L1 Informatics
Algebra 1

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Series of Exercises 02
Sets, Functions and Binary Relation

Exercice 1 *Let the following sets :* $A =]-\infty, 3], B = [-2, 8], C =]-5, +\infty[$, $D = \{x \in \mathbb{R}, |x-3| \leq 5\}$

1. *What are the equality or inclusion relationships that exist between these sets ?*
2. *Find the complement in the following cases :* $C_{\mathbb{R}}A, C_{\mathbb{R}}B, C_{\mathbb{R}}C, C_C B$.
3. *Find* $A \cap B, A \cup B, A \cap C, A \cup C, A \setminus C, (\mathbb{R} \setminus A) \cap (\mathbb{R} \setminus B)$, *and* $A \triangle B$.

Exercice 2 *Let the set* E *and* A, B, D *are three parts of* E
a. Show that :

1. $A \subset B \Rightarrow C_E B \subset C_E A$
2. $(A \setminus B) \setminus D = A \setminus (B \cup D)$
3. $C_E A \triangle C_E B = A \triangle B$
4. $(A \times D) \cup (B \times D) = (A \cup B) \times D$

b. Simplify

1. $C_E(A \cup B) \cap C_E(D \cup C_E A)$
2. $C_E(A \cap B) \cup C_E(D \cap C_E A)$. ***(homework)***

Exercice 3 *Let the functions* $f : [0, 1] \to [0, 2]$ *with* $f(x) = 2 - x$ *and* $g : [-1, 1] \to [0, 2]$ *with* $g(x) = x^2 + 1$

1. *Find* $f(\{\frac{1}{2}\}), f^{-1}(\{0\}), g([-1, 1]), g^{-1}[0, 2])$
2. *Study the injectivity and surjectivity of* f, *is the function* f *bijective ?*
3. *Study the injectivity and surjectivity of* g, *is the function* g *bijective ?*
4. *Can we calculate* $g \circ f$ *and* $f \circ g$ *Justify.*

Exercice 4 *Let* f *the application defined by :*

$$\begin{aligned} f : E &\longrightarrow \mathbb{R} \\ x &\longmapsto f(x) = \tfrac{1}{\sqrt{x^2-1}} \end{aligned}$$

1. *Find* E *so that* f *is an application.*
2. *We take :* $E =]-\infty, -1[\cup]1, +\infty[$
   a) *Determine* $f\left(\left\{-\sqrt{2}, \frac{5}{3}, \sqrt{2}\right\}\right)$ *and* $f^{-1}(\{0\})$.
   b) *Is the application* f *injective ? Is it surjective ? Justify your answer.*
3. *Show that the restriction* $g :]1, +\infty[\longrightarrow]0, +\infty[, \quad g(x) = f(x)$ *is bijective.*
4. *Determine the inverse application* g^{-1}.

Exercice 5 *Let $f : E \to F$ be a function. Let A and A' be two subsets of E, and let B and B' be two subsets of F. Show that :*

1- $A \subset f^{-1}(f(A))$	$2 - f\left(f^{-1}(B)\right) \subset B($ *homework* $)$
3- $f\left(A \cup A'\right) = f(A) \cup f\left(A'\right)$	$4-f$ *injective* $\Rightarrow f\left(A \cap A'\right) = f(A) \cap f\left(A'\right)$ (*homework*)
$5 - f^{-1}\left(B \cap B'\right) = f^{-1}(B) \cap f^{-1}\left(B'\right)$	$6 - f^{-1}\left(B \cup B'\right) = f^{-1}(B) \cup f^{-1}\left(B'\right)$ *(homework)*

Exercice 6 *We define the relation $\Re$ on $\mathbb{R}$:*

$$\forall x, y \in \mathbb{R}, \quad x \Re y \Leftrightarrow x^4 - y^4 = x^2 - y^2$$

1. *Show that $\Re$ is an equivalence relation.*
2. *Find the equivalence class of 0, and deduce that of 1.*
3. *Determine the equivalence class of x for any real x.*

Exercice 7 *We define the binary relation $\mathcal{R}$ in $\mathbb{N}^*$ by :*

$$\forall x, y \in \mathbb{N}^*, \quad x \mathcal{R} y \Leftrightarrow \exists n \in \mathbb{N} \ \text{ such that : } y^n = x$$

1. *Show that $\mathcal{R}$ is a order relation.*
2. *Is this order total ? Justify your answer.*

