Chapitre 4

Basic Principles of Linear Algebra

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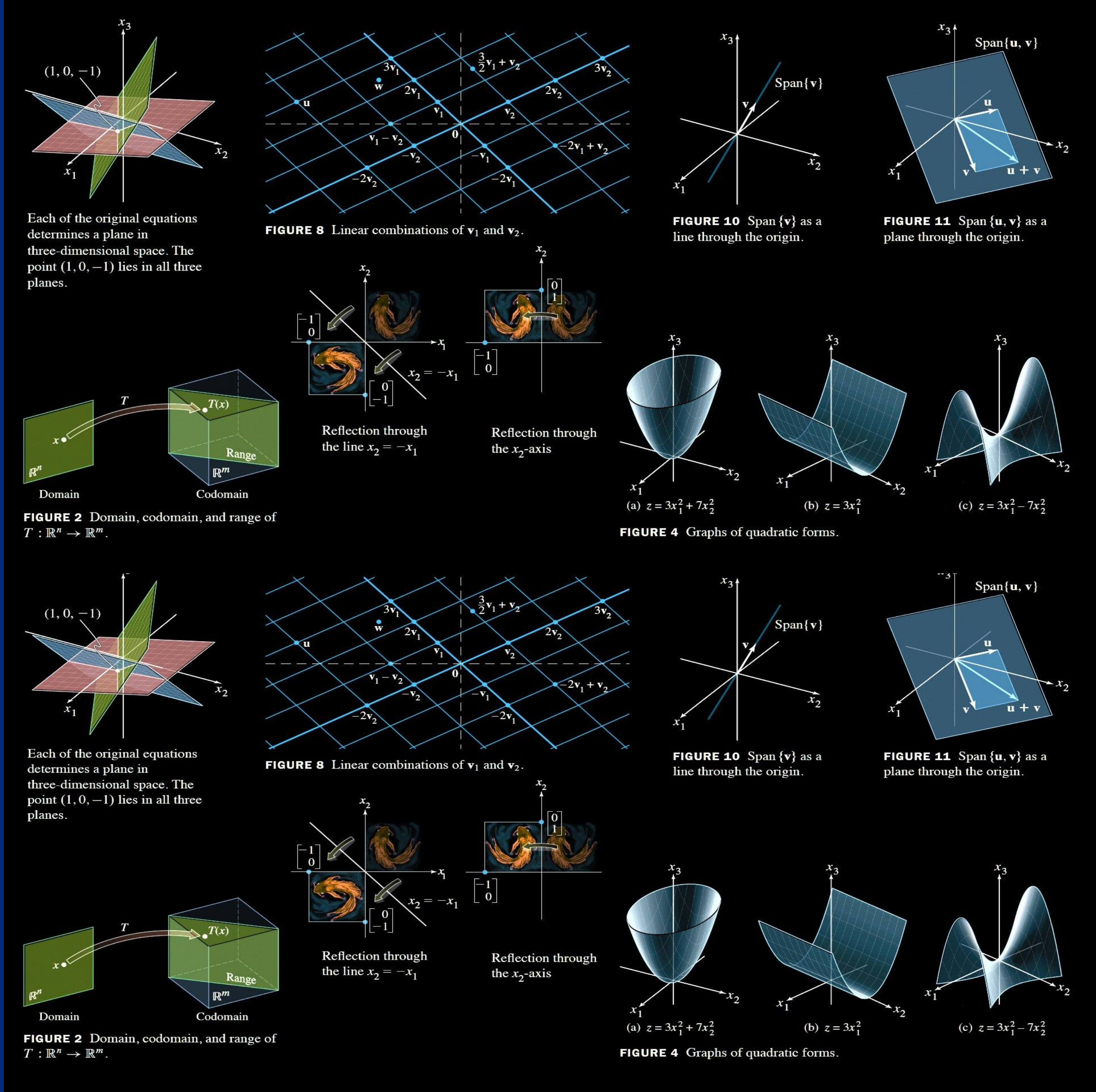
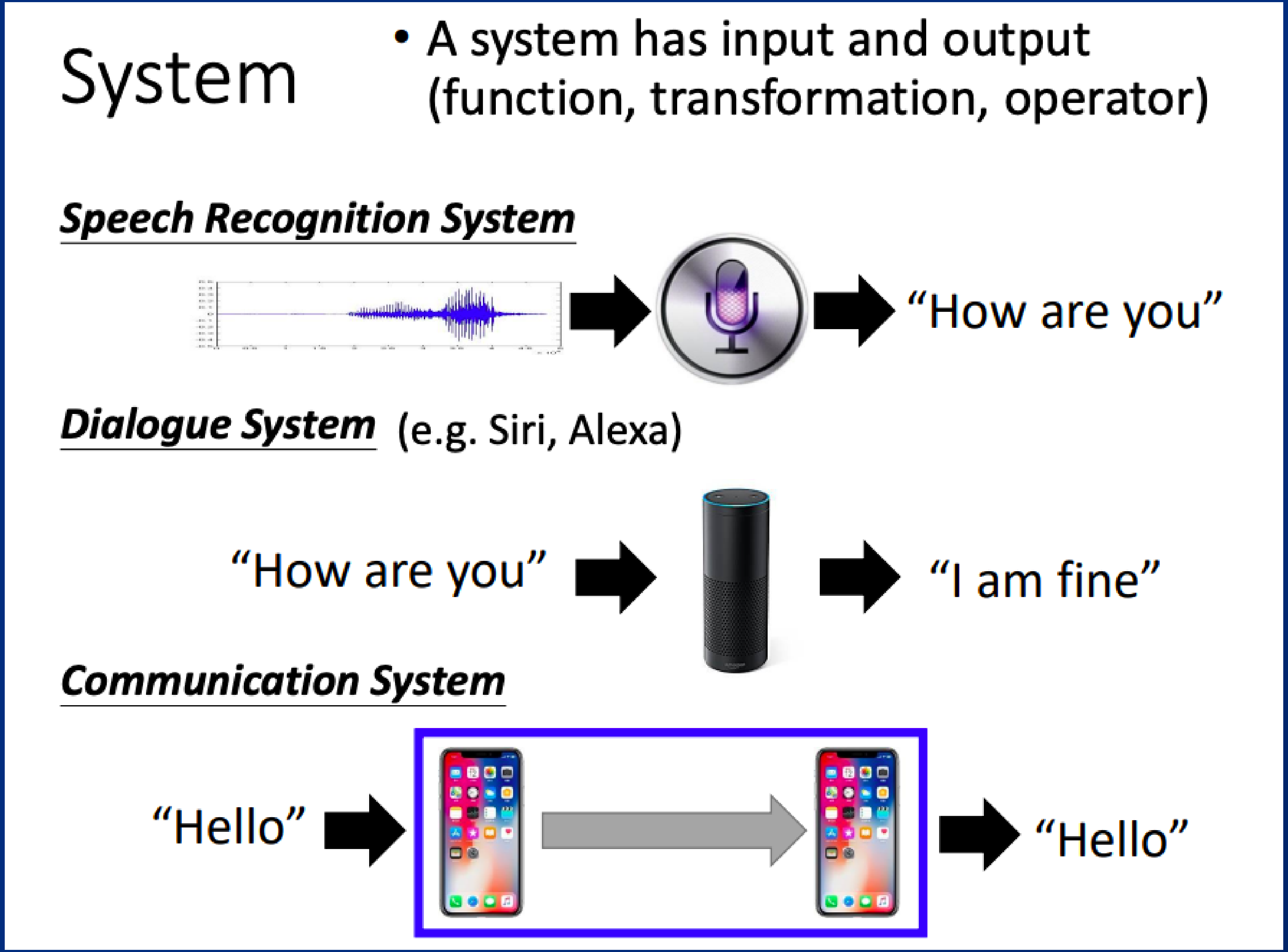
جـامعـة بـاجـي مختـار - عنـابـة



