Mathematical Logic
Series of tutorials N°01 : Calculability

Exercise 1 Let the following instructions of a Turing machine be :

q_0 s_0 D q_1
q_0 s_1 s_2 q_2
q_1 s_0 D q_1
q_1 s_2 D q_1
q_1 s_1 D q_2
q_1#
q_2 s_2 G q_3
q_3 (s_0/s_1/s_2) G q_3
q_3#

Execute this Turing machine on the following sequence : #$s_0s_0s_2s_1s_2s_1s_2s_0s_2$# considering that # represents the blank symbol.

Exercise 2 Write the Turing machine that, given a word on the tape composed of symbols a and b, determines if the word ends with a "b" or not. It writes a "T" at the end of the word if true and an "F" otherwise. Blank symbol = $.

Exercise 3 Modify the previous Turing machine to check if the input word ends with the same starting symbol (whether it is "a" or "b").

Exercise 4 The decimal values corresponding to the ASCII codes of the letters are :

- A : 65 ; B : 66 ; C : 67 ; etc,
- a : 97 ; b : 98 ; c : 99 ; etc.

To convert the ASCII code of a lowercase letter to the corresponding uppercase letter, you just need to change the 3rd bit from the left, from 1 to 0. Furthermore, the first two bits are always equal to "01" and the last 5 bits remain unchanged.

A : 65 = 01000001 and a : 97 = 01100001
C : 67 = 01000011 and c : 99 = 01100011
Write the Turing machine that transforms a lowercase letter into an uppercase one.

Exercise 5 Write the Turing machine that recognizes the sequence 0001 in a given word, considering that the tape contains multiple words and the alphabet $\Sigma = \{0, 1, \#\}$. There are multiple words on the tape separated by a single #. Two consecutive # symbols indicate the end of the sequence.

Exercise 6 Write the Turing machine that checks if a given word on the tape contains the following character sequence : « aab ». The blank symbol = # and there are multiple words on the tape separated by a single #. Two consecutive # indicate the end of the sequence. A = $\{a, b, \#\}$. We write a "T" or a "F".

Exercise 7 Write the Turing machine that replaces the "0" that comes after two "1" with a "1". A $= \{0, 1, \#\}$, q_0 is the initial stat and the tape contains one word.

Exercise 8 Write the Turing machine that transforms the word on the tape written in alphabet $\{a, b, \#\}$ so that all "a"s are at the beginning. Example : aabbaba becomes aaaabbb.

Exercise 9 Write the Turing Machine that recognizes palindromes (A palindrome is a word that reads the same backwards as forwards example : 0010100), knowing that the alphabet is $\Sigma = \{0, 1, \#\}$ and the tape contains one word.

Exercise 10 Write the Turing Machine which adds "1" to a binary number n given as input knowing that the alphabet is $\Sigma = \{0, 1, \#\}$, the tape contains multiple words separated by two consecutive # and three consecutive # indicate the end of the tape.

Exercise 11 Show that the following functions are primitive recursive :

1. The Plus function, plus = x + y,
2. The Sigma function, Sigma = $\Sigma_{i=0}^{x} i$,
3. The predecessor function (pred(x)),
4. The subtraction function (sub(x,y)) such that sub(x,y) = $\{^{x-y \text{ si } x>y}_{0 \text{ si } x\leq y}$,
5. The absolute difference function $|x - y| = \{^{x-y \text{ si } x\geq y}_{y-x \text{ si } x<y}$,
6. The Alpha function such that $\alpha(x) = \{^{1 \text{ si } x=0}_{0 \text{ si } x\neq 0}$,
7. The multiplication function, mult = x * y,
8. The factorial function, Fact(x) = $x!$.