Exercice 8 *We define the relation $\mathcal{R}$ in $\mathbb{R}^2$:*

$$\forall (x, y); (x', y') \in \mathbb{R}^2, (x, y) \mathcal{R} (x', y') \Leftrightarrow |x - x'| \leq y' - y$$

1. *Verify that : $(1,2)\mathcal{R}(4,7)$ and $(2,3)\mathcal{R}(5,3)$.*
2. *Show that $\mathcal{R}$ is a order relation.*
3. *Is the order total or partial ?*

Corrected Excercises 2

Excercise 1
Let $A =]-\infty, 3]$, $B = [-2, 8]$, $C =]-5, +\infty[$

$D = \{x \in \mathbb{R},\ |x-3| \leq 5\}$

1) The Equality or inclusion relationships between these sets:

We have: $|x-3| \leq 5 \Rightarrow -5 \leq x-3 \leq 5 \Rightarrow -2 \leq x \leq 8$

then: $x \in [-2, 8]$

So, $B = D$ and $\begin{cases} B \subset C \\ \wedge \\ D \subset C \end{cases}$

2) $C_{\mathbb{R}}A =]3, +\infty[$; $C_{\mathbb{R}}B =]-\infty, -2[\cup]8, +\infty[$

$C_{\mathbb{R}}C =]-\infty, -5]$; $C_C B =]-5, -2[\cup]8, +\infty[$

3) $A \cap B = [-2, 3]$; $A \cup B =]-\infty, 8]$; $A \cap C =]-5, 3]$

$A \cup C =]-\infty, +\infty[$; $A \setminus C =]-\infty, -5]$

$(\mathbb{R} \setminus A) \cap (\mathbb{R} \setminus B) = C_{\mathbb{R}}A \cap C_{\mathbb{R}}B =]3, +\infty[\cap (]-\infty, -2[\cup]8, +\infty[)$

$=]8, +\infty[$

$A \Delta B = (A \setminus B) \cup (B \setminus A) =]-\infty, -2[\cup]3, 8]$

Excercise 2:
Let A, B and D three parts of E

a) Show that:

1) $A \subset B \Rightarrow C_E B \subset C_E A$

Let's assume that $A \subset B$ and $x \in C_E B$. Then:

$x \in C_E B \Rightarrow x \notin B$ and since $A \subset B$, we have $x \notin A \Rightarrow x \in C_E A$

Then: $C_E B \subset C_E A$

2) $(A \setminus B) \setminus D = A \setminus (B \cup D)$

$$(A \setminus B) \setminus D = (A \cap C_E B) \cap C_E D$$
$$= A \cap [C_E B \cap C_E D]$$
$$= A \cap C_E(B \cup D) = A \setminus (B \cup D)$$

3) $C_E A \,\Delta\, C_E B = A \,\Delta\, B$

According to the definition:

$$A \Delta B = (A \setminus B) \cup (B \setminus A) = [A \cap C_E B] \cup [B \cap C_E A]$$

by replacing A with $C_E A$ and B with $C_E B$ in the previous formula:

$$C_E A \,\Delta\, C_E B = [C_E A \setminus C_E B] \cup [C_E B \setminus C_E A]$$
$$= [C_E A \cap C_E(C_E B)] \cup [C_E B \cap C_E(C_E A)]$$
$$= [C_E A \cap B] \cup [C_E B \cap A]$$
$$= [A \cap C_E B] \cup [B \cap C_E A]$$
$$= A \setminus B \;\cup B \setminus A = A \Delta B$$

4) $(A \times D) \cup (B \times D) = (A \cup B) \times D$

The Cartesian Product is: $A \times B = \{(x, y) \mid (x \in A) \wedge (y \in B)\}$

Then, we have:

$$(A \times D) \cup (B \times D) = \{(x, y) \mid (x, y) \in A \times D \;\vee\; (x, y) \in B \times D\}$$
$$= \{(x, y) \mid (x \in A \wedge y \in D) \vee (x \in B \wedge y \in D)\}$$
$$= \{(x, y) \mid (x \in A \vee x \in B) \wedge y \in D\}$$
$$= \{(x, y) \mid x \in A \cup B \wedge y \in D\}$$
$$= (A \cup B) \times D$$

b) Simplify:

1) $C_E(A \cup B) \cap C_E(D \cup C_E A) = [C_E A \cap C_E B] \cap [C_E D \cap A]$

$= [A \cap C_E A] \cap [C_E B \cap C_E D]$

$= \emptyset \cap C_E B \cap C_E D$

$= \emptyset$

Exercise 3 Let: $f : [0,1] \longrightarrow [0,2]$

$x \longmapsto f(x) = 2 - x$

$g : [-1,1] \longrightarrow [0,2]$

$x \longmapsto g(x) = x^2 + 1$

1) Find:

* $f(\{\frac{1}{2}\}) = \{ f(x) \in [0,2] \ , x \in \{\frac{1}{2}\} \} = \{ f(\frac{1}{2}) \} = \{\frac{3}{2}\}$

* $f^{-1}(\{0\}) = \{ x \in [0,1], \ f(x) \in \{0\} \}$

We have: $f(x) = 0 \Rightarrow 2 - x = 0 \Rightarrow x = 2 \notin [0,1]$

Then: $f^{-1}(\{0\}) = \emptyset$

* $g([-1,1]) = \{ g(x) \in [0,2] \ ; x \in [-1,1] \}$

We have: $x \in [-1,1] \Leftrightarrow -1 \leq x \leq 1 \Rightarrow 0 \leq x^2 \leq 1$

$\Rightarrow 1 \leq x^2 + 1 \leq 2 \Rightarrow g(x) \in [1,2] \subset [0,2]$

Then: $g([-1,1]) = [1,2]$

* $g^{-1}([0,2]) = \{ x \in [-1,1], \ g(x) \in [0,2] \}$

We have: $g(x) \in [0,2] \Leftrightarrow \quad 0 \leq g(x) \leq 2$

$$\Rightarrow 0 \leq x^2+1 \leq 2$$
$$\Rightarrow -1 \leq x^2 \leq 1$$
$$\Rightarrow 0 \leq x^2 \leq 1$$
$$\Rightarrow -1 \leq x \leq 1 \Rightarrow x \in [-1,1]$$

Then: $g^{-1}([0,2]) = [-1,1]$

2) Study the injectivity and surjectivity of f:

a) Injectivity:

f is injective: $\forall x_1, x_2 \in [0,1] : f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

$f(x_1) = f(x_2) \Leftrightarrow \quad 2 - x_1 = 2 - x_2 \Rightarrow x_1 = x_2$

Then: f is injective.

b) Surjectivity

f is surjective: $\forall y \in [0,2], \exists x \in [0,1] : y = f(x)$

AS: $f^{-1}(\{0\}) = \emptyset$ Then $0 \in [0,2]$ does not admit antecedent by f in $[-1,1]$. So, f is not surjective and consequently is not bijective

3) Study the injectivity and surjectivity of g:

a) Injectivity: The function g is even, Therfore:

$$g(-1) = g(1) \quad \text{or} \quad -1 \neq 1$$

Then g is not injective.

and consequently is not bijective.

4) We cannot calculate $g \circ f$ and $f \circ g$ because the codomain and domain are not the same in both case.

Exc 04:

$$f : E \longrightarrow \mathbb{R}$$
$$x \longmapsto f(x) = \frac{1}{\sqrt{x^2-1}}$$

1) Find E ~~set~~ that f is an application:

f is application if: $\forall x \in E$, $\exists! y \in \mathbb{R}$ such that: $f(x) = y$

Then: $E = D_f = \{x \in \mathbb{R} : x^2 - 1 > 0\} =]-\infty, -1[\cup]1, +\infty[$

2) Let: $E =]-\infty, -1[\cup]1, +\infty[$

a) $f(\{-\sqrt{2}, \frac{5}{3}, \sqrt{2}\}) = \{f(x) \in \mathbb{R} : x \in \{-\sqrt{2}, \frac{5}{3}, \sqrt{2}\}\} = \{f(-\sqrt{2}), f(\frac{5}{3}), f(\sqrt{2})\}$
$= \{1, \frac{3}{4}\}$

$f^{-1}(\{0\}) = \{x \in E : f(x) \in \{0\}\}$

$f(x) = 0 \Rightarrow \frac{1}{\sqrt{x^2-1}} = 0$ (the equation have not solution