Chapitre 4 : Principe de base de l’algèbre lineaire

- **Vectors and Vector Spaces**
  - Definition and operations on vectors
  - Vector spaces and subspaces
- **Matrices**
  - Definition, types of matrices, and operations (addition, multiplication, inversion)
  - Special matrices (diagonal, orthogonal, identity matrices)
- **Systems of Linear Equations**
  - Solving linear systems (Gauss-Jordan method, LU decomposition)

Use case of linear algebra : Using vectors for speech representation



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Use case 2 of linear algebra

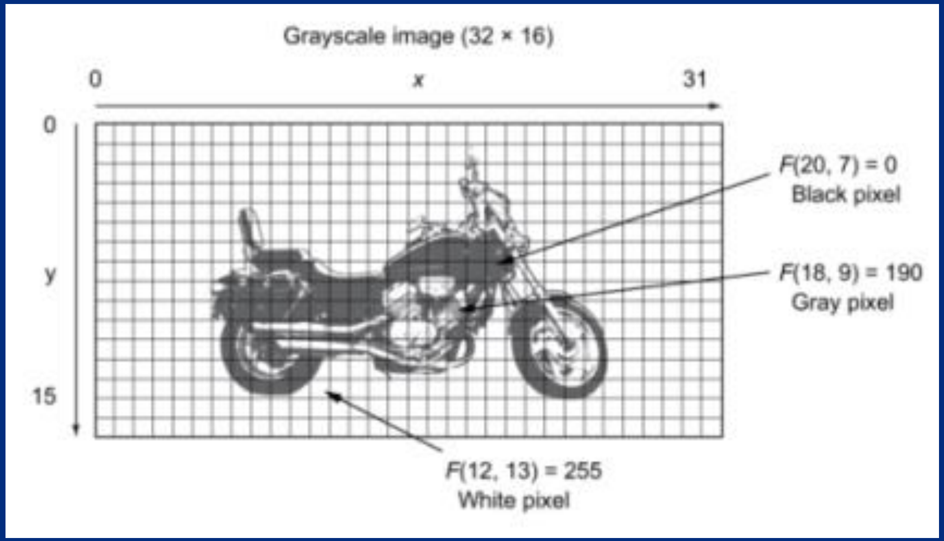


Image grayscale

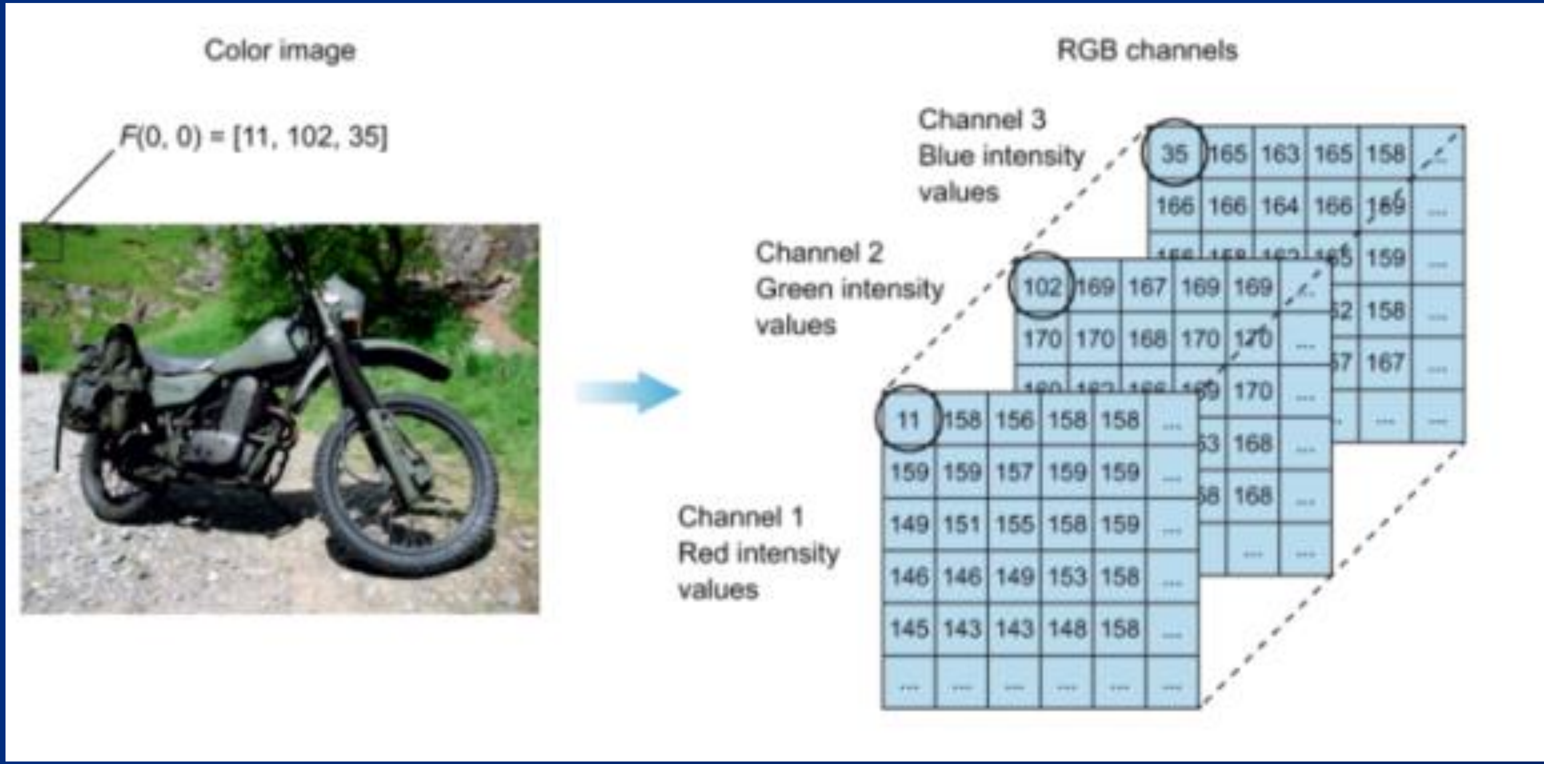


Image en couleur

Images from Deep Learning for Vision Systems book

