Chapter 4: Variable Regimes - Maxwell's Equations

4.1 Variable regime

The variable regime is characterized by specific properties related to the time dependence of the fields. These characteristics are:

The phenomenon of induction: A steady-state wire circuit carrying a constant current does not induce any electromotive force (EMF) or current in another steady-state wire circuit. However, this is not the case if the current changes or if the circuits move relative to each other: the resulting EMF or current is due to the phenomenon of induction. This phenomenon leads to the appearance of an additional electric field (called the induced field), which alters the fundamental property of the electric field.

The capacity phenomenon:A circuit containing a capacitor, powered by a time-varying voltage source, carries a variable current even though electrical continuity is interrupted by the gap between the capacitor plates. In this case, the current intensity is no longer constant throughout the circuit since it is zero in the gap between the plates. Therefore, Ampère's law can no longer be applied. To maintain the validity of this law, we will need to introduce the displacement current.

The phenomenon of propagation: Consider a system consisting of circuits carrying currents and charge distributions that vary with time: this system can be at rest or in motion. An electric field and a magnetic field exist in the vicinity of these distributions. Unlike the stationary case, these fields are not synchronous with the sources; that is, at a given time t, these fields depend on the values of the sources at an earlier time, which is a function of the distance separating the observation point from the sources. We express this fact by saying that there is finite propagation speed of the fields from the sources that give rise to them, and the delay is greater the further the point where we wish to know the fields is from the sources.

4.1.1 Lenz's Law

An electromotive force (EMF) can be induced in a thin wire circuit ($\mathcal{C}$) closed by varying the magnetic flux through the circuit: this is the phenomenon of electromagnetic induction. The cases of variations in the magnetic flux through a circuit are:

- The case of a moving circuit in a permanent magnetic field,
- The case of a fixed circuit in a varying magnetic field,
- The general case of a moving circuit in a changing magnetic field.

To qualitatively determine the direction of the induced current, we use Lenz's law, which states that: ***The direction of the induced current is such that the magnetic field it creates opposes the change in flux that gave rise to it.***

4.1.2 Faraday's Law

For a while dt, the variation of the total magnetic flux through any surface based on the circuit($\mathcal{C}$)East$d\phi$ The induced electromotive force (EMF) is expressed using Faraday's law:

$$e = -\frac{d\phi}{dt}$$

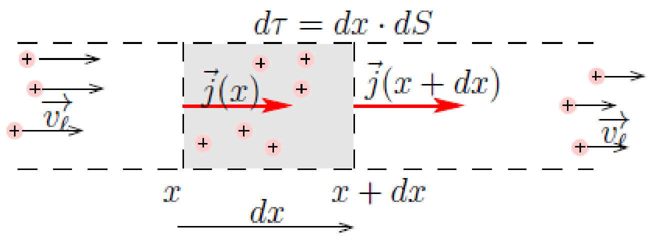
This law, established experimentally for relatively slow variations of magnetic flux as a function of time, is valid for any variable regime and serves as the basis for the study of classical electromagnetism.

4.2 Principle of conservation of charge

The conservation of electric charge is a physical. He expresses that the electrical of an isolated system is a invariant. Electric charge can therefore only be exchanged with another system, but neither created nor annihilated. It is said to be a conservative quantity.

4.3 Local charge conservation equation

Electric charge is a conservative quantity according to the principle of conservation of electricity. Between two nearby instants t and t+dt, the change in charge contained within a closed surface delimiting a system must be attributed exclusively to an exchange with the external environment.
For this, let us consider a cylindrical conductor in electrostatic disequilibrium consisting of identical free charges, of charge q, of concentration n_ and moving at a uniform speed v`.



Let's apply the principle of conservation of charge:

$$\begin{gathered}\text{la variation de } dq \text{ au cours du temps} \\ = dq_{\text{entrant}} \text{ (ce qui est entré dans } d\tau \text{ pendant la durée } dt) - \\ dq_{\text{sortant}} \text{ (ce qui est sorti de } d\tau \text{ pendant la durée } dt)\end{gathered}$$

Hence:

Equation locale de conservation de la charge :

$$dq_{\text{intérieur}} = dq_{\text{entrant}} - dq_{\text{sortant}}$$

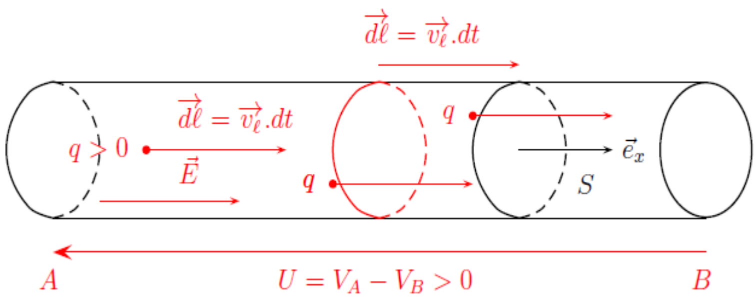
- **A une dimension, suivant l'axe** Ox **:** $-\frac{dj}{dx}.d\tau dt = \frac{\partial\rho}{\partial t}dtd\tau \Leftrightarrow -\frac{dj}{dx} = \frac{\partial\rho}{\partial t} \Leftarrow$
$$\boxed{\frac{\partial\rho}{\partial t} + \frac{dj}{dx} = 0}$$

- **Généralisation à trois dimensions :** $\boxed{\frac{\partial\rho}{\partial t} + div\vec{j} = 0}$

4.4 Ohm's Law

4.4.1 Current density vector

Let us observe what happens in a slice of length L of a cylindrical conductor of cross-section S when a potential difference U = VA - VB is applied to its terminals.
We assume that a uniform field E appears which will cause the movement of load carriers q > 0 in the following figure:



During the elementary time interval dt, the quantity of electricity dq that passes through a surface S in the positive direction, denoted by ex is the chargecontained in the cylinder with generator dl and base S.

The current intensity I is equal to the flow rate of charges passing through S:

$$I = \frac{dq}{dt}$$

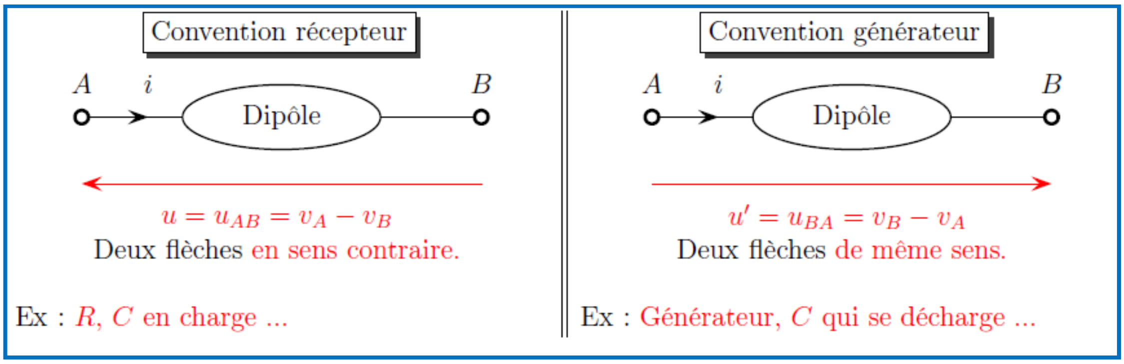
Ohm's law allows us to determine the resistance of a cylindrical conductor of length l, cross-section S, and conductivity ε.

$$I = \iint \vec{j}\,\overrightarrow{d^2S} = \gamma E_0.S \text{ et } U = \int \overrightarrow{grad}V\,\overrightarrow{d\ell} = \ell.E_0. \text{ Donc : } U = R.I \text{ avec } R = \frac{\ell}{\gamma.S}.$$

4.4.2 Receiver-generator convention:

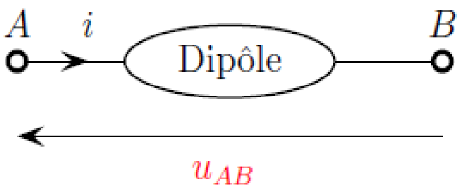
The direction of the voltage is independent of that of the current.

Once a direction has been arbitrarily fixed for the current intensity i flowing through a dipole, the voltage U across its terminals can be defined in two ways:



4.4.3 Electrical Power

By convention receiver, algebraic power received by a dipole AB is:



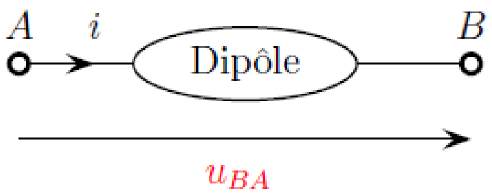
Prec = ui and u =

Remarks :

- The legal unit is the Watt W.
- If prec > 0 then the dipole receives effectively of energy, therefore of power.
- If prec < 0 then in fact, the dipole actually provides power and it functions as a generator.

