Chapter III: Electrostatic Fields and Potential

1 – Introduction

The electrostatic potential V(M) associated with the electrostatic field $\vec{E}(M)$is a scalar function unlike $\vec{E}$ We will see, in many cases, that the potential will be a convenient intermediary in the calculation of the vector field l $\vec{E}(M)$. The potential is physically linked to the concept of potential energy, hence its name.

2 - Circulation of the electrostatic field

2-1 Electrostatic potential

a/ Case of a point load

Consider a point charge q (>0) fixed at P and a point M in space (figure 1):

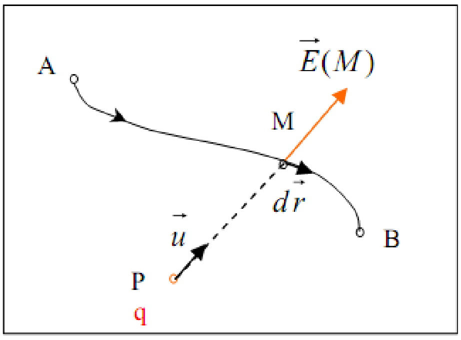

Figure 1

The point charge q fixed at P creates at every point M in space an electrostatic field given by:

$$\vec{E}(M) = \frac{q}{4\Pi\varepsilon_0}\frac{\overrightarrow{PM}}{\left\|\overrightarrow{PM}\right\|^3} = \frac{q}{4\Pi\varepsilon_0}\frac{\vec{u}}{r^2}$$

avec $\vec{u} = \frac{\overrightarrow{PM}}{\left\|\overrightarrow{PM}\right\|}$: Unit vector directed from P to M.

The elementary circulation DC of the field E corresponding to an elementary displacement $d\vec{r}$ Point M on the curve AB is:

$$dC = \vec{E}.\vec{dr} = \frac{q}{4\Pi\varepsilon_0}\frac{1}{r^2}\vec{u}.\vec{dr} \qquad (1)$$

Or, $\vec{dr} = d(\overrightarrow{PM}) = d(r\vec{u}) = dr\,\vec{u} + r\,d\vec{u}$ et $\vec{dr}.\vec{u} = (dr\,\vec{u} + r\,d\vec{u}).\vec{u} = dr + rd\vec{u}.\vec{u}$

Puisque : $d(\vec{u}.\vec{u}) = 2\vec{u}.d\vec{u} = 0$; on a : $\vec{dr}.\vec{u} = dr$

The elementary circulation DC can then be written as:

$$dC = \frac{q}{4\Pi\varepsilon_0}\frac{dr}{r^2} = -d\left(\frac{q}{4\Pi\varepsilon_0}\frac{1}{r}\right) \qquad (2)$$

Let us then state;

$$dC = \vec{E}.\vec{dr} = -dV(r)$$

V is the electrostatic potential V(M) created by the charge q fixed at M:

$$V(M) = V(r) = \frac{q}{4\Pi\varepsilon_0}\frac{1}{r} + cste \qquad (3)$$

We have just defined a new field, the electrostatic potential; it is a scalar field defined up to a constant. The value of this constant is generally chosen such that the potential is zero when point M is infinitely far from the charge.

$V(r \rightarrow \infty) = 0$. In this case, the constant is zero and the potential can be written as:

$$V(M) = V(r) = \frac{q}{4\Pi\varepsilon_0}\frac{1}{r} \qquad (4)$$

Like the field $\vec{E}$ The potential V is not defined at points Pi: $\vec{E}(P_i)$ et $V(P_i)$ are not defined.

b/ Case of a distribution of n point charges

Let n point charges q1, q2, ..., qi, ..., qn be fixed at points P1, P2, ..., Pi, ..., Pn. Let M be a point in space. (figure 2).

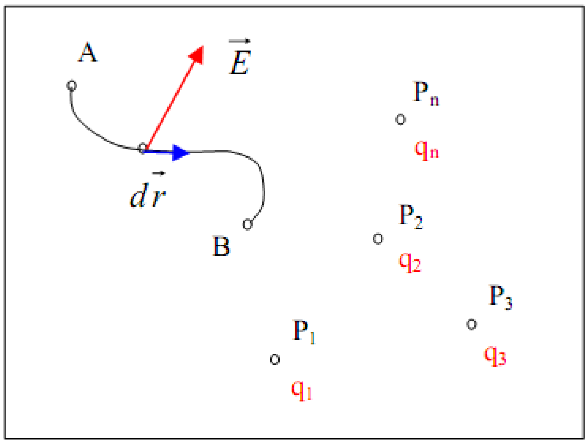

Figure 2

Let's calculate the elementary circulation dci of the field $\vec{E}_i$ created by the charge qi alone:

$$dC_i = \vec{E}_i . d\vec{r} = -dV_i(r)$$

$$\text{Avec } \vec{E}_i(M) = \frac{q_i}{4\Pi\varepsilon_0} \frac{\overrightarrow{P_iM}}{\left\|\overrightarrow{P_iM}\right\|^3} = \frac{q_i}{4\Pi\varepsilon_0} \frac{\vec{u}_i}{r_i^2} \text{ et } \overrightarrow{P_iM} = r_i\vec{u}_i$$

Thus, the electrostatic potential Vi(M) due to the charge qi.

$$V_i(M) = \frac{q_i}{4\Pi\varepsilon_0} \frac{1}{r_i}$$

$$\text{avec: } r_i = \left\|\overrightarrow{P_iM}\right\|$$

The potential V(M) due to the set of n charges is the sum of the potentials according to the superposition principle:

$$V(M) = \sum_{i=1}^{n} V_i(M) = \sum_{i=1}^{n} \frac{q_i}{4\Pi\varepsilon_0} \frac{1}{r_i} \qquad (5)$$

In this relationship, we have chosen the constant to be zero for each potential Vi created by the charge qi; this is only valid if the charges qi are distributed in a finite volume