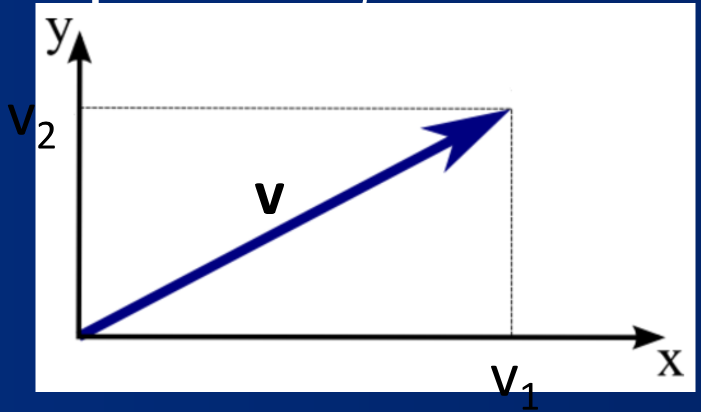

Vectors
THEY ARE EVERYWHERE
meme-arsenal.ru

What is Vector?

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

What Is a Vector?

- A vector is an ordered list of numbers, used to represent quantities that have both magnitude and direction.
- **Components**: The individual elements that make up a vector.
- The **i-th component** of a vector **v** is denoted by $\mathbf{v}_i$.
  - for example : $v_1=1$, $v_2=2$, $v_3=3$
- If a vector has fewer than four components, it can be visualized.

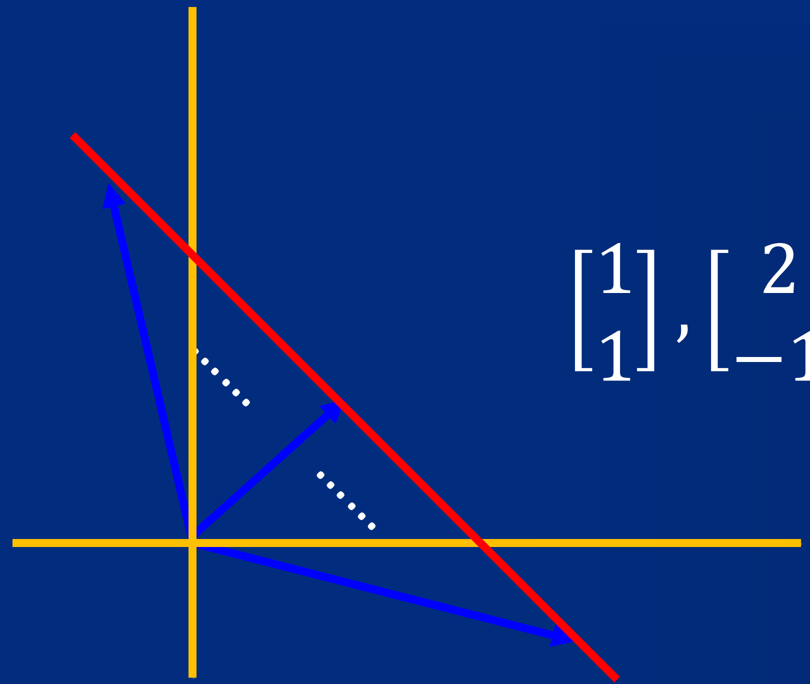


The Set of vectors

$$\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}, \begin{bmatrix}6\\8\\9\end{bmatrix}, \begin{bmatrix}9\\0\\2\end{bmatrix}\right\}$$

