Chapter 5: Propagation of the electromagnetic field

5.1 Wave Equation

The wave equation (sometimes called the wave equation or d'Alembert's equation) is the general equation that describes the spread of a wave, which can be represented by a scalar or vector quantity.

A progressive wave moves through space over time; therefore, it is governed by an equation that links variations in time to variations in space.

In the case of 1D we have:

$$\left(\frac{\partial f}{\partial z}-\frac{\partial f}{\mathrm{v}\partial t}\right)\left(\frac{\partial f}{\partial z}+\frac{\partial f}{\mathrm{v}\partial t}\right)=0$$

$$\left(\frac{\partial^2 f}{\partial z^2}-\frac{1}{\mathrm{v}^2}\frac{\partial^2 f}{\partial t^2}\right)=0$$

In the case of 3D we have:

$$\left(\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}-\frac{1}{\mathrm{v}^2}\frac{\partial^2 f}{\partial t^2}\right)=0$$

This equation uses only linear operators, which allows us to apply the superposition principle:

For a space-time dependency function f(z-vt) we have:

$$\frac{\partial f(z-\mathrm{v}t)}{\partial z}=f'(z-\mathrm{v}t) \qquad \frac{\partial f(z+\mathrm{v}t)}{\partial z}=f'(z+\mathrm{v}t)$$

$$\frac{\partial f(z-\mathrm{v}t)}{\partial t}=-\mathrm{v}f'(z-\mathrm{v}t) \quad \text{et} \quad \frac{\partial f(z+\mathrm{v}t)}{\partial t}=\mathrm{v}f'(z+\mathrm{v}t)$$

$$soit: \left(\frac{\partial f(z-\mathrm{v}t)}{\partial z}+\frac{1}{\mathrm{v}}\frac{\partial f(z-\mathrm{v}t)}{\partial t}\right)=0 \qquad soit: \left(\frac{\partial f(z+\mathrm{v}t)}{\partial z}-\frac{1}{\mathrm{v}}\frac{\partial f(z+\mathrm{v}t)}{\partial t}\right)=0$$

To obtain an equation that governs the behavior of f(z-vt) and f(z+vt) simultaneously, we must calculate the second derivative; we have:

$$\frac{\partial^2 f(z-\mathrm{v}t)}{\partial z^2} = f''(z-\mathrm{v}t) \qquad \text{et} \qquad \frac{\partial^2 f(z+\mathrm{v}t)}{\partial z^2} = f''(z-\mathrm{v}t)$$

$$\frac{\partial^2 f(z-\mathrm{v}t)}{\partial t^2} = \mathrm{v}^2 f'(z-\mathrm{v}t) \qquad \qquad \frac{\partial^2 f(z+\mathrm{v}t)}{\partial t^2} = \mathrm{v}^2 f'(z-\mathrm{v}t)$$

So, for both cases:
$$\left(\frac{\partial^2 f}{\partial z^2} - \frac{1}{\mathrm{v}^2}\frac{\partial^2 f}{\partial t^2}\right) = 0$$

In 3D this looks like:
$$\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} - \frac{1}{\mathrm{v}^2}\frac{\partial^2 f}{\partial t^2}\right) = 0$$

- The wave equation is a local equation, that is to say that we must work on the local scale (small element of space).

- Determine the physical quantity that will vary as the wave passes (electromagnetic wave: electric and magnetic field, sound wave: pressure).

- Write dynamic equations (variation in space and time) at the local scale.

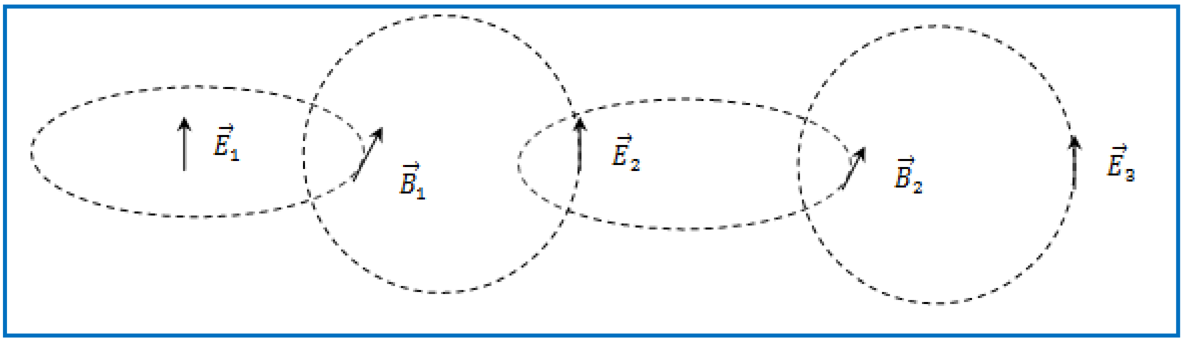
-Work with these equations to find a 3D type wave equation.

5.2 Equation of propagation of the electromagnetic field in a vacuum

The phenomenon of electromagnetic wave propagation in a vacuum can be explained by considering the last two equations of Maxwell stablished in the absence of current sources

$$\begin{cases} \overrightarrow{rot}\vec{E} \sim \partial\vec{B}/\partial t \\ \overrightarrow{rot}\vec{B} \sim \partial\vec{E}/\partial t \end{cases}$$

This means that a time-varying electric field E (or B) generates a rotating magnetic field B (or E) of the same nature. Indeed, if we initially choose a variable electric field E1, it will generate a rotating magnetic induction field B1 (see figure below). But since this field B1 is established gradually, it is time-varying. It will therefore, in turn, produce an electric field E2 which, as it establishes itself over time, will produce a magnetic induction field B2, and so on. In this way, the pair of fields (E, B) is self-sustaining over time with a progression in space.



