Chapter 3: Time-Dependent Phenomena (Quasi-Stationary Regimes)

3.1 Faraday's Law

In physics, Lenz's law, or Faraday's law, explains macroscopic phenomena of electromagnetic induction. It states that a temporal variation of the magnetic field generates an electric field; this physical effect is interpreted the electromagnetic phenomenon induction.

What happens if we move a circuit (rigid or not) in a changing magnetic field?
Which expression should be used?
In fact, we need to return to the Lorentz force in the general case of variable fields. We will then have an induced electromotive force (EMF):

$$\boxed{\begin{aligned} e &= \oint_{circuit} \left(\vec{E} + \vec{v} \wedge \vec{B}\right)\cdot \vec{dl} = \oint_{circuit} \vec{E_m} \cdot \vec{dl} - \frac{d\Phi_c}{dt} \\ &= -\frac{d\Phi}{dt} = -\iint_{circuit} \frac{\partial \vec{B}}{\partial t} \cdot \vec{dS} - \frac{d\Phi_c}{dt} \end{aligned}}$$

The first term describes the non-zero circulation of an electromotive field, associated with the time variation of the magnetic field, while the second term describes the presence of a cut flux due to the displacement of the circuit and/or its deformation.

3.2 Lenz's Law

Statement: induction produces effects that oppose the causes that gave rise to it.
This law, like the maximum flux rule, is already contained within the equations and therefore adds nothing more, except an intuition of the physical phenomena. In this case, the
Lenz's law is simply the expression of the sign "–" contained in Faraday's law.

Example: if a circuit is brought near the north pole of a magnet, the flux increases and therefore the electromotive force (EMF).
The induced current is negative. The induced current will then be negative and will itself produce a field
The induced magnetic field is opposite to that of the magnet. Two consequences:

1. The increase in flow through the circuit is reduced.
2. A negative Laplace force appears, opposing the approach of the magnet.

Note on the sign convention

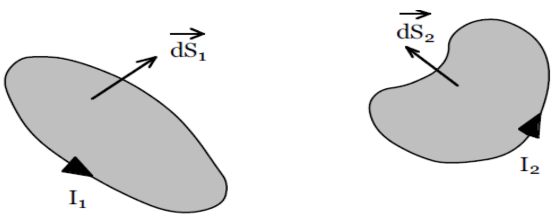
The direction of the induced current is determined as follows:

1. We arbitrarily choose a direction of travel along the circuit.
2. This direction defines, thanks to Ampère's rule, a normal to the circuit.
3. The sign of the flux is then determined by taking the dot product of the magnetic field and this normal.
4. Then, using Faraday's law, we obtain the value and sign of the fem.
5. Finally, the current is obtained from Ohm's law (its sign can also be directly known using Lenz's law).

3.3 Mutual induction and self-induction

3.3.1 Mutual induction between two closed circuits:

Consider two closed, oriented circuits, through which currents I1 and I2 flow.



The first creates a magnetic field B1 whose flux Φ12 through the can be calculated second circuit,

$$\boxed{\Phi_{12} = \iint_{S_2} \vec{B_1} \cdot \vec{dS_2} = \left[\frac{\mu_0}{4\pi} \iint_{S_2} \oint_{C_1} \frac{\vec{dl} \wedge \vec{PM}}{\left\| \vec{PM} \right\|^3} \cdot \vec{dS_2} \right] I_1}$$

where P is any point of the circuit C1 (the element of length equal to dl = dOP) and M a any point on the surface delimited by C2, through which the flux is calculated.
Similarly, we have for the flux created by circuit C2 on circuit C1

$$\boxed{\Phi_{21} = \iint_{S_1} \vec{B_2} \cdot \vec{dS_1} = \left[\frac{\mu_0}{4\pi} \iint_{S_1} \oint_{C_2} \frac{\vec{dl} \wedge \vec{PM}}{\left\| \vec{PM} \right\|^3} \cdot \vec{dS_1} \right] I_2}$$

where P is this time a point on the circuit C2 and M a point on the surface bounded by C1, at through which the flow is calculated. The terms in brackets depend on the distance between the
two circuits and purely geometric factors related to the shape of each circuit.

Since, in general, they are difficult or even impossible to calculate, it is convenient to
to set down :

$$\boxed{\begin{aligned} \Phi_{12} &= M_{12} I_1 \\ \Phi_{21} &= M_{21} I_2 \end{aligned}}$$

The sign of the coefficients depends on the respective orientation of the circuits and follows the same logic as for the induced current. Based on the choices made for the direction of flow along each circuit (see figure), the fluxes are negative for positive currents I1 and I2. Therefore, the coefficients are negative.

The mutual induction coefficient, or mutual inductance, involves a potential energy of magnetic interaction between the two circuits.

$$\boxed{W_m = -M I_1 I_2 \quad + \quad \text{Constante}}$$

3.3.2 Auto induction

If we consider an isolated circuit carrying a current I, we see that we can apply the same reasoning as above. Indeed, the current I generates a magnetic field throughout space, and therefore there is a flux of this field through the circuit itself.

$$\Phi = \iint_S \vec{B} \cdot \vec{dS} = \left| \frac{\mu_0}{4\pi} \iint_S \oint_C \frac{\vec{dl} \wedge \vec{PM}}{\left\| \vec{PM} \right\|^3} \cdot \vec{dS} \right| I$$

Which can simply be written:

$$\boxed{\mathbf{\Phi = LI}}$$

Where L is the self-induction coefficient or self-inductance (or inductance), expressed in Henrys. It does not
depends on the geometric properties of the circuit and is necessarily positive (whereas the
(the sign of the mutual inductance depends on the orientation of one circuit relative to the other).

3.4 Comparison between the steady-state regime and the quasi-steady-state regime

There are three distinct regimes in electromagnetism, each different from the other according to the variation over time.

a) **Steady state**(RS) Time-independent phenomena $\partial\ \partial t=0$; All electrical and magnetic quantities (E, H, q…) are constant.

b) Quasi-stationary regime (QSR) Phenomena vary with time $\partial\ \partial t\neq 0$ Example:cos(2πft) q 0= qIf f 1 kHz⇒Variable regime.

c) Variable regime (VR) Phenomena that vary greatly over time. Only concerns high frequencies > 1 kHz. In RV, the electromagnetic field becomes an electromagnetic wave that propagates through the air.