Set of vectors
Of 4 elements

- A vector set can contain an infinite number of elements



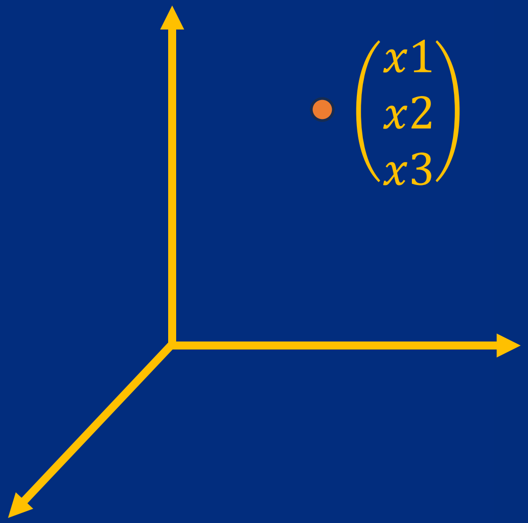
$$\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}2\\-1\end{bmatrix}, \begin{bmatrix}-0.5\\2\end{bmatrix}, \dots\dots$$

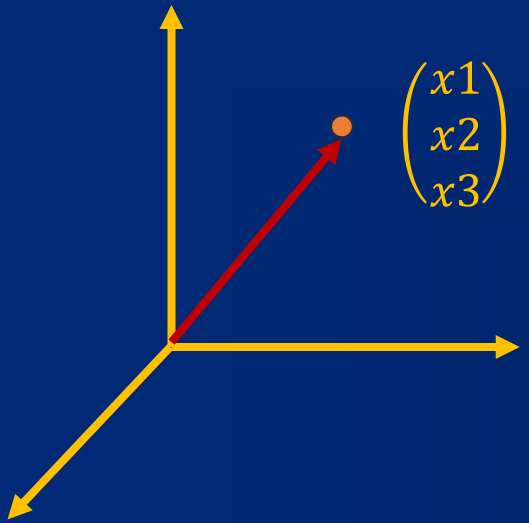
Vector operations

- The set of real numbers $\mathbb{R}$ is often represented as a line. It is a one-dimensional space.
- The plane is formed by pairs of real numbers $\begin{pmatrix} x1 \\ x2 \end{pmatrix}$. It is denoted by $\mathbb{R}^2$, and it is a two-dimensional space.
- The three-dimensional space consists of triplets of real numbers $\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix}$. It is denoted by $\mathbb{R}^3$

Vector operations

- The symbol $\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix}$ has geometric interpretation:
- Either as a point in space (left figure), or as a vector (right figure). :





Vector operations: Definition

- We generalize these notions by considering spaces of dimension *n*, for any positive integer n = 1, 2, 3, 4,..
- The elements of an *n*-dimensional space are the *n*-tuple $\begin{pmatrix} x1 \\ x2 \\ .. \\ .. \\ xn \end{pmatrix}$ of real numbers.
- The *n*-dimensional space is denoted by Rn

  As in the 2- and 3-dimensional cases, the *n*-tuple $\begin{pmatrix} x1 \\ x2 \\ .. \\ .. \\ xn \end{pmatrix}$

  - represents both a point and a vector in the *n*-dimensional space.
- Let u = $\begin{pmatrix} u1 \\ u2 \\ .. \\ .. \\ un \end{pmatrix}$ and v = $\begin{pmatrix} v1 \\ v2 \\ .. \\ .. \\ vn \end{pmatrix}$ be two vectors in R^n

Vector operations: addition, scalar multiplication, opposite vector.

- Sum of two vectors. Their sum is, by definition, the vector *u* + *v* = $\begin{pmatrix} u1+v1 \\ u2+v2 \\ .. \\ .. \\ un+vn \end{pmatrix}$
- Scalar multiplication of a vector. Soit λ ∈ R (called a scalar) : λ · u = $\begin{pmatrix} \lambda u1 \\ \lambda u2 \\ .. \\ .. \\ \lambda un \end{pmatrix}$.
- The null vector in R^n is the vector 0 = $\begin{pmatrix} 0 \\ 0 \\ .. \\ .. \\ 0 \end{pmatrix}$
- The opposite of the vector u = $\begin{pmatrix} u1 \\ u2 \\ .. \\ .. \\ un \end{pmatrix}$ *is the vector* $\begin{pmatrix} -u1 \\ -u2 \\ .. \\ .. \\ -un \end{pmatrix}$

Vector properties

The objects have the following 8 properties are “vectors”.

Pour tout vecteur **u**, **v** et **w** dans $\mathcal{R}^n$, and any scalars a and b

1. **u + v = v + u. Commutativity**
2. (**u** + **v**) + **w** = **u** + (**v** + **w**) .Associativity of addition
3. Existence of a Zero Vector, there exists a vector 0 ∈ $\mathcal{R}^n$ such that : **0 + u = u**
4. Existence of an Additive Inverse For every vector **u’** ∈ $\mathcal{R}^n$ such that **u’ + u = 0**

