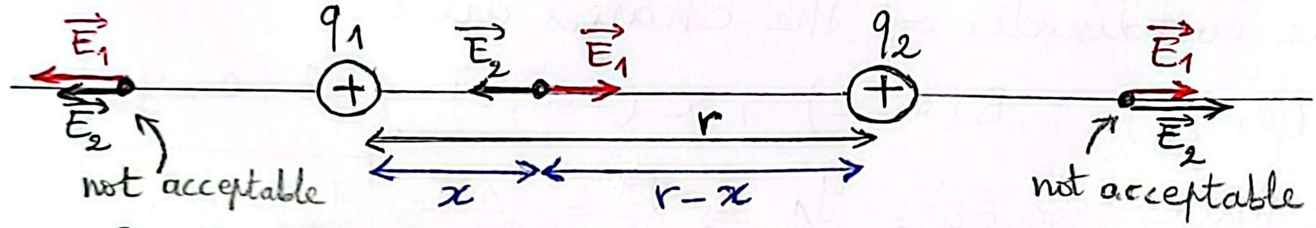



* We must find the point where:

$\boxed{\vec{E_1} + \vec{E_2} = \vec{0}} \Rightarrow \boxed{\vec{E_1} = -\vec{E_2}}$ the same electric field but in opposite direction.

This will only be achieved if we choose a point on the same line of the two charges.

- Case 1:




- On the left of $q_1$, $\vec{E_1}$ and $\vec{E_2}$ are in the same direction
$\Rightarrow$ not acceptable $(\vec{E_1} + \vec{E_2} \neq \vec{0})$
- On the right of $q_2$, $\vec{E_1}$ and $\vec{E_2}$ are in the same direction
$\Rightarrow$ not acceptable $(\vec{E_1} + \vec{E_2} \neq \vec{0})$
- In the middle of the two charges (and closer to the smaller one)
$\Rightarrow$ acceptable $(\vec{E_1} + \vec{E_2} = \vec{0})$

we have:

$\boxed{\vec{E_1} = \dfrac{K q_1}{x^2} \cdot \vec{u_1}}$ ; $\boxed{\|\vec{E_1}\| = E_1 = \dfrac{K |q_1|}{x^2}}$

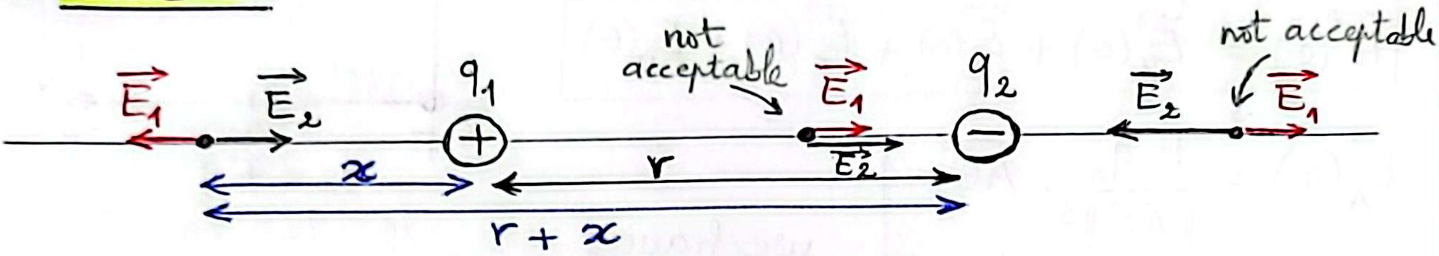
$\boxed{\vec{E_2} = \dfrac{K q_2}{(r-x)^2} \cdot \vec{u_2}}$ ; $\boxed{\|\vec{E_2}\| = E_2 = \dfrac{K |q_2|}{(r-x)^2}}$

$\vec{u_1}, \vec{u_2}$ : unit vectors of $\vec{E_1}$ and $\vec{E_2}$ respectively.

$$E_1 = E_2 \Rightarrow \frac{K|q_1|}{x^2} = \frac{K|q_2|}{(r-x)^2} \Rightarrow \sqrt{|q_1|}\cdot(r-x) = \sqrt{|q_2|}\cdot x$$

Hence : $\boxed{x = \dfrac{\sqrt{|q_1|}\cdot r}{\sqrt{|q_1|}+\sqrt{|q_2|}}}$

- Case 2 :




- In the middle of the two charges, $\vec{E_1}$ and $\vec{E_2}$ are in the same direction
$\Rightarrow$ not acceptable $(\vec{E_1}+\vec{E_2} \neq \vec{0})$.
- On the right of $q_2$, $\|\vec{E_2}\| > \|\vec{E_1}\|$ because $|q_2|>|q_1|$ and $r_1 > r_2$
$\Rightarrow$ not acceptable $(\vec{E_1}+\vec{E_2} \neq \vec{0})$
- On the left of $q_1$, $\Rightarrow$ acceptable $(\vec{E_1}+\vec{E_2} = \vec{0})$.

