Mathematical Logic
Series of tutorials N°04 : Predicate logic

Exercise 1 Model the following sentences in predicate logic, specify the vocabulary.
- All students love logic.
- Not all students like a module.
- Students who get a good grade in logic are the best.

Exercise 2 Consider the following sentences :
a- All men are mortal.
b- Socrate is a mortal.
c- Socrate is a man.

- Identify the predicates.
- Express in first order logic a, b and c.
- can we deduce the statement b from a and c ? justify.

Exercise 3 Consider the following statements :

1. People who have influenza A should take Tamiflu.
2. People who have a fever and cough have influenza A.
3. Those who have a temperature above 38 have a fever.
4. Mohamed coughs and has a temperature above 38.
5. Mohamed has to take Tamiflu.

Model the above statements in first-order logic using the following predicates :
- influenza (x) : x has influenza A.
- take (x,y) : x must take y.
- fever (x) : x has a fever.
- cough (x) : x coughs.
- temp (x, t) : x has the temperature t.
- sup (x, y) : x is greater than y.

use also the following constants : 38, Mohamed, Tamiflu.

Exercise 4 Which of the formulas below are congruent ? Justify the answer (by applying the syntactic tree method).
- $F_1 : \forall x \exists y (\forall y P(x, y) \to \exists x Q(x, y))$.
- $F_2 : \forall v \exists z (\forall u P(z, u) \to \exists u Q(u, v))$.
- $F_3 : \forall z \exists x (\forall x P(z, x) \to \exists z Q(z, x))$.

Exercise 5 Say if the following formulas are true or false, knowing that the domain D $= \{c1, c2, c3\}$, the subsets of the domain are :
- R $= \emptyset$,
- P $= \{c1, c3\}$,
- Q $= \{c1, c2, c3\}$.

and the interpretations of the constants are :
I(a) = c1.
The formulas are :

1. $\forall x \neg Q(x)$, **2.** $\forall x P(x)$, **3.** $\forall x(P(x) \rightarrow Q(x))$, **4.** $\forall x(P(x) \wedge Q(x))$, **5.** $\exists x(Q(x) \wedge \neg P(x))$, **6.** $\exists x(\neg Q(x) \rightarrow P(x))$, **7.** $\forall x Q(x) \rightarrow \neg(\exists x R(x))$, **8.** $P(a) \rightarrow R(a)$. Justify your answer.

Exercise 6 Establish the truth table of the following formulas : (knowing that the interpretation domain is $\mathrm{D} = \{1, 2\}$)
a- $\forall x(P(x) \rightarrow \exists x Q(x))$.
b- $\forall x(P(x, y) \wedge \exists x P(x))$.

Exercise 7
a- Establish the following deductions :
- $\forall x \forall y A(x, y) \vdash A(x, y)$.
- $\forall x(P(x) \rightarrow Q(x)), \forall x P(x) \vdash \forall x Q(x)$.
- $P(a), \forall x(P(x) \rightarrow Q(x)) \vdash Q(a)$.
- $\forall x S(x) \wedge \forall x R(x) \vdash \exists x(S(x) \wedge R(x))$.

b- Demonstrate that : $\vdash \forall x P(x) \rightarrow \exists x P(x)$.