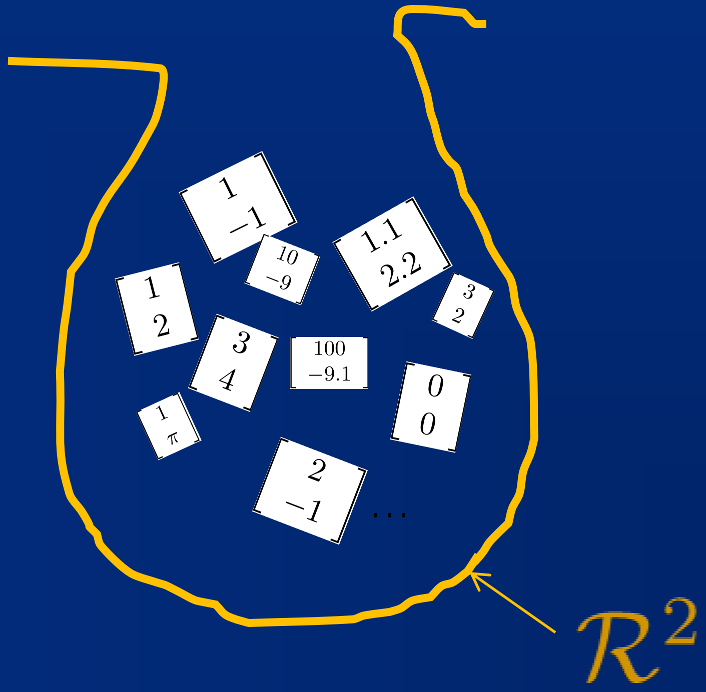
   = - **u**

5. 1**u** = **u . Multiplication by the Scalar 1**
6. (ab)**u** = a(b**u**) . associativity
7. a(**u**+**v**) = a**u** + a**v** **Distributivity of Scalar Multiplication over Vector Addition**
8. (a+b)**u** = a**u** + b**u**. **Distributivity of Scalar Addition**

Vecteur zero $\mathbf{0} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$

Vector space

- The set of all vectors with n entries is denoted by $\mathcal{R}^n$



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Vector space and sub-vector

- A vector space is a set of vectors where two operations are defined: vector addition and scalar multiplication (by a real or complex number), satisfying the eight axioms of linear algebra (such as associativity, commutativity, the existence of a zero vector, etc.). Essentially, it is a set of vectors where these basic operations are possible.

- Examples:

- $\mathcal{R}^n$: The set of n-tuples of real numbers.

Vector subspace

- A vector subspace is a subset of a vector space that is itself a vector space.
- To be a subspace, it must be closed under vector addition and scalar multiplication, and it must contain the zero vector.
- Examples:
  Subset of $\mathcal{R}^3$ All vectors of the form (x, 0, 0), which form a line passing through the origin.

Example of a vector subspace:

Let's consider the vector space $\mathbb{R}^3$ (the set of all 3-dimensional vectors).

An example of a vector subspace is the set of vectors $S \subseteq R^3$ that satisfy the equation:

$$z = 0, \quad \text{where } v = (x, y, z)$$

Let's verify that S is a subspace:

1. **Addition**: If two vectors $u = (x_1, y_1, 0)$ and $v = (x_2, y_2, 0)$ belong to S, their sum is:

   $$u + v = (x_1 + x_2, y_1 + y_2, 0)$$

   This also belongs to S because the z-component remains 0.

2. **Scalar multiplication**: If $u = (x, y, 0)$ is in S and a is a scalar, then:

   $$au = (a \cdot x, a \cdot y, a \cdot 0) = (ax, ay, 0)$$

   This also belongs to S.

3. **Zero vector**: The zero vector $0 = (0, 0, 0)$ clearly belongs to S.

Therefore, S is a vector subspace of $\mathbb{R}^3$.

Practice about vectors

- Installation of the necessary library for practicing linear algebra and importing it.

```
# Installation de la bibliothèque numpy
!pip install numpy
```

```
[1]:
import numpy as np
print(np.__version__)
```

```
1.26.4
```

Creating 2 vectors and performing the addition operation

Creating 2 vectors

[2]:

```
A = np.array([1, 2, 3])
B = np.array([5, 6, 7])
```

Addition de deux vecteurs

[3]:

```
C = A+B
print(f"Addition : {C}")
D= A-B
print(f"Soustraction {D}")
```

Dot product (scalar product) between two vectors

Dot product (scalar product)

```
[12]:
np.dot(A, B)
[12]:
70
[21]:
np.dot(3, A)
[21]:
array([3, 6, 9])
```

Cross product (vector product)

Vector product

$\vec{C}$ = A*B= ($a_2 b_3 - a_3 b_2 , a_3 b_1 - a_1 b_3 , a_1 b_2 - a_2 b_1$)

[20]:

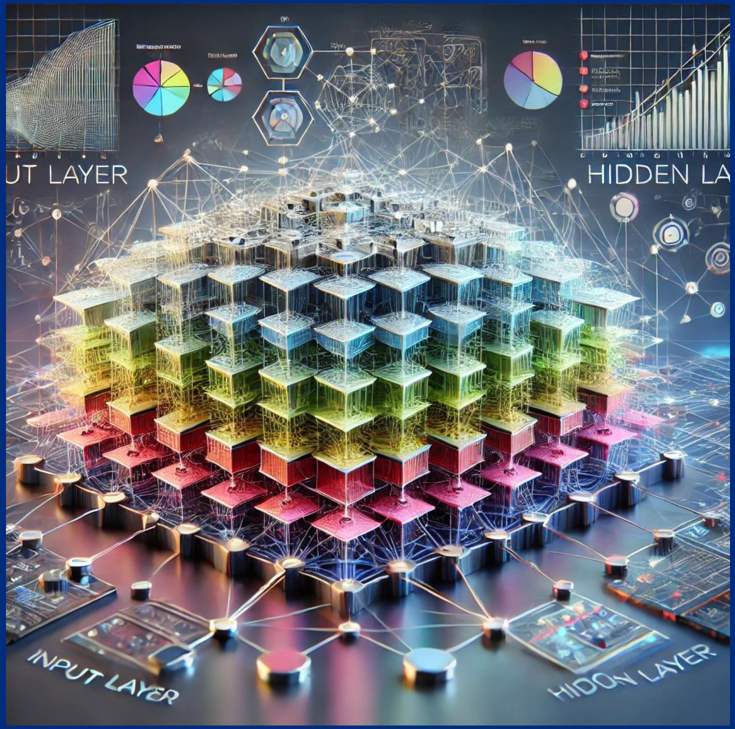
```python
C = np.cross(A,B)
print("Vecteur A :", A)
print("Vecteur B :", B)
print("Produit vectoriel A x B :", C)
```

```
Vecteur A : [1 2 3]
Vecteur B : [5 6 7]
Produit vectoriel A x B : [-4  8 -4]
```