Then: $f^{-1}(\{0\}) = \emptyset$

b) - f is not injective because: $\exists x_1 = -\sqrt{2}, x_2 = \sqrt{2} \in E$

Such that: $x_1 \neq x_2$ but $f(x_1) = f(x_2) = 1$

- f is not surjective because: $\exists y = 0 \in \mathbb{R}$

Such that $f^{-1}(\{0\}) = \emptyset$

3) $g :]1, +\infty[\longrightarrow]0, +\infty[$ such that: $g(x) = \frac{1}{\sqrt{x^2-1}}$

show that g is bijective:

* g injective: $\forall x_1, x_2 \in]1,+\infty[: g(x_1) = g(x_2) \overset{?}{\Rightarrow} x_1 = x_2$

$$g(x_1) = g(x_2) \Leftrightarrow \frac{1}{\sqrt{x_1^2-1}} = \frac{1}{\sqrt{x_2^2-1}} \Rightarrow \sqrt{x_1^2-1} = \sqrt{x_2^2-1} \Rightarrow x_1^2-1 = x_2^2-1$$

$$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = x_2 \left(\begin{array}{c}\text{because} \\ x_1, x_2 \in]1,+\infty[\end{array}\right)$$

Then g is injective.

* g Surjective: $\forall y \in]0,+\infty[, \exists x \in]0,+\infty[: y = g(x)$

$$y = g(x) \Leftrightarrow \frac{1}{\sqrt{x^2-1}} = y \Rightarrow \sqrt{x^2-1} = \frac{1}{y} \Rightarrow x^2-1 = \frac{1}{y^2}$$

$$\Rightarrow x^2 = \frac{1}{y^2}+1 \Rightarrow x = \sqrt{\frac{1}{y^2}+1} \left(\begin{array}{c}\text{because} \\ x \in]1,+\infty[\end{array}\right)$$

for: $y \in]0,+\infty[$ we have $\sqrt{\frac{1}{y^2}+1} \in]1,+\infty[\Rightarrow x \in]1,+\infty[$

Then: g Surjective

Consequently: g is bijective.

4) The inverse Application g^{-1}:

$$g^{-1} :]0,+\infty[\longrightarrow]1,+\infty[$$
$$y \longmapsto x = g^{-1}(y) = \sqrt{\frac{1}{y^2}+1}$$

Exo 5: $f : E \longrightarrow F$, $\begin{cases} A, A' \subset E \\ B, B' \subset F \end{cases}$

Show that:

1) $A \overset{?}{\subset} f^{-1}(f(A))$

$\forall x \in A \Rightarrow f(x) \in f(A) \Rightarrow x \in f^{-1}(f(A))$

Then: $A \subset f^{-1}(f(A))$

2) $f(A \cup A') = f(A) \cup f(A')$

- $f(A \cup A') \overset{?}{\subset} f(A) \cup f(A')$

$\forall y \in f(A \cup A')$: $\exists x \in A \cup A'$ such that: $y = f(x)$

$x \in A \cup A' \Rightarrow \begin{cases} x \in A \\ \vee \\ x \in A' \end{cases} \Rightarrow \begin{cases} f(x) \in f(A) \\ \vee \\ f(x) \in f(A') \end{cases} \Rightarrow f(x) \in f(A) \cup f(A')$

Then: $y \in f(A) \cup f(A')$

So, $f(A \cup A') \subset f(A) \cup f(A')$

- $f(A) \cup f(A') \overset{?}{\subset} f(A \cup A')$

$\forall y \in f(A) \cup f(A') \Rightarrow \begin{cases} y \in f(A) \Rightarrow \exists x \in A : y = f(x) \\ \vee \\ y \in f(A') \Rightarrow \exists x \in A' : y = f(x) \end{cases}$

Then: $\begin{cases} x \in A \\ \vee \\ x \in A' \end{cases} \Rightarrow x \in A \cup A' \Rightarrow f(x) \in f(A \cup A')$

Then: $y \in f(A \cup A')$

So, $f(A) \cup f(A') \subset f(A \cup A')$

Consequently: $f(A \cup A') = f(A) \cup f(A')$

5) $f^{-1}(B \cap B') = f^{-1}(B) \cap f^{-1}(B')$

- $f^{-1}(B \cap B') \subset f^{-1}(B) \cap f^{-1}(B')$

$\forall x \in f^{-1}(B \cap B') \Rightarrow f(x) \in B \cap B' \Rightarrow \begin{cases} f(x) \in B \\ \wedge \\ f(x) \in B' \end{cases}$