By convention generator, algebraic power received by a dipole AB is:

Prec = u'i and u' =



A dipole can be a generator at one instant and a receiver at another instant (capacitor, car battery for example).

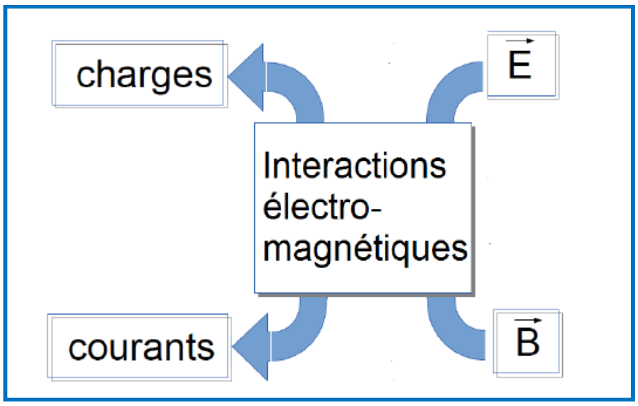
4.5 Maxwell's Equations

4.5.1 Introduction

Maxwell's equations, also called Maxwell-Lorentz equations, are fundamental laws of the physical They constitute, along with the expression of theelectromagnetic forceofLorentz, THE postulates basic of the electromagnetism.

The equations translate into local form different theorems(Gauss,Ampere,Faraday) which governed electromagnetism before Maxwellnot to bring them together in the form of equations They thus provide a frame work mathematical precise to the fundamental concept Offield introduced in physics by Faraday.

These equations show in particular that in steady state, the electric and magnetic fields are independent of each other, whereas they are not in variable state.



4.5.2 Fundamental constants of electromagnetism

- **Permittivity of a vacuum:**

The permittivity of free space, dielectric permittivity of free space, or (di)electric constant is aphysical constant.It is denoted ε0 (pronounced "epsilon zero").

ε0= 854,187,812 8(13) × 10−12HAS²s⁴kg⁻¹m⁻³.

- **Permeability of a vacuum:**

The permeability of free space, also called magnetic permeability of free space or magnetic constant, is a physical symbolized by µ0.

The magnetic constant is often expressed inhenryper meter:

µ0 = 4π × 10−7H m−1.

- **Speed of light in a vacuum:**

The speed of light in the empty, noted c for "speed" is a constant universal and relativistic. This concept is used to describe in optical the re speed of a wave in the empty and a mechanical wave in matter; it is also called pseudo-velocity.

c=3 × 108m/s or 300,000 km/s

4.5.3 Maxwell's equations in a vacuum

- **Maxwell-Gauss equation**

This local equation gives the divergence of the electric field as a function of the electric charge density:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad \text{aussi notée} \quad \operatorname{div} \vec{E} = \frac{\rho}{\varepsilon_0}.$$

- **Maxwell-Thomson equation**

This equation is also called the Maxwell-flux equation; it expresses that the flux of the magnetic field through a closed surface {\display style (\Sigma)} is always zero:

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \text{aussi notée} \quad \operatorname{div} \vec{B} = 0.$$

- **Maxwell-Faraday equation**

This local equation describes the fundamental phenomenon electromagnetic discovered by Faraday.; she gives the rotational of the electric field as a function of the time derivative of the magnetic field. It indicates that the change in the magnetic field creates an electric field.

$$\vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{aussi notée} \quad \overrightarrow{\text{rot}}\, \vec{E} = -\frac{\partial \vec{B}}{\partial t}.$$

- **Maxwell-Ampère equation**

This equation is inherited from Ampere's. In local form, it can be written in terms of the current density vector.

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

In summary, in a vacuum, Maxwell's equations can be written as:

	Forme locale	Forme intégrale
Théorème de Gauss pour $\vec{E}$ ou équation de *Maxwell-Gauss*	$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_o}$	$\oiint_{(S)} \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \iiint_{(\tau)} \rho \, d\tau$
Equation du flux magnétique ou équation de *Maxwell-Thomson*	$\vec{\nabla} \cdot \vec{B} = 0$	$\oiint_{(S)} \vec{B} \cdot d\vec{S} = 0$
Equation de *Maxwell-Faraday*	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint_{(\Gamma)} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint_{(S)} \vec{B} \cdot d\vec{S}$
Equation de *Maxwell-Ampère*	$\vec{\nabla} \times \vec{B} = \mu_0 \left[\vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right]$	$\oint_{(\Gamma)} \vec{B} \cdot d\vec{\ell} = \iint_{(S)} \mu_0 \left[\vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right] \cdot d\vec{S}$