The matrics


THE MATRIX IS EVERYWHERE.
YOU CAN SEE IT WHEN
YOU LOOK OUT YOUR WINDOW
OR WHEN YOU TURN ON
YOUR TELEVISION.

Why are matrices important?



Handling large amounts of data in data science, and feeding data into neural networks

Image processing (Computer Vision, CV)

From the book Deep Learning for Vision Systems

Matrices – Definition and Types

Definition of matrices:

- A rectangular array of numbers organized in rows and columns.
- Example: A grayscale image is a matrix of pixels.

Example of matrix

Example of a 3×3 matrix (3 rows and 3 columns):

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 1 & -1 \\ -2 & 1 & 1 \end{bmatrix}$$

Matrix

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 1 & -1 \\ -2 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} 3 \text{ X } 1 \quad \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \; 1 \text{ X } 3$$

- If a matrix has m rows and n columns, we say that the size of the matrix is m by n, denoted $m \times n$.
- We use M_{mxn} to denote the set of all matrices of size $m \times n$.

3 colonnes

$$2 \text{ lignes} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \in M_{2x3}$$

2 X 3

2 colonnes

$$3 \text{ lignes} \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \in M_{3x2}$$

3 X 2

Matrices

- Component index: The scalar located in the (i)-th row and j-th column is called the (i, j) element of the matrix.

(1,2)-entry

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 1 & -1 \\ -2 & 1 & 1 \end{bmatrix}$$

(3,1)-entry

(3,3)-entry

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$A = [\boldsymbol{a}_1 \quad \boldsymbol{a}_2 \quad \dots \quad \boldsymbol{a}_n]$$

Vectors

Matrix: Properties

- Two matrices of the same size can be added or subtracted.
- A matrix can be multiplied by a scalar.

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 6 & 9 \\ 8 & 0 \\ 9 & 2 \end{bmatrix} \quad 9B = \begin{bmatrix} 54 & 81 \\ 72 & 0 \\ 81 & 18 \end{bmatrix}$$

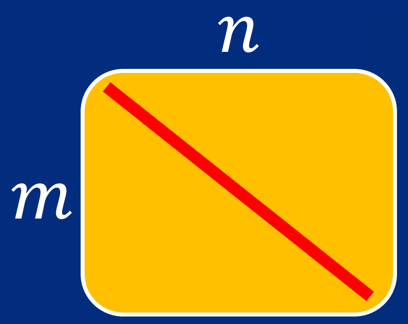
$$A + B = \begin{bmatrix} 7 & 13 \\ 10 & 5 \\ 12 & 8 \end{bmatrix} \quad A - B = \begin{bmatrix} -5 & -5 \\ -6 & 5 \\ -6 & 4 \end{bmatrix}$$

Matrices properties

- A, B, C are matrices of dimensions mxn, and s and t are scalars
  - $A + B = B + A$
  - $(A + B) + C = A + (B + C)$
  - $(st)A = s(tA)$
  - $s(A + B) = sA + sB$
  - $(s+t)A = sA + tA$

Types of matrices: square and triangular matrices

- A matrix is called square if the number of rows is equal to the number of columns (n x n).



$m = n$

diagonal

$$\begin{bmatrix} 2 & 3 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Upper triangular matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$$

Lower triangular matrix

Types of matrices

Est ce que la matrice I_3 est une matrice diagonal? **YES**

La matrice $O_{3\times3}$ est elle diagonal? **YES**

- Diagonal matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

All off-diagonal elements are equal to 0.

- Identity matrix

$$\mathrm{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

It is a special square matrix that plays a role similar to the number 1.

- Zero matrix

$$O_{2\times3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Denoted by O ou $O_{m\times n}$

Transpose

- If A is an mxn, A^T (the transpose of A) is an nxm matrix whose (i,j) entry is the (j, i) entry of A.

(1,2)

$$A = \begin{bmatrix} 6 & 9 \\ 8 & 0 \\ 9 & 2 \end{bmatrix}$$

(3,2)

Transpose →

$$A^T = \begin{bmatrix} 6 & 8 & 9 \\ 9 & 0 & 2 \end{bmatrix}$$

(2,1) (2,3)

Transpose: proprieties

- A and B are two matrices of m x n dimensions, and s is a scalar.
- $(A^T)^T = A$
- $(sA)^T = sA^T$
- $(A+B)^T = A^T + B^T$

} This is a linear system ☺

Symmetric Matrix $A^T = A$

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & -1 \\ 4 & -1 & 5 \end{bmatrix} = A^T \qquad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \neq B^T$$

Matrice Orthogonale

- A square matrix A is said to be orthogonal if it satisfies the following property: A^T A= I, where A^T is the transpose of A and I is the identity matrix,
- This means that the rows (or columns) of A form a set of orthonormal vectors.