$\Rightarrow \begin{cases} x \in f^{-1}(B) \\ \wedge \\ x \in f^{-1}(B') \end{cases} \Rightarrow x \in f^{-1}(B) \cap f^{-1}(B')$

Then: $f^{-1}(B \cap B') \subset f^{-1}(B) \cap f^{-1}(B')$

- The same for inverse inclusion $f^{-1}(B) \cap f^{-1}(B) \subset f^{-1}(B \cap B')$

So, $f^{-1}(B \cap B') = f^{-1}(B) \cap f^{-1}(B')$

Esc 6:

$\forall x, y \in \mathbb{R}: \quad x R y \Leftrightarrow x^4 - y^4 = x^2 - y^2$

1) Show that R is equivalence relation:

C1) Reflexive: R reflexive: $\forall x \in \mathbb{R}: x R x$

$x R x \Leftrightarrow x^4 - x^4 = x^2 - x^2 \Rightarrow 0 = 0 \Rightarrow R$ reflexive.

C2) Symetric: R symetric: $\forall x, y \in \mathbb{R}: x R y \overset{?}{\Rightarrow} y R x$

$x R y \Leftrightarrow x^4 - y^4 = x^2 - y^2 \xrightarrow{\times(-1)} y^4 - x^4 = y^2 - x^2 \Leftrightarrow y R x$

$\Rightarrow R$ Symetric

C3) Transitive: R transitive: $\forall x, y, z \in \mathbb{R}: \begin{cases} x R y \\ \wedge \\ y R z \end{cases} \Rightarrow x R z$

$\begin{cases} x R y \\ \wedge \\ y R z \end{cases} \Leftrightarrow \begin{cases} x^4 - y^4 = x^2 - y^2 \quad \text{①} \\ \wedge \\ y^4 - z^4 = y^2 - z^2 \quad \text{②} \end{cases} \xrightarrow{\text{①}+\text{②}} x^4 - z^4 = x^2 - z^2 \Leftrightarrow x R z$

$\Rightarrow R$ transitive

So, R is equivalence Relation.

2) $C_0 = \{x \in \mathbb{R} : x R 0\}$

$x R 0 \Leftrightarrow x^4 - 0^4 = x^2 - 0^2 \Rightarrow x^4 - x^2 = 0$

$x^2(x^2-1) = 0 \Rightarrow \begin{cases} x^2 = 0 \Rightarrow x = 0 \\ x^2 - 1 = 0 \Rightarrow x = 1 \vee x = -1 \end{cases}$

$C_0 = \{-1, 0, 1\}$

$C_1 = C_0 = \{-1, 0, 1\}$

3) $C_x = \{y \in \mathbb{R} : x R y\}$

$x R y \Leftrightarrow x^4 - y^4 = x^2 - y^2$

$(x^2 - y^2)(x^2 + y^2) - (x^2 - y^2) = 0$

$(x^2 - y^2)[x^2 + y^2 - 1] = 0 \Rightarrow \begin{cases} x^2 - y^2 = 0 \Rightarrow y = x \vee y = - \\ x^2 + y^2 - 1 = 0 \Rightarrow y^2 = 1 - x \end{cases}$

- if $1 - x^2 < 0 \Rightarrow y^2 \neq 1 - x^2$
- if $1 - x^2 \geqslant 0 \Rightarrow y^2 = 1 - x^2 \Rightarrow y = \sqrt{1-x^2} \vee y = -\sqrt{1-x^2}$

$C_x = \{-x, x, \sqrt{1-x^2}, -\sqrt{1-x^2}\}$

Exc 7: $\forall x, y \in \mathbb{N}^* : x R y \Leftrightarrow \exists n \in \mathbb{N} : y^n = x$

1) Show that R is order relation:

C1) R reflexive: $\forall x \in \mathbb{N}^* : x R x$

$x R x \Leftrightarrow \exists n = 1 \in \mathbb{N}$ such that: $x^1 = x \Rightarrow R$ reflexive

C2) R antisymetric: $\forall x, y \in \mathbb{N}^*: \begin{cases} xRy \\ \wedge \\ yRx \end{cases} \Rightarrow x = y$

$\begin{cases} xRy \Leftrightarrow \exists n \in \mathbb{N}: y^n = x \\ yRx \Leftrightarrow \exists m \in \mathbb{N}: x^m = y \end{cases} \Rightarrow (x^m)^n = x \Rightarrow x^{m \times n} = x$

Then: $m \times n = 1 \Rightarrow n = m = 1$

So, $x = y \Rightarrow R$ antisymetric.