$$E_1 = E_2 \Rightarrow \frac{K|q_1|}{x^2} = \frac{K|q_2|}{(r+x)^2} \Rightarrow \sqrt{|q_1|}\cdot(r+x) = \sqrt{|q_2|}\cdot x$$

Hence : $\boxed{x = \dfrac{\sqrt{|q_1|}\cdot r}{\sqrt{|q_2|}-\sqrt{|q_1|}}}$

**2** According to the first question, this correspond to the first case : $\boxed{x = \dfrac{\sqrt{|q_1|}\cdot r}{\sqrt{|q_1|}+\sqrt{|q_2|}}}$

D.A : $q_1 = +2\,nC$ ; $q_2 = +18\,nC$ ; $r = 25\ cm$

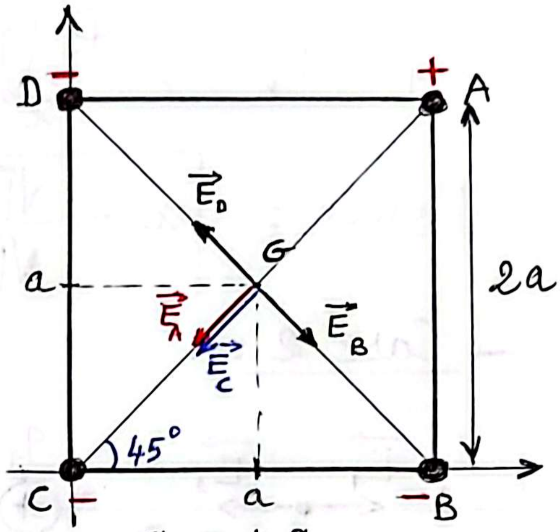
$$x = \frac{\sqrt{2.10^{-9}}\cdot 25.10^{-2}}{\sqrt{2.10^{-9}}+\sqrt{18.10^{-9}}} = \frac{\cancel{\sqrt{2.10^{-9}}}\cdot 25.10^{-2}}{\cancel{\sqrt{2.10^{-9}}}\,(1+3)}$$

$$\Rightarrow \boxed{x = 6{,}25.10^{-2}\ m = 6{,}25\ cm}$$

# EX4:

**1** In presence of multiple charges:

$$\boxed{E = \sum_i E_i = \frac{K\, q_i}{r_i^2}}$$




$q_A = +q$

$q_B = q_C = q_D = -q$

At the point G we have:

$$\boxed{\vec{E}(G) = \vec{E}_A(G) + \vec{E}_B(G) + \vec{E}_C(G) + E_D(G)}$$

$$\left.\begin{aligned}
\vec{E}_A(G) &= \frac{K\, q_A}{\|\vec{AG}\|^3} \cdot \vec{AG} \\
\vec{E}_B(G) &= \frac{K\, q_B}{\|\vec{BG}\|^3} \cdot \vec{BG} \\
\vec{E}_C(G) &= \frac{K\, q_C}{\|\vec{CG}\|^3} \cdot \vec{CG} \\
\vec{E}_D(G) &= \frac{K\, q_D}{\|\vec{DG}\|^3} \cdot \vec{DG}
\end{aligned}\right\} \Rightarrow$$

we have:

$\|\vec{AG}\| = \|\vec{BG}\| = \|\vec{CG}\| = \|\vec{DG}\|$

$$\boxed{\vec{E}_{(G)} = \frac{Kq}{\|\vec{AG}\|^3}\left(\vec{AG} - \vec{BG} - \vec{CG} - \vec{DG}\right)}$$

* G the center of the square $\Rightarrow G(a, a)$

$$\sin\frac{\pi}{4} = \frac{a}{CG} \Rightarrow \frac{1}{\sqrt{2}} = \frac{a}{CG} \Rightarrow \boxed{CG = \sqrt{2}\cdot a}$$

Or: by calculating the modulus of the vector $\vec{AG}$.

$$\|\vec{AG}\| = AG = \sqrt{a^2 + a^2} \Rightarrow \boxed{AG = \sqrt{2}\cdot a}$$

Now, we find the different vectors:

Exp: $\vec{AG} = (x_G - x_A)\cdot\vec{i} + (y_G - y_A)\cdot\vec{j}$

$$\left.\begin{aligned}
\vec{AG} &= -a\,\vec{i} - a\,\vec{j} \\
\vec{BG} &= -a\,\vec{i} + a\,\vec{j} \\
\vec{CG} &= a\,\vec{i} + a\,\vec{j} \\
\vec{DG} &= a\,\vec{i} - a\,\vec{j}
\end{aligned}\right\} \Rightarrow \boxed{\vec{AG} - \vec{BG} - \vec{CG} - \vec{DG} = -2a\,\vec{i} - 2a\,\vec{j}}$$

$$\boxed{\|\vec{AG}\|^3 = AG^3 = 2\sqrt{2}\cdot a^3}$$

by replacing in the expression of $\vec{E}(G)$, we find:

$$\vec{E}(G) = -\frac{K q \cdot 2a}{2\sqrt{2} \cdot a^3} \cdot (\vec{i} + \vec{j}) \Rightarrow \boxed{\vec{E}(G) = -\frac{K \cdot q}{\sqrt{2} \cdot a^2} \cdot (\vec{i} + \vec{j})}$$

* The modulus of the electric field vector is:

$$\boxed{E(G) = \|\vec{E}(G)\| = \frac{K \cdot q}{a^2}}$$ its unit is: $(V/m)$ or $(N/C)$

**2** The force exerted by the other charges on $q_G$:

$$\boxed{\vec{F}(G) = q_G \cdot \vec{E}(G)}$$

we have: $q_G = +q \Rightarrow \boxed{\vec{F}(G) = -\frac{K \cdot q^2}{\sqrt{2} \cdot a^2} \cdot (\vec{i} + \vec{j})}$

* The modulus of the electric force vector is:

$$\boxed{F(G) = \|\vec{F}(G)\| = \frac{K \cdot q^2}{a^2}}$$ its unit is: $(N)$

**3** The electric potential V:

In the presence of multiple charges:

$$\boxed{V = \sum_i V_i = \sum_i \frac{K \cdot q_i}{r_i}}$$ is a scalar

* $V(G)$ is the electric potential created by all the charges at the point $G$:

$$\boxed{V(G) = V_A(G) + V_B(G) + V_C(G) + V_D(G)}$$

$$\left.\begin{aligned} V_A(G) &= \frac{K q_A}{AG} \\ V_B(G) &= \frac{K q_B}{BG} \\ V_C(G) &= \frac{K q_C}{CG} \\ V_D(G) &= \frac{K q_D}{DG} \end{aligned}\right\} \Rightarrow V(G) = \frac{K q}{\sqrt{2} \cdot a}(+1-1-1-1) \Rightarrow \boxed{V(G) = -\frac{\sqrt{2} K \cdot q}{a}}$$

* $V(C)$ is the electric potential created at the origin of the coordinates ≡ at the point $C(0,0)$:

$$\boxed{V(C) = V_A(C) + V_B(C) + V_D(C)}$$

We note that we only concider the charges located at the vertices of the square ABCD.

$$\left.\begin{aligned} V_A(C) &= \frac{K q_A}{AC} \\ V_B(C) &= \frac{K q_B}{BC} \\ V_D(C) &= \frac{K q_D}{DC} \end{aligned}\right\} \Rightarrow V(C) = \frac{K.q}{2a}\cdot\left(\frac{1}{\sqrt{2}} - 1 - 1\right) = \frac{K.q}{2a}\left(\frac{1-2\sqrt{2}}{\sqrt{2}}\right)$$

We have: $AC = \sqrt{(2a)^2 + (2a)^2} = 2\sqrt{2}.a$

$BC = DC = 2a$

$$\boxed{V(C) = \frac{K.q}{2\sqrt{2}.a}\cdot(1 - 2\sqrt{2})}$$

D.A :

$q = 2\ \mu C \quad ; \quad a = 10\ cm \quad ; \quad K = 9.10^9\ (SI)$

* $\boxed{E(G) = \frac{K.q}{a^2}}$

$$E(G) = \frac{9.10^9 .\ 2.10^{-6}}{(10.\ 10^{-2})^2} \Rightarrow \boxed{E(G) = 18.10^5\ V/m}$$

* $\boxed{F(G) = \frac{K.q^2}{a^2}}$

$$F(G) = \frac{9.10^9 .\ (2.10^{-6})^2}{(10.\ 10^{-2})^2} \Rightarrow \boxed{F(G) = 3,6\ N}$$

* $\boxed{V(G) = -\frac{\sqrt{2}.Kq}{a}}$

$$V(G) = -\frac{\sqrt{2}.\ 9.10^9 .\ 2.10^{-6}}{10.\ 10^{-2}} \Rightarrow \boxed{V(G) = -2,54.10^5\ V}$$

* $\boxed{V(C) = \frac{K.q}{2\sqrt{2}.a}(1 - 2\sqrt{2})}$

$$V(C) = \frac{9.10^9 .\ 2.10^{-6}(1 - 2\sqrt{2})}{2\sqrt{2}.\ 10.\ 10^{-2}} \Rightarrow \boxed{V(C) = -1,16.10^5\ V}$$