Considérons la matrice suivante :

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Calculons la Transposée de A

$$A^T = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$A^T A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Effectuons les calculs :

$$A^T A = \begin{pmatrix} (\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}) & (\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot (-\frac{1}{\sqrt{2}})) \\ (\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + (-\frac{1}{\sqrt{2}}) \cdot \frac{1}{\sqrt{2}}) & (\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + (-\frac{1}{\sqrt{2}}) \cdot (-\frac{1}{\sqrt{2}})) \end{pmatrix}$$

Cela simplifie à :

$$A^T A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Matrix Operations

- Matrix operations are fundamental in linear algebra and serve as the foundation for many fields, such as:
  - solving systems of linear equations,
  - numerical computation, and
  - machine learning.

1. Addition of matrices

- Two matrices of the same dimension can be added by summing their corresponding elements.
- Exemple :

If A = $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 4 \\ 5 & 7 \\ 1 & 9 \end{bmatrix}$, then :

A+B = $\begin{bmatrix} 1+2 & 4+4 \\ 2+5 & 5+7 \\ 3+1 & 6+9 \end{bmatrix}$ = $\begin{bmatrix} 3 & 8 \\ 7 & 12 \\ 4 & 15 \end{bmatrix}$

2. Subtraction of Matrices

- As with addition, two matrices of the same dimension can be subtracted by subtracting their corresponding elements.
- Exemple :

If A = $\begin{bmatrix} 3 & 5 \\ 5 & 9 \\ 3 & 11 \end{bmatrix}$ et B = $\begin{bmatrix} 2 & 4 \\ 5 & 7 \\ 1 & 9 \end{bmatrix}$, Then :

$$A-B = \begin{bmatrix} 3-2 & 5-4 \\ 5-5 & 9-7 \\ 3-1 & 11-9 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 2 & 2 \end{bmatrix}$$

3. Multiplication par un scalaire

- A matrix can be multiplied by a scalar by multiplying all of its elements by that number.
- Exemple :

$$A = \begin{bmatrix} 3 & 5 \\ 5 & 9 \\ 3 & 11 \end{bmatrix}$$

And rhe scalar k =2, then :

$$k.A = 2 * A = \begin{bmatrix} 3 & 5 \\ 5 & 9 \\ 3 & 11 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ 10 & 18 \\ 6 & 22 \end{bmatrix}$$

4. Multiplying two matrices

- Two matrices A and B can be multiplied if the number of columns in A is equal to the number of rows in B.
- A = $\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 2 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & 5 & 6 \\ 2 & 4 & 7 \end{bmatrix}$
- A*B = $\begin{bmatrix} 1*3+1*2 & 1*5+1*4 & 1*6+1*7 \\ 0*3+2*2 & 0*5+2*4 & 0*6+2*7 \\ 2*3+2*2 & 2*5+2*4 & 2*6+2*7 \end{bmatrix} = \begin{bmatrix} 5 & 9 & 13 \\ 4 & 8 & 14 \\ 10 & 18 & 26 \end{bmatrix}$

5. Transposition

- The transposition of a matrix A consists of swapping its rows and columns.
  In other words, each row of A becomes a column in A^T, and each column becomes a row.

- A = $\begin{bmatrix} 3 & 5 & 2 \\ 2 & 3 & 7 \end{bmatrix}$

$$A^T = \begin{bmatrix} 3 & 2 \\ 5 & 3 \\ 2 & 5 \end{bmatrix}$$

6. Determinant of matrix

6. Calcul du déterminant (pour les matrices carrées)

Le déterminant est un **scalaire** associé à une matrice carrée. Il est utile pour vérifier si une matrice est inversible.

Exemple : Pour $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, le déterminant est donné par :

$$\det(A) = (1 \cdot 4) - (2 \cdot 3) = 4 - 6 = -2$$

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Inverse Matrix

7. Inversion de matrice (matrice inversible uniquement)

La matrice inverse A^{-1} d'une matrice A carrée satisfait $A \cdot A^{-1} = I$, où I est la matrice identité. Elle existe uniquement si le déterminant de A est différent de 0.

Exemple : Pour $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, l'inverse est donnée par :

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

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Some Practical Examples of Matrix Manipulation Using the NumPy Library

- Numpy library

```
[2]: # Installation de la bibliothèque numpy
     !pip install numpy

     Requirement already satisfied: numpy in /opt/miniconda3/lib/python3.12/site-packages (1.26.4)

[20]: import numpy as np
      print(np.__version__)

      1.26.4
```

Addition and soustraction of matrices

```
[4]: import numpy as np

     # Définir les matrices A et B
     A = np.array([[1, 2, 3], [4, 5, 6], [7, 8, 9]])
     B = np.array([[9, 8, 7], [6, 5, 4], [3, 2, 1]])

     # Addition de matrices
     C = A + B
     print("Addition de A et B:\n", C)

     # Soustraction de matrices
     D = A - B
     print("Soustraction de A et B:\n", D)
```

```
Addition de A et B:
 [[10 10 10]
 [10 10 10]
 [10 10 10]]
Soustraction de A et B:
 [[-8 -6 -4]
 [-2  0  2]
 [ 4  6  8]]
```

Multiplication of matrices par scalar and par matrice

Multiplication par a scalar

```
[5]: # Définir la matrice A
     A = np.array([[1, 2, 3], [4, 5, 6], [7, 8, 9]])

     # Scalaire
     k = 2

     # Multiplication par un scalaire
     B = k * A
     print("Multiplication de A par le scalaire 2:\n", B)

     Multiplication de A par le scalaire 2:
      [[ 2  4  6]
      [ 8 10 12]
      [14 16 18]]
```

Multiplication of 2 matrices

```
[6]: # Définir les matrices A et B
     A = np.array([[1, 2], [3, 4], [5, 6]])
     B = np.array([[7, 8, 9], [10, 11, 12]])

     # Multiplication de matrices
     C = np.dot(A, B)
     print("Multiplication de A et B:\n", C)
```