C3) R Transitive: $\forall x, y, z \in \mathbb{N}^*: \begin{cases} xRy \\ \wedge \\ yRz \end{cases} \Rightarrow xRz$

$\begin{cases} xRy \Leftrightarrow \exists n \in \mathbb{N}: y^n = x \\ yRz \Leftrightarrow \exists m \in \mathbb{N}: z^m = y \end{cases} \Rightarrow (z^m)^n = x \Rightarrow z^{m \times n} = x$

for: $p = m \times n \in \mathbb{N}$ we have: $z^p = x \Leftrightarrow xRz$

$\Rightarrow R$ transitive.

<u>Consequently</u>: R is order Relation.

2) Total order: $\forall x, y \in \mathbb{N}^*: \begin{cases} xRy \\ \vee \\ yRx \end{cases}$

$\exists\ x = 2,\ y = 3 \in \mathbb{N}^*$ but: $\begin{cases} 2 \not R 3 \\ \wedge \\ 3 \not R 2 \end{cases}$

Then: the order is partiel.

<u>Exo 8</u>: $\forall (x, y); (x', y') \in \mathbb{R}^2: (x, y) R (x', y') \Leftrightarrow |x - x'| \leq y' - y$

1) $(1, 2) R (4, 7) \Leftrightarrow |1 - 4| \leq 7 - 2 \Rightarrow |-3| \leq 5 \Rightarrow 3 \leq 5$ (tr

$(2, 3) R (5, 3) \Leftrightarrow |2 - 5| \leq 3 - 3 \Rightarrow |-3| \leq 0 \Rightarrow 3 \leq 0$ (false

2) R order Relation!

C1) R reflexive; $\forall (x,y) \in \mathbb{R}^2 : (x,y) R (x,y)$

$(x,y) R (x,y) \Leftrightarrow |x-x| \leq y-y \Rightarrow 0 \leq 0$ (True)

C2) R antisymetric; $\forall (x,y) ; (x',y') \in \mathbb{R}^2 : \begin{cases} (x,y) R (x',y') \\ \wedge \\ (x',y') R (x,y) \end{cases} \Rightarrow (x,y) = (x',y')$

$$\begin{cases} (x,y) R (x',y') \Leftrightarrow |x-x'| \leq y'-y \\ (x',y') R (x,y) \Leftrightarrow |x'-x| \leq y-y' \end{cases} \overset{\oplus}{\Rightarrow} 2|x-x'| \leq 0 \Rightarrow |x-x'| \leq 0$$

$$\Rightarrow |x-x'| = 0 \Rightarrow \boxed{x = x'}$$

and $\begin{cases} y'-y \geq 0 \\ \wedge \\ y-y' \geq 0 \end{cases} \Rightarrow \begin{cases} y'-y \geq 0 \\ \wedge \\ y'-y \leq 0 \end{cases} \Rightarrow y'-y = 0 \Rightarrow \boxed{y = y'}$

Then: $(x,y) = (x',y') \Rightarrow$ R antisymetric

C3) R Transitive; $\forall (x,y) ; (x',y') ; (x'',y'') \in \mathbb{R}^2 : \begin{cases} (x,y) R (x',y') \\ \wedge \\ (x',y') R (x'',y'') \end{cases}$

$\Rightarrow (x,y) R (x'',y'')$

$$\begin{cases} (x,y) R (x',y') \Rightarrow |x-x'| \leq y'-y \\ (x',y') R (x'',y'') \Rightarrow |x'-x''| \leq y''-y' \end{cases} \Rightarrow \begin{cases} -y'+y \leq x-x' \leq y'-y \\ \oplus \\ -y''+y' \leq x'-x'' \leq y''-y' \end{cases}$$

$$\Rightarrow y-y'' \leq x-x'' \leq y''-y \Rightarrow |x-x''| \leq y''-y$$

$$\Leftrightarrow (x,y) R (x'',y'')$$

$\Rightarrow$ R Transitive $\Rightarrow$ R is order relation.

3) R is Partiel order because: $\exists (x,y) = (2,3) ; (x',y') = (5,3)$ then:

$(2,3) \not R (5,3)$