## I-4 PRINCIPLE OF SUPERPOSITION

Consider three-point charges q1, q2 and q fixed respectively at P1, P2 and M (Figure I-2).

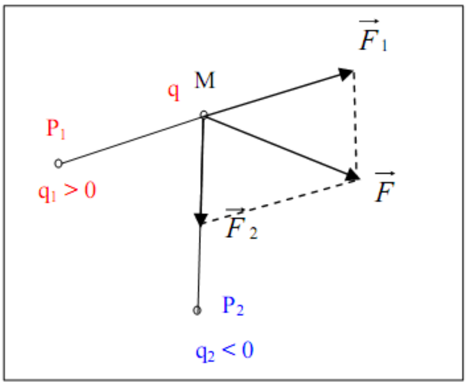


**Figure 2**

What is the force F exerted on charge q when it is in the presence of charges q1 and q2? Coulomb's law allows us to calculate the force F1 exerted on charge q when it is only in the presence of q1. Similarly, we can calculate F2, the force exerted on q when only q2 is in the presence of charge q.

Experience shows that the force F experienced by q when it is in the presence of the two charges q1 and q2 is the vector sum of the forces $\vec{F}_1$And $\vec{F}_2$:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = \frac{q\,q_2}{4\Pi\varepsilon_0}\frac{\overrightarrow{P_1M}}{\left\|\overrightarrow{P_1M}\right\|^3} + \frac{q\,q_2}{4\Pi\varepsilon_0}\frac{\overrightarrow{P_2M}}{\left\|\overrightarrow{P_2M}\right\|^3} \qquad (3)$$

This result holds true regardless of the number of charges present. The force resultant subjected to a load q placed at M, in the presence of n charges q1, q2, ..., qi, ..., qn fixed at P1, P2, ..., Pi, ..., Pn is the vector sum of the forces due to the interaction of the charges with q, calculated separately:

$$\vec{F} = \sum_{i=1}^{n}\vec{F}_i = q\sum_{i=1}^{n}\frac{q_i}{4\Pi\varepsilon_0}\frac{\overrightarrow{P_iM}}{\left\|\overrightarrow{P_iM}\right\|^3} \qquad (4)$$

This expression conveys the principle of superposition. The total force F due to a set of charges is the vector sum of the effect of each charge taken individually. This implies that the force exerted between two charges is not modified by the presence of a third charge.

The solution is simply the sum of the solutions calculated for each pair of charges. It follows that the equations of electrostatics are linear equations. The superposition principle applies to electromagnetic phenomena: Maxwell's equations, the basic equations of electromagnetism, are linear equations.

## I-5 The Electrostatic Field

Consider the force F defined by (I-4). Dividing expression (I-4) by the charge q, we obtain a vector quantity that depends on the structure of the n charges and the position of point M: this quantity is called the electrostatic field, E(M), created at point M by the system of charges q1, q2, ..., qi, ..., qn fixed at P1, P2, ..., Pi, ..., Pn.

$$\vec{E}(M) = \frac{\vec{F}}{q} = \sum_{i=1}^{n} \frac{q_i}{4\Pi\varepsilon_0} \frac{\overrightarrow{P_iM}}{\left\|\overrightarrow{P_iM}\right\|^3} = \sum_{i=1}^{n} \frac{q_i}{4\Pi\varepsilon_0} \frac{\vec{u}_i}{r_i^2} \qquad (5)$$

$$\vec{u}_i = \frac{\overrightarrow{P_iM}}{\left\|\overrightarrow{P_iM}\right\|} \text{ et } r_i = \left\|\overrightarrow{P_iM}\right\|$$

The electrostatic field $\vec{E}(M)$ which results from $\vec{F}$ is the vector sum of the fields $\vec{E}_i(M)$ created by the charges $q_i$:

$$\vec{E}(M) = \sum_{i=1}^{n} \vec{E}_i(M) \qquad (6)$$

Or $\vec{E}_i(M)$ is the field created at M by the charge $q_i$ point placed at $P_i$(Figure I-3)

$$\vec{E}_i(M) = \frac{q_i}{4\Pi\varepsilon_0} \frac{\overrightarrow{P_iM}}{\left\|\overrightarrow{P_iM}\right\|^3} = \frac{q_i}{4\Pi\varepsilon_0} \frac{\vec{u}_i}{r_i^2} \qquad (7)$$

We have just defined a vector quantity, a function of the point M, characteristic of the system of charges q1, q2, ..., qi, ..., qn, sources of the field E. At each point in space we associate a vector E, a function of the point considered (Figure I-3).