Matrices Transposition and determinant

4. Transposition de Matrices

[7]:
```
# Définir la matrice A
A = np.array([[1, 2, 3], [4, 5, 6]])
print("La matrice A:\n", A)
# Transposition
A_T = np.transpose(A)
print("Transposée de A:\n", A_T)
```

```
La matrice A:
 [[1 2 3]
 [4 5 6]]
Transposée de A:
 [[1 4]
 [2 5]
 [3 6]]
```

5. Déterminant d'une Matrice

21]:
```
# Définir la matrice A
A = np.array([[1, 2], [3, 4]])
# Calcul du déterminant
det_A = np.linalg.det(A)
print("Déterminant de A:", det_A)
```

```
Déterminant de A: -2.0000000000000004
```

Matrix Inversion and Decomposition

Code | JupyterLab | Python 3 (ipykernel)

6. Inversion d'une Matrice

[19]:
```
# Définir la matrice A
A = np.array([[1, 2], [3, 4]])
print("La mtarice A:\n", A)

# Calcul de l'inverse
A_inv = np.linalg.inv(A)
print("Inverse de A:\n", A_inv)
```

```
La mtarice A:
 [[1 2]
 [3 4]]
Inverse de A:
 [[-2.   1. ]
 [ 1.5 -0.5]]
```

7. Décomposition LU

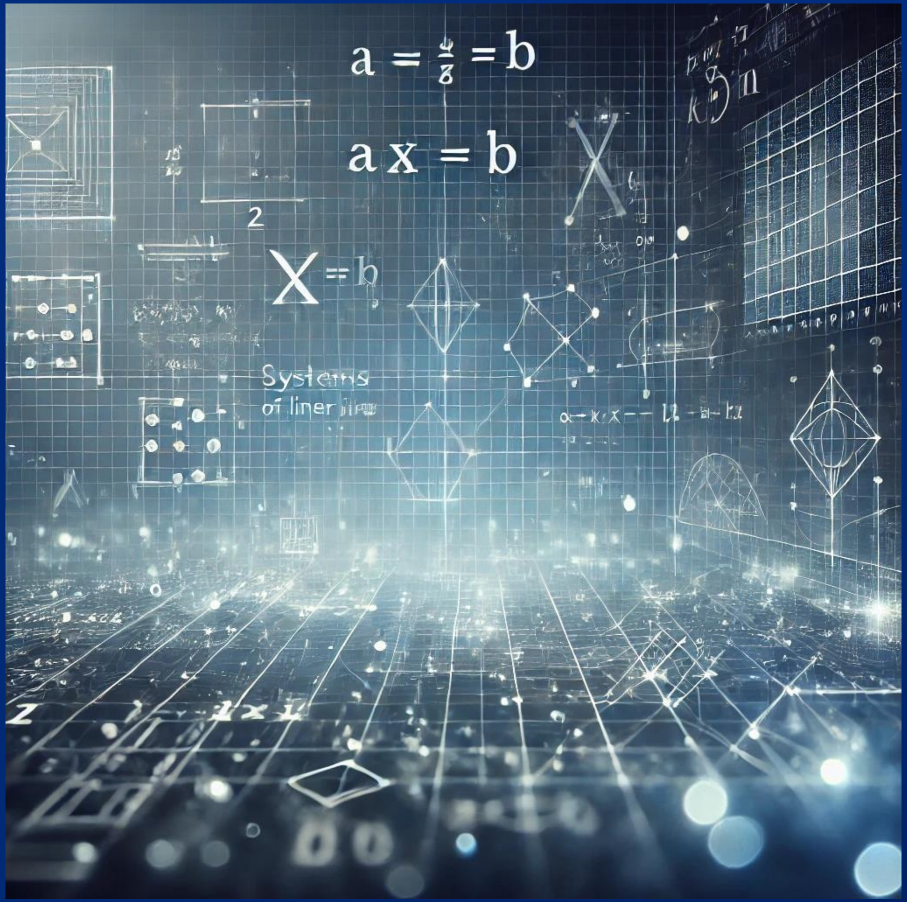
[15]:
```
from scipy.linalg import lu

# Définir la matrice A
A = np.array([[4, 3], [6, 3]])
print("Matrice A : ", A)

# Décomposition LU
P, L, U = lu(A)
print("Matrice L:\n", L)
print("Matrice U:\n", U)
```

Systèmes d’équations linéaires

Systèmes d’équations linéaires :

Résolution des systèmes linéaires

- Méthode de Gauss-Jordan
- Décomposition LU



Linear Systems of Equations

- Linear systems of equations and their resolution are fundamental components of linear algebra. They provide the basis for understanding and solving real-world problems by using the concepts of vectors and matrices.

- Linear systems of equations are used in many fields—such as physics, engineering, economics, and computer science—to model and solve complex problems.

Introduction to Systems of Linear Equations

Definition

A system of linear equations is a set of equations where each equation is a linear combination of the variables. It is written in the form :

$$Ax=b$$

- A: matrix of coefficients (taille n × m)
- x: Vector of unknown variables
- b: Vector of constants

Linear Systems of Equations

- Let the following simple linear equation be given to solve:

$$2x + 3y = 5 \quad \text{and} \quad x - 4y = -2.$$

- A system of equations can be represented in matrix form as follows:

$$Ax = b,$$

- where A is the coefficient matrix, x is the vector of unknowns, and B is the result vector.

$$A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix} \qquad X = \begin{pmatrix} x \\ y \end{pmatrix} \qquad B = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

Example of a system of linear equations

$$\begin{cases} x+2y+3z=6 \\ 4x+5y+6z=15 \\ 7x+8y+9z=24 \end{cases}$$

$Ax = b$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$b= \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \begin{pmatrix} 6 \\ 15y \\ 24 \end{pmatrix}$$

- **Linear systems use matrices to organize coefficients.**
- **Vectors represent the unknowns and the constants of the problem.**