The equations describing the propagation of electromagnetic waves in a vacuum are:

$$\begin{cases} \Delta\vec{E} - \mu_0\varepsilon_0 \dfrac{\partial^2\vec{E}}{\partial t^2} \equiv 0 \\ \Delta\vec{B} - \mu_0\varepsilon_0 \dfrac{\partial^2\vec{B}}{\partial t^2} \equiv 0 \end{cases}$$

We note that these equations are of the type of d'Alembert's equation which is found for any propagation phenomenon in physics.

5.3 Characteristics of plane waves

5.3.1 Definition

The plane wave is a concept from the physics of propagation. It’s a wave including fronts are plans infinite, all perpendicular to the same direction of propagation designated by its vector unitary.

We need to solve the wave equation: $\Delta\psi - \dfrac{1}{v^2}\dfrac{\partial^2\psi}{\partial t^2} = 0$

In the plane perpendicular to the direction of wave propagation, we then have a constant amplitude and phase. These are called plane harmonic waves. For a wave propagating along increasing r:

$$\psi = A\cos(\vec{k}\vec{r} - wt) \qquad \vec{k} \text{ est le vecteur d'onde } |\vec{k}| = \frac{2\pi}{\lambda} n(\lambda)$$

$$\omega = 2\pi\nu \text{ où } \nu = \textit{fréquence optique}$$

$$\vec{r} = \text{vecteur position d'un point de l'onde}$$

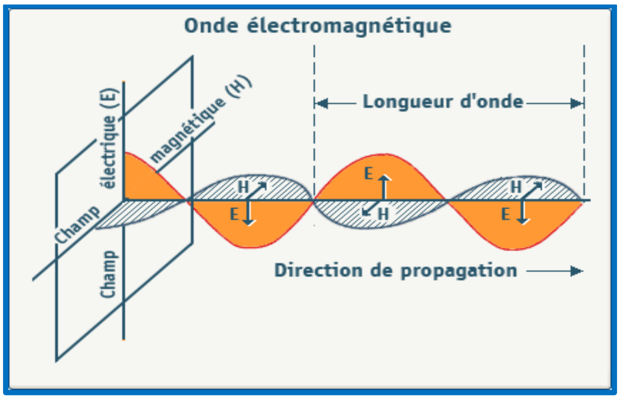
$$\nu = \frac{C}{\lambda} \text{ avec } C = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \text{ où } C = \text{vitesse de la lumière dans le vide.}$$

5.3.2 Characteristics of plane waves

- One direction of propagation
- Constant in the plane of the equation:ΨPerturbation: kxx + ky y+kz z= constant
- This plane is called the wavefront and is perpendicular to the direction of energy propagation.
- The wave has infinite extension = energy throughout the plane transverse to the direction of propagation

5.4 Speed and Wavelength

An electromagnetic wave can therefore be conceived as an electrical disturbance of matter that propagates. It is characterized by its frequency f and its wavelength λ.



Frequency (Hz)

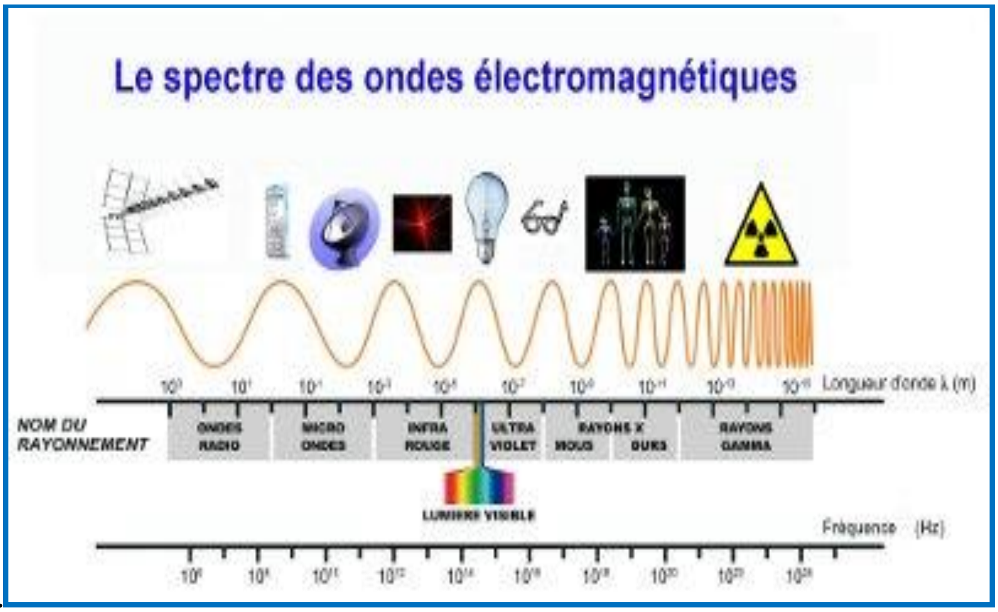
$$f = \frac{1}{T}$$

Wavelength (m)

$$\lambda = \frac{c}{f} = cT$$

5.5 Electromagnetic Radiation Spectrum

The electromagnetic spectrum encompasses all sources of electromagnetic radiation, classified by their frequency, wavelength, and energy. Theoretically, it extends continuously from zero to infinity in frequency (or wavelength) and includes both ionizing and non-ionizing radiation.



Non-ionizing radiation is further subdivided into static fields (0 Hz), extremely low frequency (ELF) fields, intermediate frequency (IF) fields, radio frequency (RF) fields and microwaves.