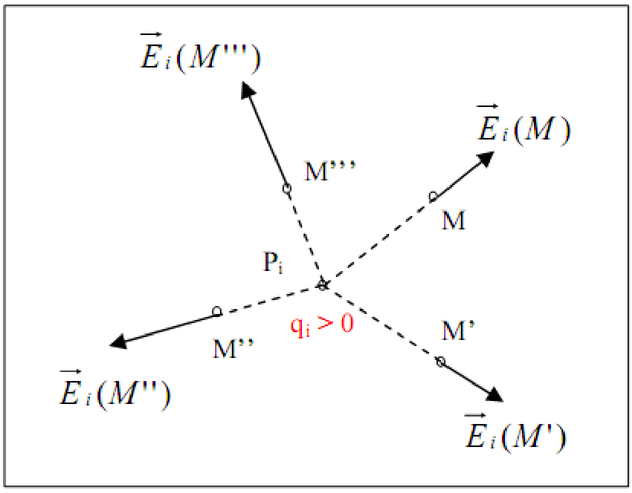


**Figure 3**

The set of vectors $\vec{E}$ constitutes a vector field. The field $\vec{E}$ once determined, the force $\vec{F}$ The effect of a charge q placed at a point M is given by the relation:

$$\boxed{\vec{F} = q\,\vec{E}(M) \qquad (8)}$$

The introduction of the field E leads to a new description of the electrostatic interaction. We have replaced the action at a distance contained in Coulomb's law with the notion of an electrostatic field, a local quantity.

Instead of considering the charges qi and q present interacting via the Coulomb force:

Charge $q_i$ en $P_i$ $\Rightarrow$ Charge q en M soumise à $\vec{F} = q\sum_{i=1}^{n} \frac{q_i}{4\Pi\varepsilon_0} \frac{\overrightarrow{P_iM}}{\left\|\overrightarrow{P_iM}\right\|^3}$

We express the field $\vec{E}_i$ created by the charge $q_i$ throughout the space surrounding this charge. This field exists independently of whether or not another charge q exists in the presence of the charge qi, source of the field $\vec{E}_i$ The force F experienced by q placed at M results from the existence of an electrostatic field at that point:

Charge $q_i$ en $P_i$ : source du champ electrostatique $\Rightarrow$ $\vec{E}(M) = \sum_{i=1}^{n} \frac{q_i}{4\Pi\varepsilon_0} \frac{\overrightarrow{P_iM}}{\left\|\overrightarrow{P_iM}\right\|^3}$ $\Rightarrow$ Agit sur la charge q : $\vec{F} = q\,\vec{E}$

## I-6 Conclusion

The electrostatic field created at a point M by a point charge q placed at O is:

$$\vec{E}(M) = \frac{q}{4\Pi\varepsilon_0}\frac{\overrightarrow{OM}}{\left\|\overrightarrow{OM}\right\|^3} = \frac{q}{4\Pi\varepsilon_0}\frac{\vec{u}_r}{r^2}$$

$$\text{où : } \vec{u}_r = \frac{\overrightarrow{OM}}{\left\|\overrightarrow{OM}\right\|} \text{ et } r = \left\|\overrightarrow{OM}\right\|$$

The field $\vec{E}(M)$ It has two characteristics:

- The first lies in the fact that $\vec{E}(M)$ is of the form $f(r)\vec{u}_r$, a property that we will exploit in the calculation of the circulation of E and which will lead to the definition of the electrostatic potential.

- The second characteristic is the form of f(r), in $1/r^2$, a property that we will exploit in calculating the flux of the field and which will lead to Gauss's theorem. The results that we will obtain will be valid for any field of the form $f(r)\vec{u}_r = \frac{\vec{u}_r}{r^2}$, in particular the gravitational field.